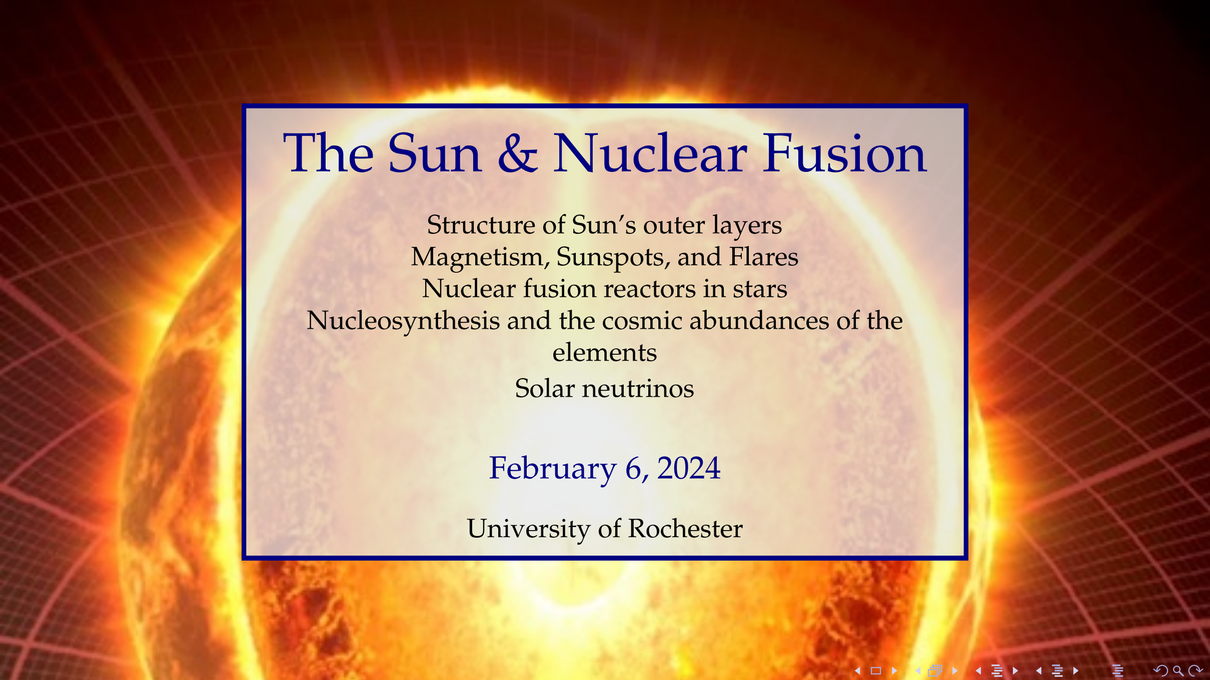


The Sun & Nuclear Fusion

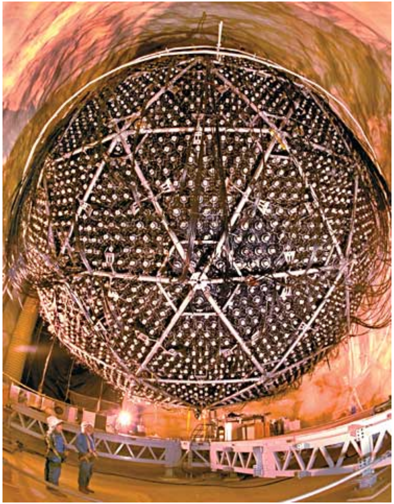
Structure of Sun’s outer layers
Magnetism, Sunspots, and Flares
Nuclear fusion reactors in stars
Nucleosynthesis and the cosmic abundances of the elements
Solar neutrinos

February 6, 2024

University of Rochester

The Sun & Nuclear Fusion

- The structure of the Sun's outer layers: convection zone, photosphere, chromosphere, and corona
- Solar activity: magnetism, sunspots, and flares
- Solar energy
- Review of some fundamental physics
- Nuclear fusion reactors in stars
- Temperature dependence of the fusion rate in stars
- Nucleosynthesis and the cosmic abundances of the elements

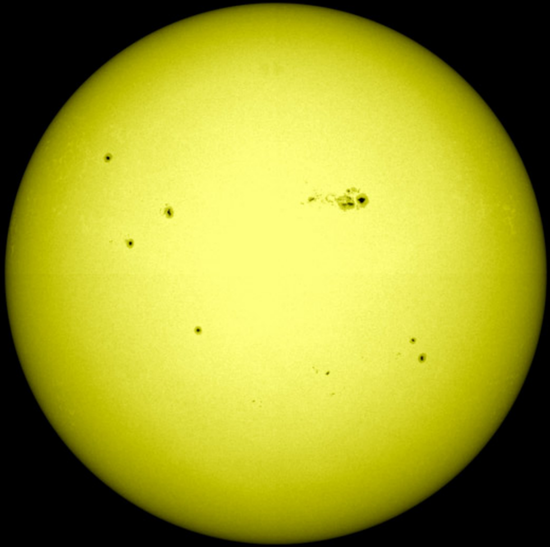


Sudbury Neutrino Observatory (SNO), from Brookhaven Neutrino and Nuclear Chemistry Group.

Reading: Kutner Ch. 9.2–9.3, Ryden Sec. 15.3

The solar photosphere

- The photosphere is the observable surface of the Sun at visible wavelengths.
- It is extremely thin ($\sim$400 km), resulting in the Sun appearing to have a very well-defined edge.



The solar photosphere

As we have noted, the spectrum of the Sun closely resembles a blackbody.

- From the total energy flux at Earth — total solar irradiance, (TSI), or "solar constant" —

  $$f_\odot = 1.361 \times 10^6 \text{ erg s}^{-1}\text{ cm}^{-2}$$

  we get the Sun's luminosity

  $$L_\odot = 3.828 \times 10^{33} \text{ erg/s}$$

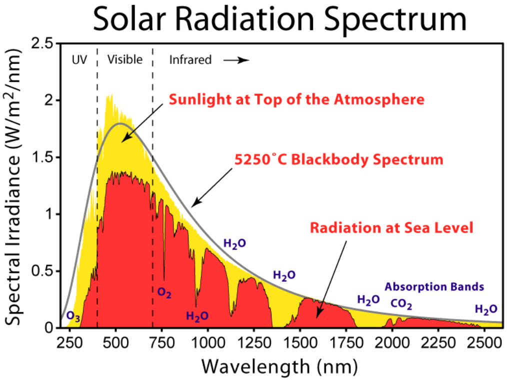
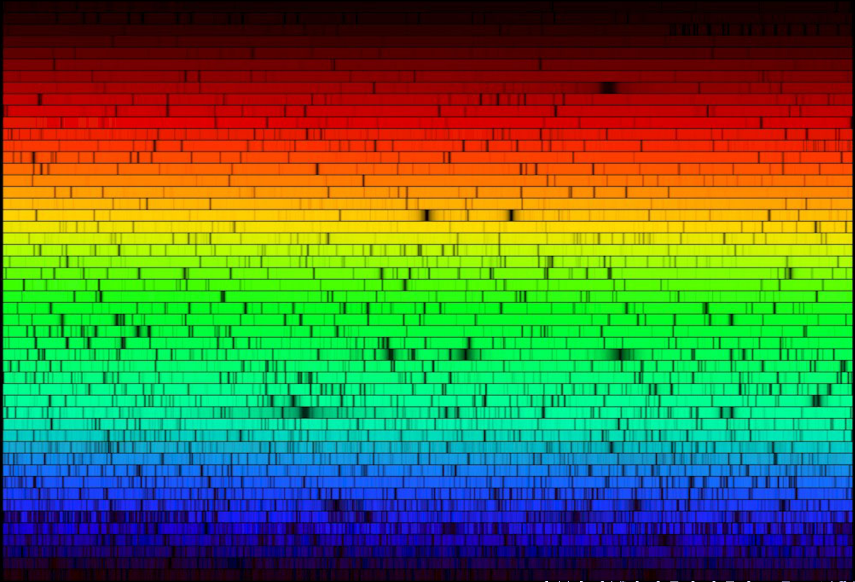


Image from Mediawiki.

- TSI, *L*, and solar flux at most wavelengths vary little with time.
- At very long and very short wavelengths, **flares** can change the Sun's brightness by huge factors.

The solar photosphere



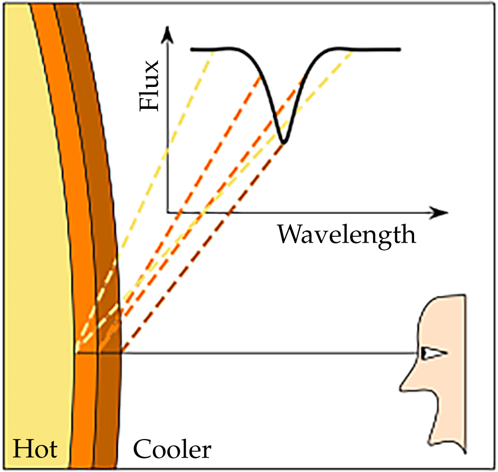
Absorption lines are also seen in the solar spectrum. They match up with many known transitions of atoms, ions, and molecules.

The solar spectrum. Nigel Sharp, using data from Bob Kurucz et al. (NOAO/NSO/Kitt Peak FTS/AURA/NSF).

The solar photosphere

Spectral-line absorption by atoms and molecules is a hallmark of stars.

- Gases absorb strongly at the wavelengths of **spectral lines** (transitions between the quantum mechanical states) of the atoms and molecules of which they are composed.
- Stars are heated from inside and are cooler on the outside.
- The absorption lines in the photosphere are then a result of the cooler temperature of the photosphere (relative to the interior).
- This temperature gradient also results in limb darkening, where the edges of the star are fainter than the center.



The Solar convection zone

Convection becomes the dominant energy transport mechanism when the material is too opaque for photons to escape.

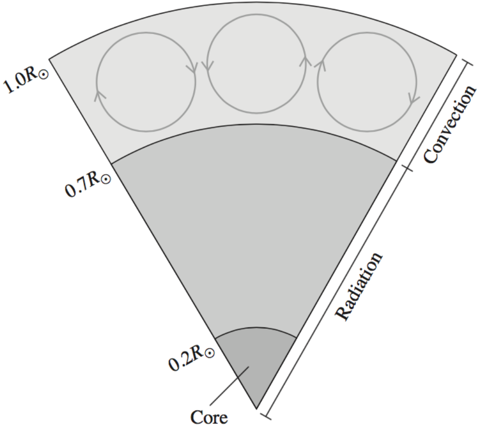
- Being fully ionized, the Sun's interior has an adiabatic index

$$\gamma = \frac{C_P}{C_V} = \frac{5}{3}$$

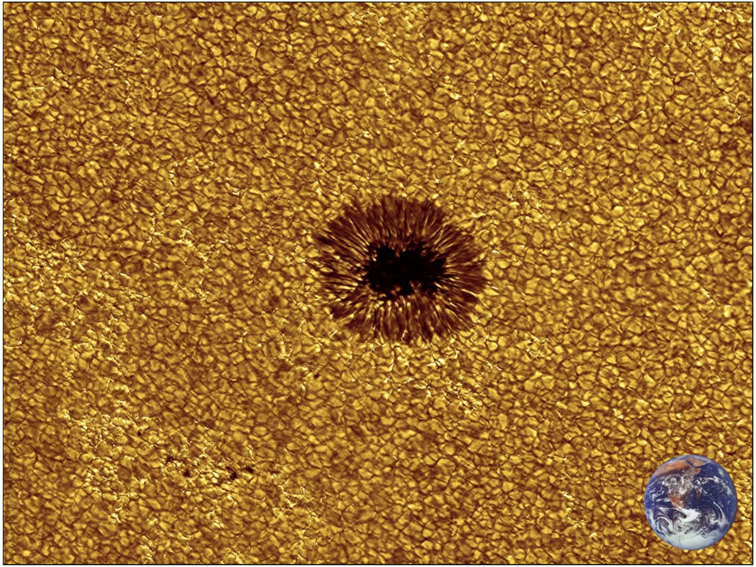
- Gas is unstable to convection if

$$\frac{T}{P}\frac{dP}{dT} < \frac{\gamma - 1}{\gamma} = \frac{2}{5}$$

- In the Sun, this is true for $\frac{2}{3}R_\odot < r < R_\odot$, so it has a large outer convection layer.



The Solar convection zone

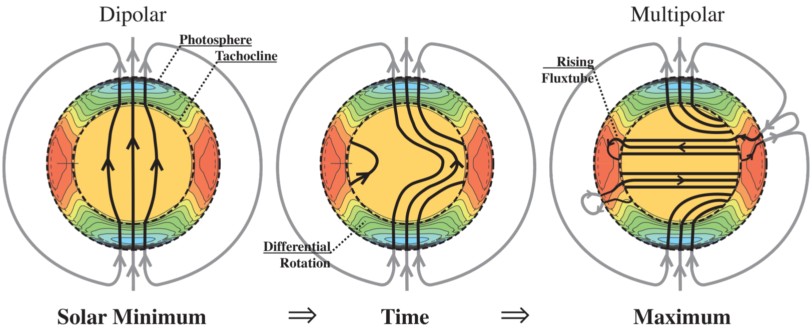


Sunspot and solar granularity observed by Dutch Open Telescope (Rutten et al. 2017, APOD 2005). Each grain is the top of a convection cell.

A 10-minute time lapse of the photosphere (NSO/NSF/AURA); see also here.

The solar dynamo & the solar cycle

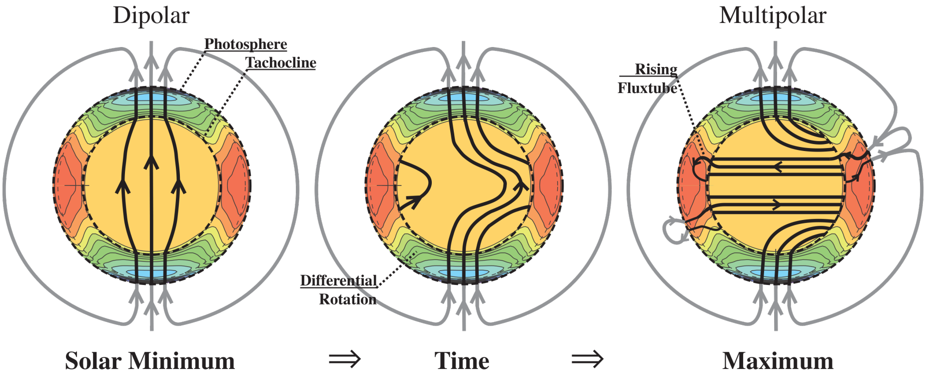
Below the Sun's convection zone lies the radiative zone, which rotates like a solid body; the convection zone rotates *differentially*.



The differential rotation winds and amplifies a poloidal solar magnetic field, turning it into a more toroidal field (Higgins 2012). This occurs because the field is **frozen** into the ionized material by the Lorentz force.

The solar dynamo & the solar cycle

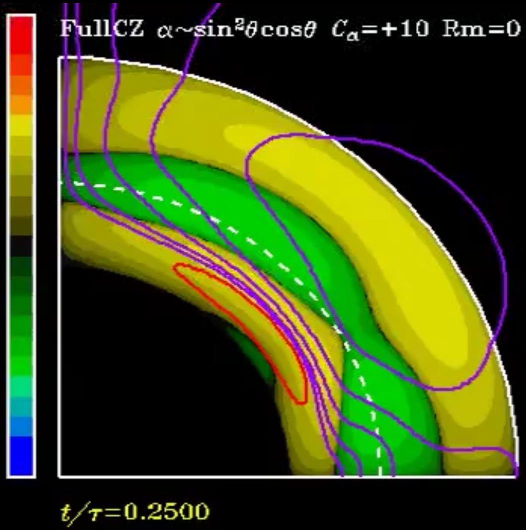
Convection makes the field lines twist out of the surface and loop through the lower atmosphere, creating **sunspot** pairs and **prominences** connecting them.



The twisting and winding of the field lines eventually causes the poloidal field to reappear but with the N and S poles reversed.

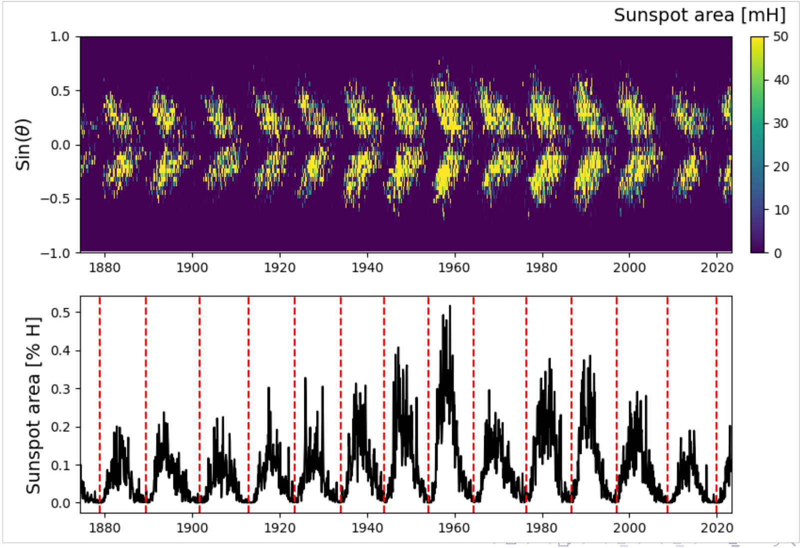
The solar dynamo & the solar cycle

- The reversal of the magnetic field repeats in a regular cycle. There are 22 years between identical configurations of the field.
- The self-generation process of the field is called **dynamo** action.
- For the Sun, there are 11 years between sunspot number maxima.



From P. Charbonneu, U. Montreal (Charbonneau 2010)

The 11-year sunspot cycle

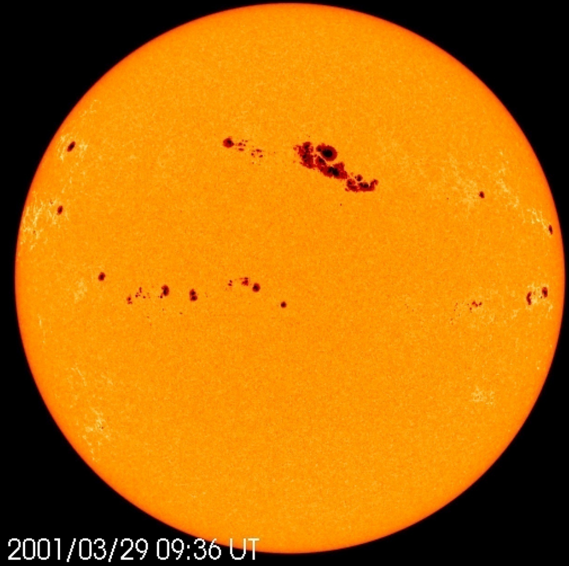


The first sunspots in a cycle form near ∼30° latitude, and the last near the equator, producing the "butterfly diagram."

Matthew Owens

Sunspots & solar activity

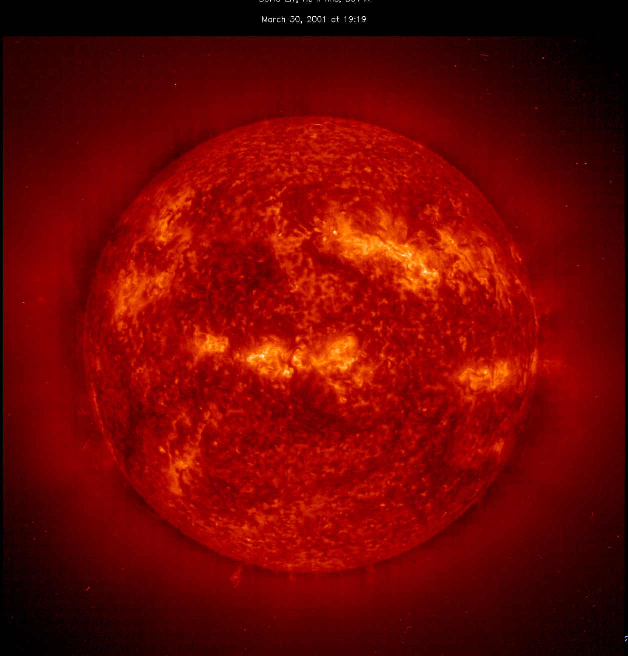
- Sunspots appear dark because they are slightly cooler than the rest of the solar surface.
- They are surrounded by hotter-than-average regions called **faculae**.
- Zeeman effect measurements show that they are maxima of the magnetic field.



SOHO/NASA.

Sunspots & solar activity

- Solar images from March 29, 2001:
  - Visible light
  - He II at 30.4 nm
  - Fe IX at 17.1 nm
- Note the X-ray bright faculae surrounding the sunspots. These regions add more to the Sun's luminosity than the spots take away.



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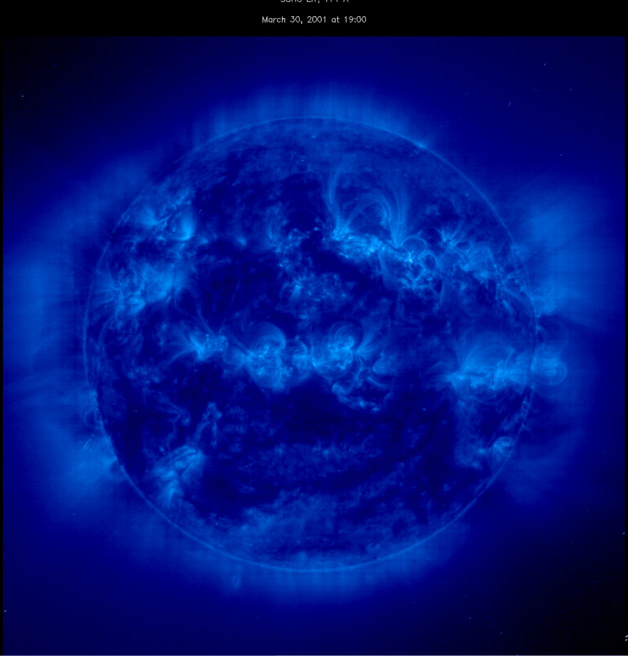
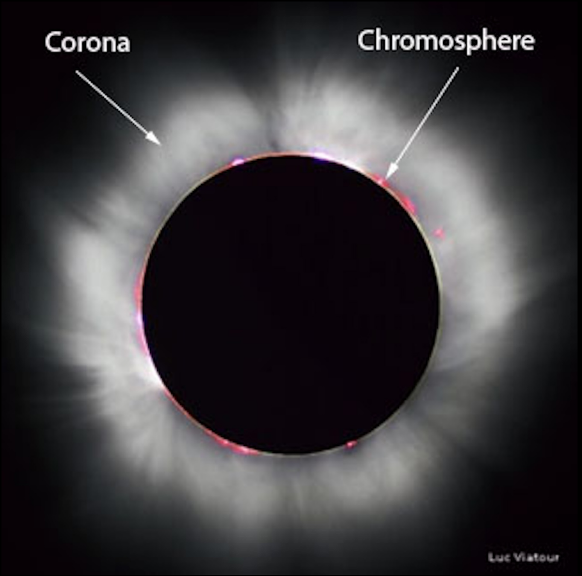


Chromosphere & corona

- The chromosphere is the layer of the Sun's atmosphere immediately above the photosphere.
- The chromosphere produces an emission spectrum (primarily Hα and He), as it is a warm diffuse gas.
- The corona is the outer layer of the Sun's atmosphere.
- The coronal density is so low that it cannot cool efficiently, so it reaches temperatures $> 10^6$ K.



Energy & the Sun

A thought experiment:

- Hydrostatic equilibrium and ideal gas behavior ensure that the center of the Sun is very hot, and energy (in the form of light) is radiated from the center.
- The high photon opacity of the Sun determines the rate at which energy leaks out. As we have seen, it takes a long time for photons to diffuse from the center to the surface.
- This cannot go on forever without the Sun cooling down or replacing the energy that leaks away.
- We know that the solar system is about 4.6×10^9 years old from many radioisotope measurements of meteorites, and that life has existed here for at least 3.5×10^9 years.
- Therefore the Sun must have had close to its present luminosity for billions of years for liquid water to be present on Earth to facilitate life.

How long would the Sun's present heat last?

The energy density at the center of the Sun is given by the energy density of the electron gas there, considered to be an ideal gas:

$$u_e = \frac{3}{2} n_e k T$$

The energy density of the radiation (light) about to leak away is

$$u_r = \frac{4}{c} f = \frac{4\sigma T^4}{c}$$

We have shown that it takes $t \approx 31,000$ years for a photon to leak from the center to the surface, so the heat lasts

$$\frac{u_e}{du_e/dt} \approx \frac{u_e}{u_r/t} = \frac{3}{2} n_e k T \frac{c}{4\sigma T^4} t \approx \frac{3}{8} \frac{kc}{\sigma m_p} \frac{\rho}{T^3} t \approx 3 \times 10^7 \text{ yr}$$

This is much less than the Sun's age, so some process must be replacing the energy that leaks away.

Energy source: Mass energy mc^2

Suppose a small fraction (0.1%) of the mass of the Sun could be converted to radiation. Then

$$\Delta E = 0.001 M_\odot c^2 = 2 \times 10^{51}\ \mathrm{erg}$$

$$\Delta t = \frac{\Delta E}{L_\odot} = 4.6 \times 10^{17}\ \mathrm{s} = \boxed{1.5 \times 10^{10}\ \mathrm{yr}}$$

How could the Sun convert its mass-energy to radiation?

- **Nuclear fusion**, the liberation of mass energy, stored as potential energy of the strong nuclear interaction (Bethe 1939, Nobel Prize 1967).

Could this work under the conditions in the center of the Sun?

$$\rho_c = 150\ \mathrm{g/cm^3} \qquad P_c = 2.1 \times 10^{17}\ \mathrm{dyne/cm^2}$$

$$T = 1.57 \times 10^7\ \mathrm{K} \qquad \text{Mostly hydrogen}$$

Yes. In fact, fusion requires such conditions.

Fundamental physics: The four forces of nature

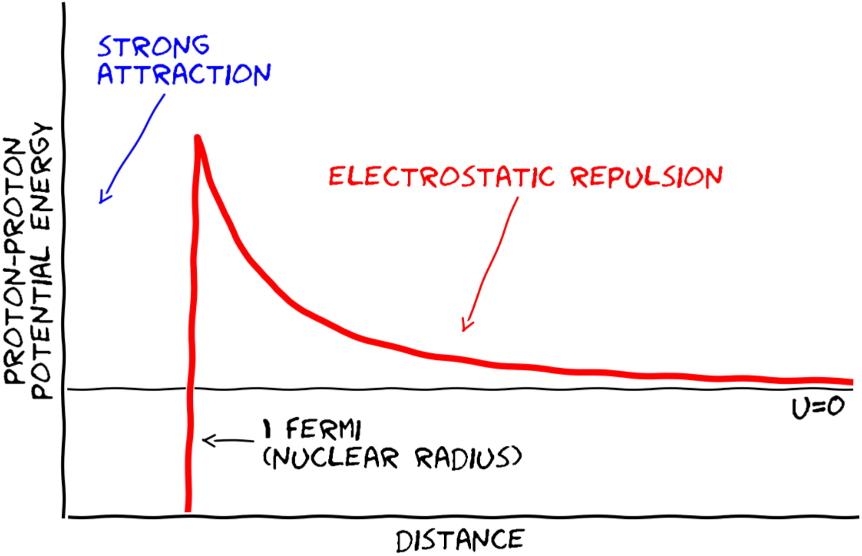
To understand nuclear fusion, we need to review some fundamental physics. First, we have the concept that there are two kinds of particles:

1. The particles which make up matter
2. The particles which carry forces

This represents a different paradigm for forces than what you learned in introductory physics classes. Rather than Newtonian "action at a distance," forces are mediated by an exchange of force-carrying particles ($\vec{F} = d\vec{p}/dt$) between matter particles.

Force	**Acts on...**	**Carrier**	**Range**	**Strength**
Strong nuclear	quarks, gluons	gluons	10^{-15} m	1
Electromagnetism	charge	photon (γ)	∞; $1/r^2$	10^{-2}
Weak nuclear	fermions	$W^{\pm}, Z$	10^{-18} m	10^{-5}
Gravity	mass	graviton?	∞; $1/r^2$	10^{-39}

Nuclear potential as a function of distance

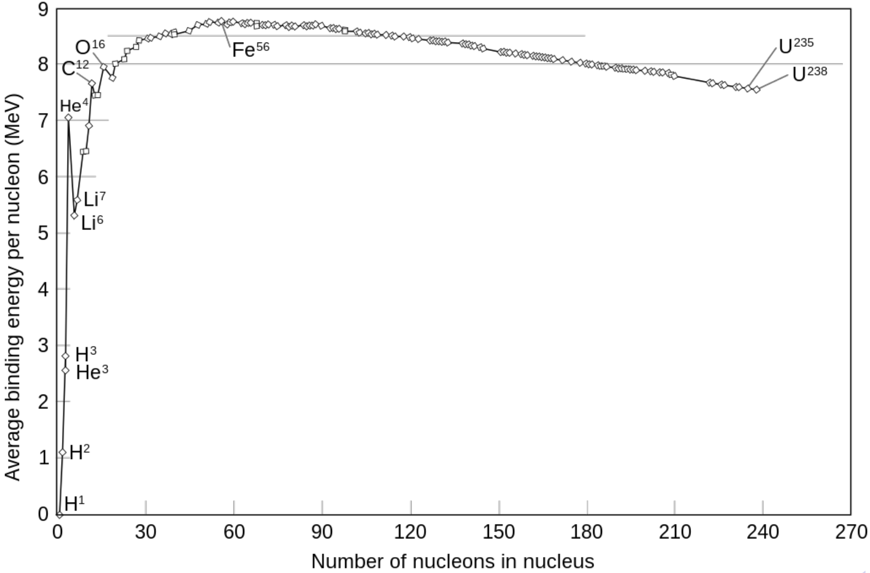


Hotter fusion and heavier elements

In principle, could stars live forever simply by gravitationally contracting and increasing their temperature to ignite the next heavier source of nuclear fuel whenever they run out of the lighter elements?

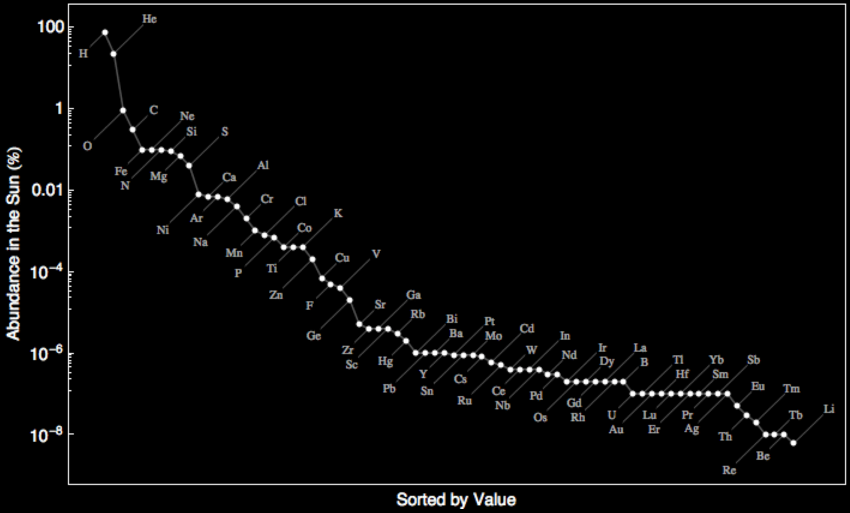
- **No**. The strong interaction range is smaller than the diameters of all but the smallest nuclei, but the range of the Coulomb interaction still covers the whole nucleus.
- If nuclei get large enough, the increase in electrostatic repulsion of the protons becomes greater than the increase in the binding energy from the strong interaction.
- Thus, there is a peak in the relationship between binding energy per baryon vs. atomic mass number.
- The peak turns out to lie at iron (Fe: $A = 56, Z = 26$).

Hotter fusion and heavier elements



Once a star's core is composed completely of iron, it can no longer replenish its energy losses (from luminosity) by fusion. Stars must therefore die, eventually.

Abundances of the elements in the Sun



Fundamental physics: Matter

Matter is made of **quarks** and **leptons**. They come in three generations with two types of particle per generation:

Family	**Generations**			**Notes**
Quarks	u	c	t	Participate in all 4 interactions
	d	s	b	(strong, weak, E&M, gravity)
Leptons	e^-	μ^-	τ^-	Abstain from strong interaction
	ν_e	ν_μ	ν_τ	

All of these particles have spin-$\frac{1}{2}$ (i.e., $\frac{\hbar}{2}$). Thus:

- They are all **fermions**.
- They obey the Pauli exclusion principle.

Note: Each particle also has a corresponding antiparticle with some reversed quantum numbers. More on that in a moment.