

# Astronomy 142 — Recitation #6

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## Formulas to remember

### Reddening and extinction

**Magnitude reminder**  $U = m_U$ ,  $B = m_B$ ,  $V = m_V$  (apparent magnitudes)

$$B - V = m_B - m_V \quad (\text{color index}) \quad (1)$$

**Visual extinction** toward a star:

$$A_V = R \cdot E(B - V) \quad (2)$$

where the star's **color excess** (“**reddening**”)  $E(B - V)$  is the difference between that star's  $B - V$  color index and the  $B - V$  color index of an unextinguished main-sequence star of the same spectral type, and where  $R$ , the ratio of total to selective extinction, is 3.06 for the extinction provided by the Milky Way's diffuse interstellar clouds.

On average, for that same extinction,

$$\frac{E(U - B)}{E(B - V)} = 0.72 \quad (3)$$

The following works for any pair of wavelengths, but we will use the example of  $B$  and  $V$ .

Extinction correction: If you measure magnitudes  $B$  and  $V$  for a star and determine  $E(B - V)$  and  $A_V$ , then the extinction corrected color and  $V$  magnitude are

$$(B - V)_0 = (B - V) - E(B - V) \quad V_0 = V - A_V \quad (4)$$

The **distance modulus (DM)** can be reliably converted to distance if the extinction-corrected apparent magnitudes are used (recall  $DM = 0$  corresponds to 10 pc!):

$$DM = m - M = V_0 - M_V \quad (5)$$

### Jeans mass

Weight and pressure in hydrostatic equilibrium for a uniform, spherical gas cloud with mass density  $\rho$ , temperature  $T$ , and average molecular mass  $\mu$  if its mass is

$$M = M_J = \left( \frac{kT}{\mu G} \right)^{3/2} \left( \frac{3}{4\pi\rho} \right)^{1/2} \quad (6)$$

### Ideal gas reminder

$$P = nkT = \frac{\rho kT}{\mu} \quad E = \frac{3}{2}NkT \quad (7)$$

where  $N$  is the number of molecules in the system ( $= M/\mu = nV$ ).

## Free-fall time

$$t_{ff} = \sqrt{\frac{3\pi}{32G\rho_0}} \quad (8)$$

## Workshop problems

**Remember!** The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in ASTR 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem in some sort of bound notebook.

1. Extinction at some wavelengths besides those of the astronomical  $B$  and  $V$  filters are given in the table below.

Band	Wavelength, $\lambda$ [ $\mu\text{m}$ ]	$A_\lambda/A_J$
$U$	0.36	5.53
$B$	0.44	4.70
$V$	0.54	3.55
$R$	0.70	2.66
$I$	0.90	1.70
$J$	1.25	1.00
$H$	1.65	0.624
$K$	2.21	0.382
$L$	3.40	0.182
$M$	4.80	0.095

- (a) An A0 V star (absolute  $V$  magnitude  $M_V = 0.6$ , visible color index  $B - V = 0$ , all visible and near-infrared color indices similarly zero) is placed at the distance of the Galactic center ( $r = 8.0 \times 10^3$  pc). What is its apparent  $V$  magnitude and visible color index?
  - (b) The same star is placed in the *real* Galactic center, toward which the visible extinction is  $A_V = 28$ . Now what is its apparent  $V$  magnitude and visible color index?
  - (c) What is its magnitude at  $K$  ( $\lambda = 2.2 \mu\text{m}$ )?
  - (d) With ground-based telescopes, we can detect stars as faint as about the 30th magnitude at  $V$  and about 24th at  $K$ . At what wavelength should we search for stars in our Galactic center?
2. A uniform-density, pure molecular hydrogen, spherical star has mass  $M = 2M_\odot$ , temperature  $T = 50$  K, and molecular-hydrogen number density  $n = 10^5 \text{ cm}^{-3}$ .
    - (a) Show that it is not unstable (it is not gravitationally collapsing).
    - (b) At what temperature would it become gravitationally unstable if its density remained at its original value?
    - (c) Suppose that it were possible to compress the cloud from the outside (by a passing supernova shock, for example). By what factor would the volume need to be compressed for the cloud to become gravitationally unstable and proceed to collapse on its own (without further squeezing) if the temperature remained at its original value?
  3. A constant-density, spherical molecular cloud undergoes free-fall collapse. Consider molecules at two particular initial radii: those at the initial cloud radius  $r_0$  and those halfway between the center and the edge, at  $r_0/2$ . Which molecules reach the center first? Explain (in detail).

**Learn your way around the sky** (A feature *exclusive* of ASTR 142 recitations.) You may find the lab's celestial globes and the program Stellarium useful in answering these questions about the celestial sphere and the constellations.

4. In a previous recitation, we calculated local sidereal times in Rochester and in the location at which the vernal equinox took place with the Sun on the meridian. We compared this with an accurate calculation and saw that all of those times were off by several minutes, and all by the same amount. Here is the first part of the reason why; the second part will be in two weeks, and the complete result is called the Equation of Time.
  - (a) First: suppose that Earth's orbit was circular, and that Earth's obliquity (angle between rotation axis and the Solar System's rotation axis) was zero. At what angular speed  $\omega$  would the Sun appear to move through the sky against the background of fixed stars, and along what path would it move? (We shall call this speed, and this path, that of the average, or mean, Sun.) Express your answer both in radians per second and in minutes of right ascension per day.  
If the Earth's orbit were really like this, then sundials and clocks would always be in sync.
  - (b) The Earth's orbit is not really circular, though. Its semimajor axis length and eccentricity are

$$a = 1.00000102 \text{ AU} \quad \varepsilon = 0.0167086$$

It reached perihelion on January 4, and aphelion half a year later, on July 3.

- (a) At what angular speeds  $\omega_p$  and  $\omega_a$  does the Sun appear to move through the sky when the Earth is near perihelion and aphelion?
- (b) Suppose that the Sun's apparent angular speed oscillates between these two bounds repeatedly every orbit, in the simplest way possible. Construct a formula for the difference  $\Delta\omega_\varepsilon(t)$  between the Sun's apparent angular frequency and that of the mean Sun.
- (c) Suppose the Sun and the mean Sun coincide at perihelion. Integrate this expression to obtain the angular distance  $\Delta\theta_\varepsilon$  between the Sun and the mean Sun.

#### Intro to Python (A feature *exclusive* of ASTR 142 recitations.)

5. The data file `Hyades.txt` (found on the course website) contains data on 158 candidate members of the nearest large open cluster, the Hyades. The data file `ZAMS.txt` (also found on the course website) contains data on the Zero-Age Main sequence: the absolute magnitudes of main sequence stars of all spectral types constructed from observations of very nearby (and therefore unextinguished) stars and accurate stellar models. Here, we will use the ZAMS to measure and correct the small extinction toward the Hyades (which also turns out to be fairly close), and to measure the distance of the Hyades by main sequence fitting, the technique you will use on the next problem set to measure the distances to a number of further-off open clusters. Unlike the clusters used in the homework, though, we can check our main-sequence fitting distance to the Hyades by comparison to measured parallax for its members.
  - (a) Begin by writing a script that reads in `Hyades.txt` and plots the color-color diagram (a scatter plot of  $(U - B)$  v.  $(B - V)$ ) of the stars.
  - (b) Go back and also import `ZAMS.txt`. Add a line of this data to the same color-color diagram.
  - (c) Now, add two variables to your code: "EBV" and "EUB" (or something along those lines) to represent the color excesses  $E(B - V)$  and  $E(U - B)$ . You will be changing these values each time you run your script, so put the assignments somewhere close to the top of the file (so that they are easy to find).
  - (d) Assign your initial guesses for  $E(B - V)$  and  $E(U - B)$  to the respective variables, and calculate  $(B - V)_0$  and  $(U - B)_0$  based on the color excess variables.
  - (e) Add your extinction-corrected colors to the color-color diagram (you should now have three data sets plotted: the original color-color data points for the Hyades cluster, the extinction-corrected color-color data points, and the ZAMS curve).
  - (f) Adjust your values of  $E(B - V)$  and  $E(U - B)$  until you get the best match between the extinction-corrected data points and the ZAMS.

- (g) Make a second plot, this time a color-magnitude (HR) diagram of the Hyades cluster:  $(B - V)$  on the  $x$ -axis and  $M_V$  on the  $y$ -axis. Add the ZAMS data as a curve on this plot as well. You will want to flip the vertical axis, so that the values increase from top to bottom (this way, the brighter objects will be towards the top of the diagram).
- (h) Create a new variable for the reddening-correction,  $A_V$ . Calculate the value of  $A_V$  based on your best guess of  $E(B - V)$ . (Do not hard-code any values into your calculations! Do everything with the variables.) Also, create a distance-modulus variable — you will be changing this value, so it is probably best to place the assignment somewhere close to your color excess variable assignments.
- (i) Assign your initial guess for the distance modulus, and calculate the extinction-corrected absolute  $V$  magnitude,  $M_V = V - A_V - DM$ .
- (j) Add your extinction-corrected absolute magnitude and color to the HR diagram, so again, you will have three data sets plotted. Adjust the value of the distance modulus until you get the best match between the extinction-corrected data points and the ZAMS. Give the highest weight to points with  $(B - V)_0 \approx 0.5 - 1$ .
- (k) How far do you conclude that the Hyades cluster is from us? (Recall that  $r = 10 \times 10^{DM/5}$ .)
- (l) Included in the `Hyades.txt` file are *Gaia* measurements of parallax for members of the Hyades. Compare the median value of these parallax distances to the cluster distance that you determined by main-sequence fitting. Note that the parallax is given in milliarcseconds (0.001 arcsec).
- (m) Calculate the absolute  $V$  magnitudes for the stars using the parallax ( $M_V = V - A_V - 5 \log(r/10\text{pc})$ ). Add these new  $M_V$  plotted as a function of  $(B - V)_0$  (as a scatter plot) to your HR diagram plot. Why did I ask you to give the greatest weight to the intermediate color excess values in your main sequence fitting? Why do the Hyades points systematically depart from the ZAMS in the way that they do for both the bluest and reddest stars?