The Sun & Nuclear Fusion

Structure of Sun's outer layers Magnetism, Sunspots, and Flares Nuclear fusion reactors in stars Nucleosynthesis and the cosmic abundances of the elements Solar neutrinos

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DAG

The Sun & Nuclear Fusion

- The structure of the Sun's outer layers: convection zone, photosphere, chromosphere, and corona
- Solar activity: magnetism, sunspots, and flares
- Solar energy
- Review of some fundamental physics
- Nuclear fusion reactors in stars
- Temperature dependence of the fusion rate in stars
- Nucleosynthesis and the cosmic abundances of the elements

Reading: Kutner Ch. 9.2-9.3, Ryden Sec. 15.3



Sudbury Neutrino Observatory (SNO), from Brookhaven Neutrino and Nuclear Chemistry Group.

Opacity and luminosity in stars

How many steps (in number of mean free paths) does it take for a photon to get from the center of the sun to the surface?

Let us work in 1D. Suppose a photon starts off at the center of the star and has an equal chance to go right or left after each absorption and re-emission step. The average position after *N* steps is

$$\langle x_N \rangle = (x_1 + x_2 + \dots + x_N)/N = 0$$

However, the average value of the *square* of position is nonzero. Consider step N + 1, assuming an equal chance of going left or right:

$$\begin{split} \langle x_{N+1}^2 \rangle &= \frac{1}{2} \langle (x_N - \ell)^2 \rangle + \frac{1}{2} \langle (x_N + \ell)^2 \rangle \\ &= \frac{1}{2} \langle x_N^2 - 2x_N \ell + \ell^2 \rangle + \frac{1}{2} \langle x_N^2 + 2x_N \ell + \ell^2 \rangle \\ &= \langle x_N^2 \rangle + \ell^2 \end{split}$$

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Opacity and luminosity in stars

If this expression for $\langle x_{N+1}^2 \rangle$ is true for all *N*, then we can find $\langle x_N^2 \rangle$ by starting at zero and adding $\ell^2 N$ times (using induction):

$$\langle x_N^2 \rangle = N \ell^2$$

Thus, to randomly walk a distance $\sqrt{\langle x_N^2 \rangle} = L$ the photon needs to take, on average,

$$N = L^2/\ell^2$$
 steps

In 3D, the photon needs to take 3 times as many steps, so to travel a distance R it needs

$$N = \frac{3R^2}{\ell^2} \text{ steps}$$

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Opacity and luminosity in stars

For the Sun, assuming a constant mean free path $\ell = 0.5$ cm and using $R = 6.96 \times 10^{10}$ cm,

$$N = \frac{3(6.96 \times 10^{10})^2}{(0.5)^2} = 5.81 \times 10^{22} \text{ steps}$$

This is very opaque!

Each step should take a time $\Delta t = \ell/c$, so the average time required for a photon to diffuse from the center of the Sun to the surface is

$$t = N\Delta t = \frac{3R_{\odot}^2}{\ell c} = 9.7 \times 10^{11} \text{ s} \approx 31,000 \text{ yr}$$

Note that the same trip takes only $t = R_{\odot}/c = 2.3$ s for a photon traveling in a straight line!

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- The photosphere is the observable surface of the Sun at visible wavelengths.
- It is extremely thin (~400 km), resulting in the Sun appearing to have a very well-defined edge.



As we have noted, the spectrum of the Sun closely resembles a blackbody.

 From the total energy flux at Earth — total solar irradiance, (TSI), or "solar constant" —

$$f_{\odot} = 1.361 imes 10^6 \ {
m erg \ s^{-1} \ cm^{-2}}$$

we get the Sun's luminosity

 $L_{\odot} = 3.828 \times 10^{33} \text{ erg/s}$

- TSI, L, and solar flux at most wavelengths vary little with time.
- At very long and very short wavelengths, flares can change the Sun's brightness by huge factors.



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Image from Mediawiki.

Absorption lines are also seen in the solar spectrum. They match up with many known transitions of atoms, ions, and molecules.

The solar spectrum. Nigel Sharp, using data from Bob Kurucz et al. (NOAO/NSO/Kitt Peak FTS/AURA/NSF).



Spectral-line absorption by atoms and molecules is a hallmark of stars.

- Gases absorb strongly at the wavelengths of spectral lines (transitions between the quantum mechanical states) of the atoms and molecules of which they are composed.
- Stars are heated from inside and are cooler on the outside.
- The absorption lines in the photosphere are then a result of the cooler temperature of the photosphere (relative to the interior).
- This temperature gradient also results in limb darkening, where the edges of the star are fainter than the center.



Astronomy Today

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The Solar convection zone

Convection becomes the dominant energy transport mechanism when the material is too opaque for photons to escape.

 Being fully ionized, the Sun's interior has an adiabatic index

$$\gamma \equiv rac{C_P}{C_V} = rac{5}{3}$$

Gas is unstable to convection if

$$\frac{T}{P}\frac{dP}{dT} < \frac{\gamma - 1}{\gamma} = \frac{2}{5}$$

► In the Sun, this is true for ²/₃R_☉ < r < R_☉, so it has a large outer convection layer.



The Solar convection zone



Sunspot and solar granularity observed by Dutch Open Telescope (Rutten et al. 2017, APOD 2005). Each grain is the top of a convection cell. A 10-minute time lapse of the photosphere (NSO/NSF/AURA); see also here.

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The solar dynamo & the solar cycle

Below the Sun's convection zone lies the radiative zone, which rotates like a solid body; the convection zone rotates *differentially*.



The differential rotation winds and amplifies a poloidal solar magnetic field, turning it into a more toroidal field (Higgins 2012). This occurs because the field is **frozen** into the ionized material by the Lorentz force.

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The solar dynamo & the solar cycle

Convection makes the field lines twist out of the surface and loop through the lower atmosphere, creating **sunspot** pairs and **prominences** connecting them.



The twisting and winding of the field lines eventually causes the poloidal field to reappear but with the N and S poles reversed.

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The 11-year sunspot cycle

- The reversal of the magnetic field repeats in a regular cycle. There are 22 years between identical configurations of the field.
- The self-generation process of the field is called **dynamo** action.
- For the Sun, there are 11 years between sunspot number maxima.
- The first sunspots in a cycle form near ~30° latitude, and the last near the equator, producing the "butterfly diagram."



Matthew Owens

Sunspots & solar activity

- Sunspots appear dark because they are slightly cooler than the rest of the solar surface.
- They are surrounded by hotter-than-average regions called faculae.
- Zeeman effect measurements show that they are maxima of the magnetic field.



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March 30, 2001 at 19:19

Sunspots & solar activity

- Solar images from March 29, 2001:
 - Visible light
 - He II at 30.4 nm
 - Fe IX at 17.1 nm
- Note the X-ray bright faculae surrounding the sunspots. These regions add more to the Sun's luminosity than the spots take away.



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March 30, 2001 at 19:00

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Chromosphere & corona

- The chromosphere is the layer of the Sun's atmosphere immediately above the photosphere.
- The chromosphere produces an emission spectrum (primarily Hα and He), as it is a warm diffuse gas.
- The corona is the outer layer of the Sun's atmosphere.
- The coronal density is so low that it cannot cool efficiently, so it reaches temperatures > 10⁶ K.

2023 total solar eclipse, by Reinhold Wittich



Energy & the Sun

A thought experiment:

- Hydrostatic equilibrium and ideal gas behavior ensure that the center of the Sun is very hot, and energy (in the form of light) is radiated from the center.
- The high photon opacity of the Sun determines the rate at which energy leaks out. As we have seen, it takes a long time for photons to diffuse from the center to the surface.
- This cannot go on forever without the Sun cooling down or replacing the energy that leaks away.
- We know that the solar system is about 4.6 × 10⁹ years old from many radioisotope measurements of meteorites, and that life has existed here for at least 3.5 × 10⁹ years.
- Therefore the Sun must have had close to its present luminosity for billions of years for liquid water to be present on Earth to facilitate life.

How long would the Sun's present heat last?

The energy density at the center of the Sun is given by the energy density of the electron gas there, considered to be an ideal gas:

$$u_e = \frac{3}{2}n_ekT$$

The energy density of the radiation (light) about to leak away is

$$u_r = \frac{4}{c}f = \frac{4\sigma T^4}{c}$$

We have shown that it takes $t \approx 31,000$ years for a photon to leak from the center to the surface, so the heat lasts

$$rac{u_e}{du_e/dt} pprox rac{u_e}{u_r/t} = rac{3}{2}n_e kT rac{c}{4\sigma T^4} t pprox rac{3}{8}rac{kc}{\sigma m_p}rac{
ho}{T^3} t pprox 3 imes 10^7 \ {
m yr}$$

This is much less than the Sun's age, so some process must be replacing the energy that leaks away.

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Energy source: Mass energy mc^2

Suppose a small fraction (0.1%) of the mass of the Sun could be converted to radiation. Then

$$\Delta E = 0.001 M_{\odot} c^2 = 2 \times 10^{51} \text{ erg}$$

 $\Delta t = \frac{\Delta E}{L_{\odot}} = 4.6 \times 10^{17} \text{ s} = \boxed{1.5 \times 10^{10} \text{ yr}}$

How could the Sun convert its mass-energy to radiation?

Nuclear fusion, the liberation of mass energy, stored as potential energy of the strong nuclear interaction (Bethe 1939, Nobel Prize 1967).

Could this work under the conditions in the center of the Sun?

$$ho_c = 150 ext{ g/cm}^3$$
 $P_c = 2.1 \times 10^{17} ext{ dyne/cm}^2$
 $T = 1.57 \times 10^7 ext{ K}$
Mostly hydrogen

Yes. In fact, fusion requires such conditions.

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