# Astronomy 142 — Recitation 6

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## Formulas to remember

### The main sequence (theoretically)

$$\ell \propto \begin{cases} \frac{T^{3.5}}{\bar{\rho}^2} & \text{if } M \leq M_\odot \\ \frac{1}{\bar{\rho}} & \text{if } M \geq M_\odot \end{cases} \qquad L \propto \begin{cases} \frac{M^{5.5}}{R^{0.5}} & \text{low mass} \\ M^3 & \text{high mass} \end{cases}$$

$$L = L_\odot \left(\frac{M}{M_\odot}\right)^4 \qquad R = R_\odot \left(\frac{M}{M_\odot}\right)$$

$$T_e = T_{e,\odot}\sqrt{\frac{M}{M_\odot}} \qquad L = L_\odot \left(\frac{T_e}{T_{e,\odot}}\right)^8$$

### Mean molecular weight

$$\frac{m_p}{\mu} \cong 2X + \frac{3}{4}Y + \frac{1}{2}Z \tag{1}$$

where $X$, $Y$, and $Z$ are the fractions of the star's mass in the form of hydrogen, helium, and all other elements, respectively.

## Workshop problems

**Remember!** The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in ASTR 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem in some sort of bound notebook.

1. *Evolution on the main sequence for a fully convective star.* A M3 main sequence star has mass $0.3M_\odot$, radius $0.3R_\odot$, and luminosity $0.036L_\odot$. Stars of low mass like this one turn out to be fully convective, so the elemental abundances are uniform throughout the star. For simplicity, suppose that the mass fractions of hydrogen ($X$) and helium ($Y$) start off at 0.75 and 0.25, respectively; neglect the heavier elements.
   (a) How many helium nuclei are produced per second? What, therefore, are the rates of change of the hydrogen and helium mass fractions, in terms of the star's luminosity?
   (b) If the star remains fully convective for its entire life, $\mu$ is uniform for its entire life; this means that no core of helium "ash" builds up, and thus there is no reason for the center to gradually heat up. It is safe to then assume that the central temperature stays constant. Obtain an expression for the radius of the star as a function of $\mu$.
   (c) What, then, will be the radius of the star if all the hydrogen is converted to helium? How does the star move in the HR diagram while this change is taking place?

2. *A uranium star.* The longest-lived isotope of element 92 has a mass number 238. (The atomic mass unit $u = 1.661 \times 10^{-24}$ g.) Suppose that it were possible to build a $1M_\odot$ star out of pure $^{238}$U.
   (a) What is the average molecular mass, $\mu$?
   (b) What is the central temperature, presuming that the functional behavior of density and pressure are the same as in the Sun (i.e. $P_0 = 19GM^2/R^4$, etc.)?
   (c) Qualitatively describe the evolution of this star. What nuclear processes can it use to generate energy, and what effect do the nuclear processes have on the average molecular mass? (Note that $^{238}$U is not "fissile" like $^{235}$U.)

3. In an attempt to identify the most important components of mass loss from giant and supergiant stars, several researchers have postulated parameterizations of the mass loss rate that are based on fitting observed rates for a specified set of stars with some general equation that includes measurable quantities associated with the stars in the sample. One of the most popular equations for AGB stars is that by Reimers:
$$\frac{dM}{dt} = -\frac{\eta LR}{GM} \tag{7}$$
   where $L$, $M$, and $R$ are the star's luminosity, mass, and radius, respectively, and $\eta$ is a dimensionless fitting parameter that turns out to be about $1.3 \times 10^{-5}$. The minus sign indicates that the star's mass is decreasing.
   (a) Explain qualitatively why $L$, $M$, and $R$ enter this equation in the way that they do.
   (b) Estimate the mass-loss rate for a $1M_\odot$ AGB star that has luminosity $7000L_\odot$ and effective temperature 3000 K.
   (c) If the mass-loss rate stays constant at this value, how long does it take the star to be reduced to its degenerate carbon-oxygen core ($0.6M_\odot$)?

**Learn your way around the sky, lesson 6.** (A feature *exclusive* of ASTR 142 recitations.) You may find the lab's celestial globes and the program Stellarium useful in answering these questions about the celestial sphere and the constellations.

4. In a previous recitation, we calculated local sidereal times in Rochester and in the location at which the vernal equinox took place with the Sun on the meridian. We compared this with an accurate calculation and saw that all of those times were off by several minutes, and all by the same amount. Here is the first part of the reason why; the second part will be in two weeks, and the complete result is called the Equation of Time.
   (a) First: suppose that Earth's orbit was circular, and that Earth's obliquity (angle between rotation axis and the Solar System's rotation axis) was zero. At what angular speed $\omega$ would the Sun appear to move through the sky against the background of fixed stars, and along what path would it move? (We shall call this speed, and this path, that of the average, or mean, Sun.) Express your answer both in radians per second and in minutes of right ascension per day.

       If the Earth's orbit were really like this, then sundials and clocks would always be in sync.
   (b) The Earth's orbit is not really circular, though. Its semimajor axis length and eccentricity are
$$a = 1.00000102 \text{ AU} \qquad \varepsilon = 0.0167086$$
       It reaches perihelion on January 4, and aphelion half a year later, on July 6.
       (a) At what angular speeds $\omega_p$ and $\omega_a$ does the Sun appear to move through the sky when the Earth is near perihelion and aphelion?
       (b) Suppose that the Sun's apparent angular speed oscillates between these two bounds repeatedly every orbit, in the simplest way possible. Construct a formula for the difference $\Delta\omega_\varepsilon(t)$ between the Sun's apparent angular frequency and that of the mean Sun.

(c) Suppose the Sun and the mean Sun coincide at perihelion. Integrate this expression to obtain the angular distance $\Delta\theta_\varepsilon$ between the Sun and the mean Sun.

**Intro to Python, lesson 6.** (A feature *exclusive* of ASTR 142 recitations.)

5. Plot $\Delta\theta_\varepsilon$ as a function of time through its one-year period of oscillation. Express $\Delta\theta_\varepsilon$ in minutes of RA and time in days.

   Comment on the plotted result. What does it mean for the real Sun to reach the meridian at a different time than the mean Sun? (Hint: Where is the mean Sun when the clocks read noon?) What does it mean for time-telling by sundials as compared to (good) mechanical clocks?