

Inside Stars

Nuclear fusion reactors in stars
Nucleosynthesis & the cosmic abundances of elements
Solar neutrinos
Pulsations in stars & the instability strip
Helioseismology
The standard solar model

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Fundamental physics: The four forces of nature

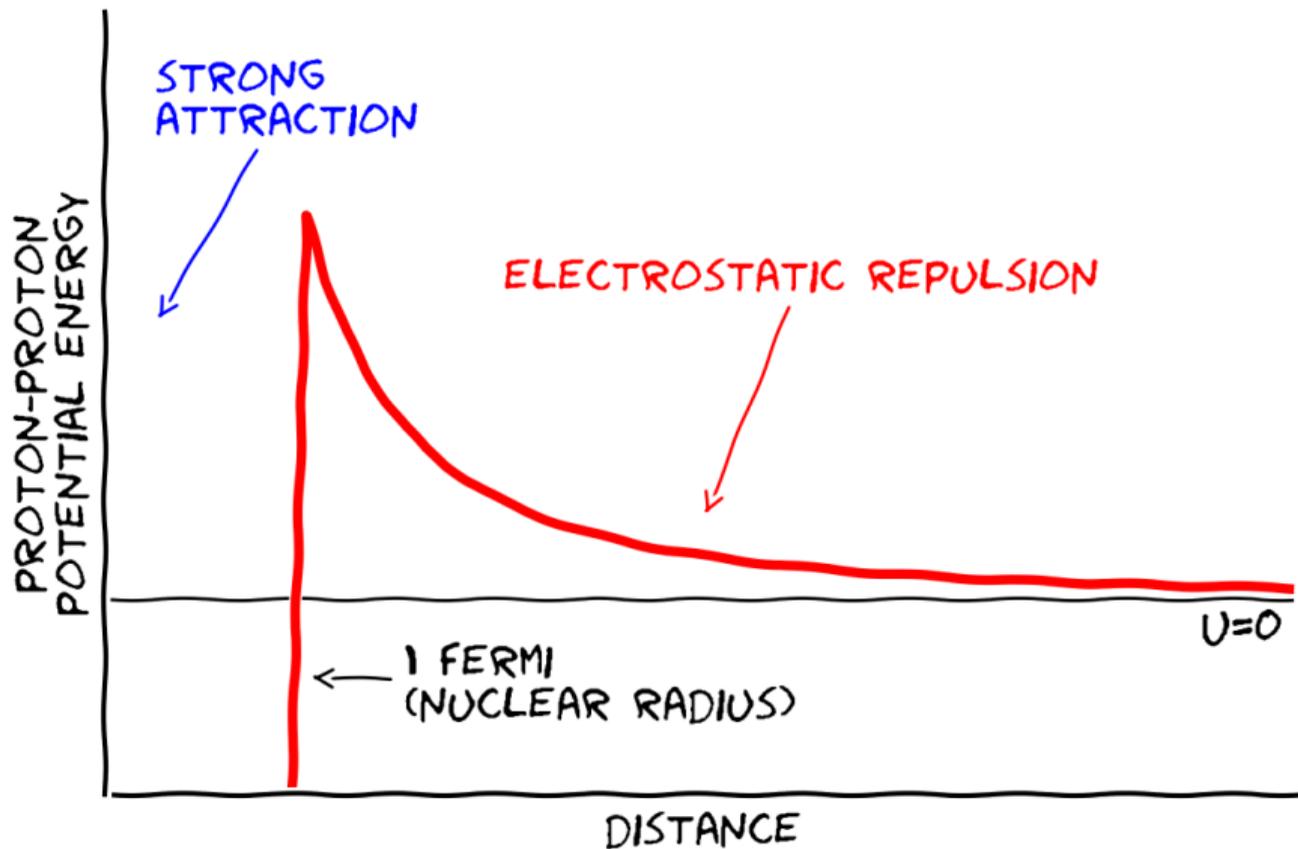
To understand nuclear fusion, we need to review some fundamental physics. First, we have the concept that there are two kinds of particles:

1. The particles which make up **matter**
2. The particles which carry **forces**

This represents a different paradigm for forces than what you learned in introductory physics classes. Rather than Newtonian “action at a distance,” forces are mediated by **an exchange of force-carrying particles** ($\vec{F} = d\vec{p}/dt$) between matter particles.

Force	Acts on...	Carrier	Range	Strength
Strong nuclear	quarks, gluons	gluons	10^{-15} m	1
Electromagnetism	charge	photon (γ)	$\infty; 1/r^2$	10^{-2}
Weak nuclear	fermions	W^{\pm}, Z	10^{-18} m	10^{-5}
Gravity	mass	graviton?	$\infty; 1/r^2$	10^{-39}

Nuclear potential as a function of distance



Fundamental physics: Matter

Matter is made of **quarks** and **leptons**. They come in **three generations** with two types of particle per generation:

Family	Generations			Notes
Quarks	u	c	t	Participate in all 4 interactions (strong, weak, E&M, gravity)
	d	s	b	
Leptons	e^-	μ^-	τ^-	Abstain from strong interaction
	ν_e	ν_μ	ν_τ	

All of these particles have spin- $\frac{1}{2}$ (i.e., $\frac{\hbar}{2}$). Thus:

- ▶ They are all **fermions**.
- ▶ They obey the Pauli exclusion principle.

Note: Each particle also has a corresponding **antiparticle** with some reversed quantum numbers. More on that in a moment.

Fundamental physics: Quarks

- ▶ Quarks have fractional electric charge of $\pm\frac{1}{3}e$, where e is the electron charge.
- ▶ They come in three different kinds of the strong-interaction analog of charge (**color**): **red**, **green**, and **blue**.
- ▶ Individual quarks are never observed (“confinement”).
- ▶ Nuclear particles are made of 2 or 3 quarks and are always *color-neutral*.
 - ▶ **Mesons**: quark-antiquark pairs (e.g., π^\pm , π^0)
 - ▶ **Baryons**: three quarks (e.g., protons, neutrons)
- ▶ More exotic 4 and 5 quark states have also been created in accelerators ([Swanson 2013](#), [Aaij et al. 2014](#), [Aaij et al. 2015](#)).

Quantities conserved in the four interactions

- ▶ Energy, linear momentum, angular momentum, etc.
- ▶ Electric charge
- ▶ Lepton number, separately for each generation of leptons

Particles	l_e	l_μ	l_τ
e^-, ν_e	1	0	0
μ^-, ν_μ	0	1	0
τ^-, ν_τ	0	0	1

- ▶ Baryon number. For example:

Particle	Spin (\hbar)	Charge (e)	Baryon #
u	1/2	+2/3	1/3
d	1/2	-1/3	1/3
proton (uud)	1/2	1	1
neutron (udd)	1/2	0	1

Antiparticles

Each of the quarks and leptons has a corresponding **antiparticle**.

- ▶ Antiparticles have exactly the same mass and spin as the corresponding particle.
- ▶ They have the **opposite** electric charge, lepton number, and baryon number of the corresponding particle.

Notation used in this class (not standard):

baryon #
charge **Particle**

baryon #
charge **Antiparticle**

Examples:

$$\text{proton} = {}^1_1p \text{ or } {}^1_1\text{H}$$

$$\text{electron} = {}^0_{-1}e$$

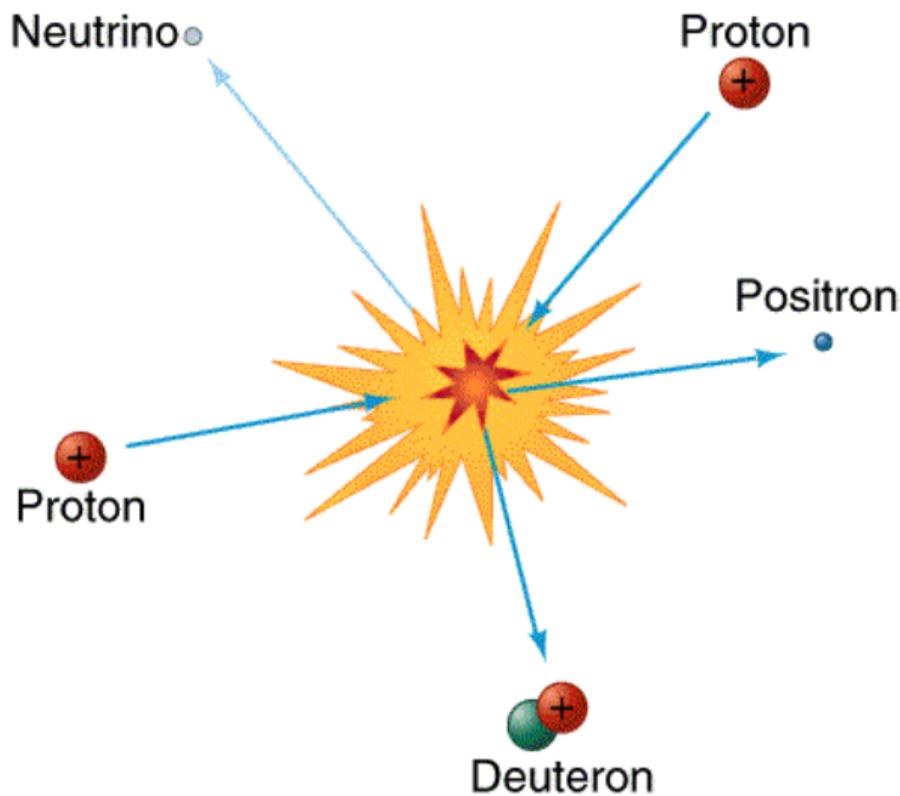
$$\text{photon} = {}^0_0\gamma$$

$$\text{antiproton} = {}^{-1}_{-1}\bar{p} \text{ or } {}^{-1}_{-1}\bar{\text{H}}$$

$$\text{positron} = {}^0_1e^+$$

$$\text{electron antineutrino} = {}^0_0\bar{\nu}_e$$

Fusion of two protons: $p + p \rightarrow {}^2\text{H} + e^+ + \nu_e$



From Chaisson & McMillan, Astronomy Today

Fusion and $E = mc^2$

Mass is a form of energy. Even when a body is at rest and is far from other attracting or repelling bodies, it has a total **rest energy**

$$E_0 = m_0c^2$$

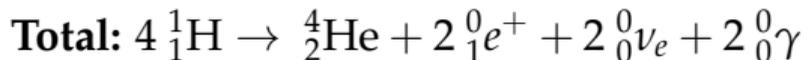
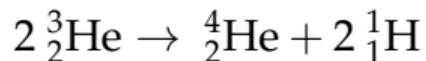
where m_0 is the rest mass. In pp fusion:

- ▶ Two protons fuse to make a deuteron and two lightweight particles.
- ▶ The deuteron has a proton and neutron, both of which are about the same mass, but also a large **negative potential energy** from the strong interaction between them.
- ▶ Thus, the rest energy of the deuteron is *less* than the sum of the rest energies of the two protons. Equivalently, its rest mass is less than that of the protons.
- ▶ This suggests a convenient method for accounting for the energy released in fusion processes...

The proton-proton chains

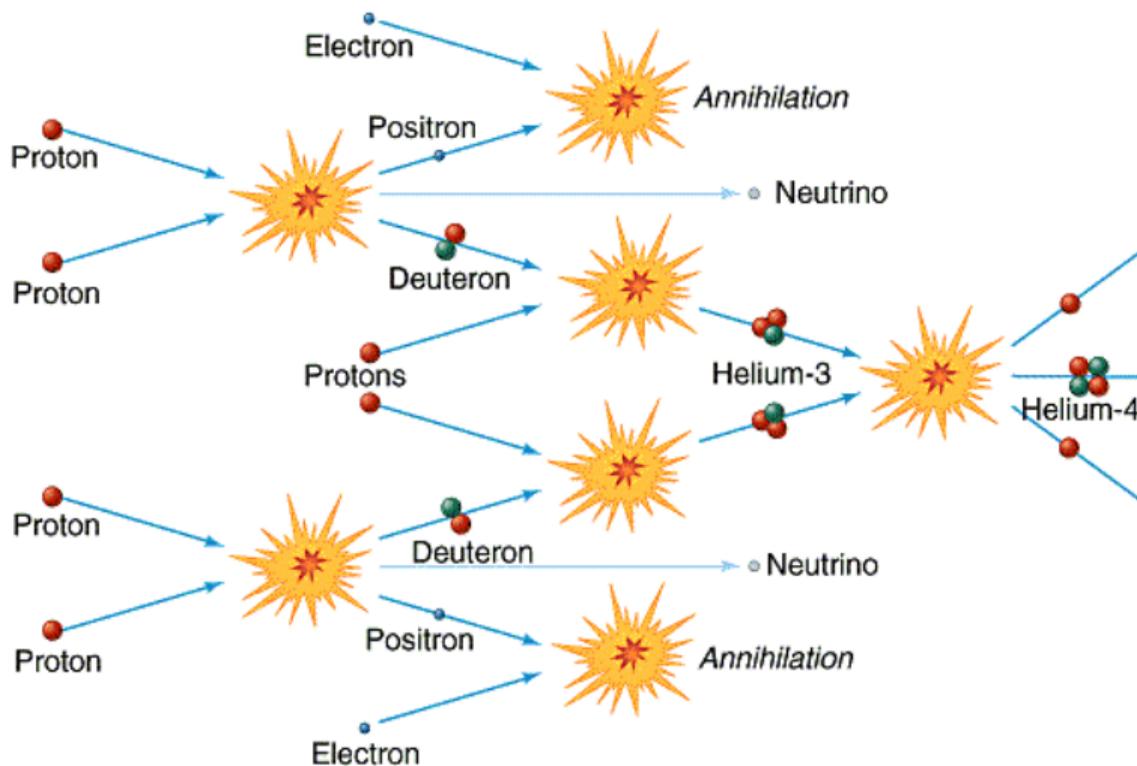
Several different sequences of reactions starting with the fusion of two protons drive nuclear fusion in main sequence stars. Collectively, they are called the **proton-proton chains**.

- ▶ Here is pp chain I (70% of pp chain reactions):



- ▶ The rest mass of the products is less than the reactants, so the products have more kinetic energy than the reactants (heat!).

Proton-proton chain I (PPI)



From Chaisson and McMillan, Astronomy Today

The proton-proton chains

Note the application of all conservation laws:

$2 {}^1_1\text{H} \rightarrow {}^2_1\text{H} + {}^0_1e^+ + {}^0_0\nu_e$ Mass and baryon number are conserved in the ${}^2_1\text{H}$. The extra charge (+) is carried off by a non-baryon, the positron (an anti-lepton). An electron neutrino is needed to conserve lepton number.

${}^2_1\text{H} + {}^1_1\text{H} \rightarrow {}^3_2\text{He} + {}^0_0\gamma$ Energy and momentum cannot both be conserved unless there is more than one particle in the final state. A neutrino/antineutrino pair would also work here but would happen much less frequently.

The proton-proton chains

How much kinetic energy do the products have? The difference in rest mass between products and reactants gives the difference in binding (potential) energy:

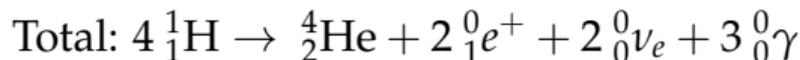
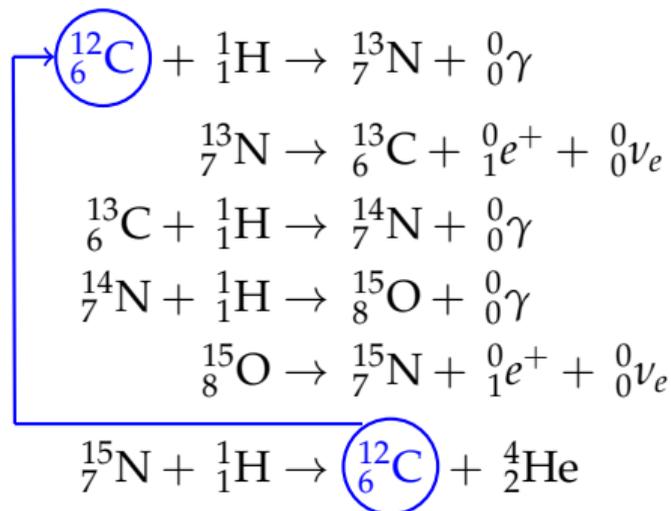
$$\begin{aligned}\Delta W &= m_0(\text{He})c^2 + 2m_0(e)c^2 - 4m_0(\text{H})c^2 \\ &\approx m_0(\text{He})c^2 - 4m_0(\text{H})c^2 && \text{(Note: } \frac{m_p}{m_e} = 1836 \text{)} \\ &= 3.97m_p c^2 - 4m_p c^2 && \text{(mass of He nucleus)} \\ &= -0.03m_p c^2 \\ &= \boxed{-4.5 \times 10^{-5} \text{ erg}}\end{aligned}$$

Compare to the **average kinetic energy of particles in an ideal gas** at 1.57×10^7 K:

$$\langle KE \rangle = \frac{3}{2}kT = 3 \times 10^{-9} \text{ erg}$$

Nuclear catalysis: the CNO bi-cycle

One branch, the CN cycle:



${}^{12}_6\text{C}$ is a **catalyst**; it is *not* used up in the reactions.

This has the same rest mass difference, and therefore kinetic energy in the products, as the pp chains. It requires higher T , though.

Hot fusion

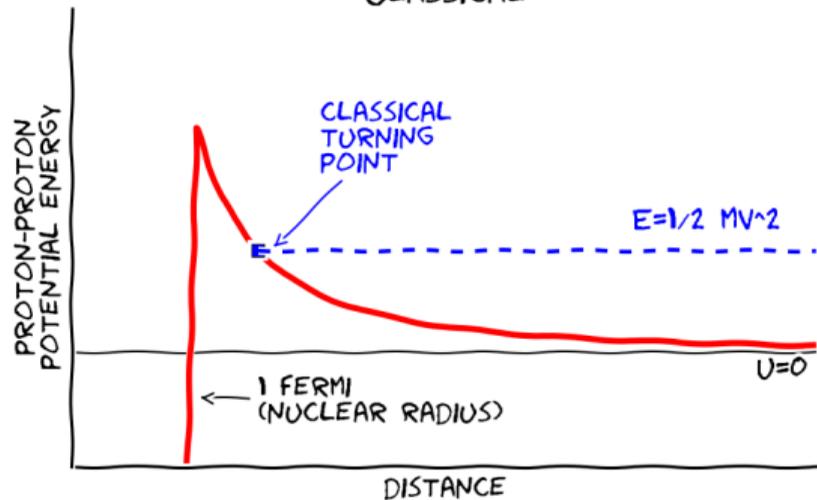
Why do pp chain reactions not take place in the ocean, where there is plenty of hydrogen?

- ▶ The **Coulomb barrier** prevents fusion. The average kinetic energy of oceanic H is *much* less than the height of the potential energy barrier due to electrostatic repulsion. Repulsion keeps the protons too far apart for strong interactions to take over.
- ▶ In fact, even at tens of millions of degrees, as in the centers of main sequence stars, the average kinetic energy is *still* too small for classical collisions to result in protons penetrating the Coulomb barrier.

Quantum mechanical **tunneling** through the Coulomb barrier is still required for fusion to occur, even at stellar core temperatures!

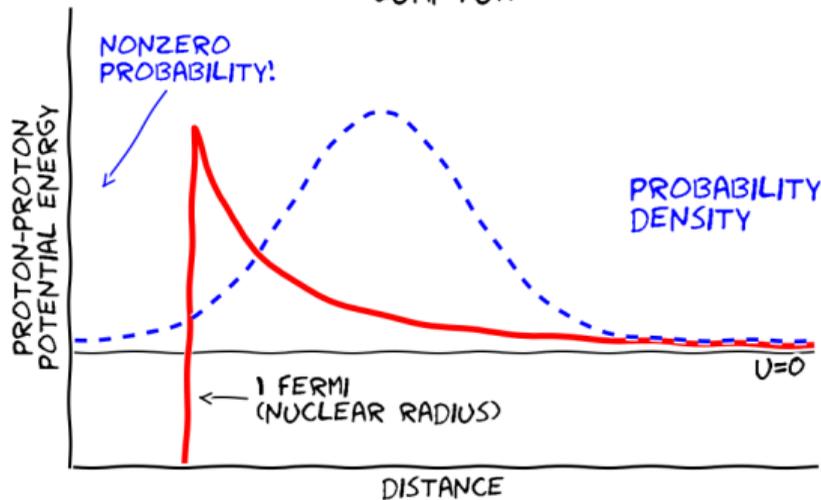
Fusion by tunneling

CLASSICAL



A classical particle of energy E encountering a Coulomb barrier of potential U will scatter off the barrier if $E < U$.

QUANTUM



A quantum mechanical particle has a nonzero probability of penetrating the barrier because its wave function has a nonzero evanescent component in the region $E < U$.

Temperature dependence of proton fusion rate

Let us do a simplified version of a calculation first done by George Gamow (Gamow 1928) and also in Ryden pp. 362-366 and Shu p. 115.

Consider in 1D the fusion of two particles with masses m_1 and m_2 , charges q_1 and q_2 , speeds v_1 and v_2 , and separation r . Their reduced mass and relative speed is

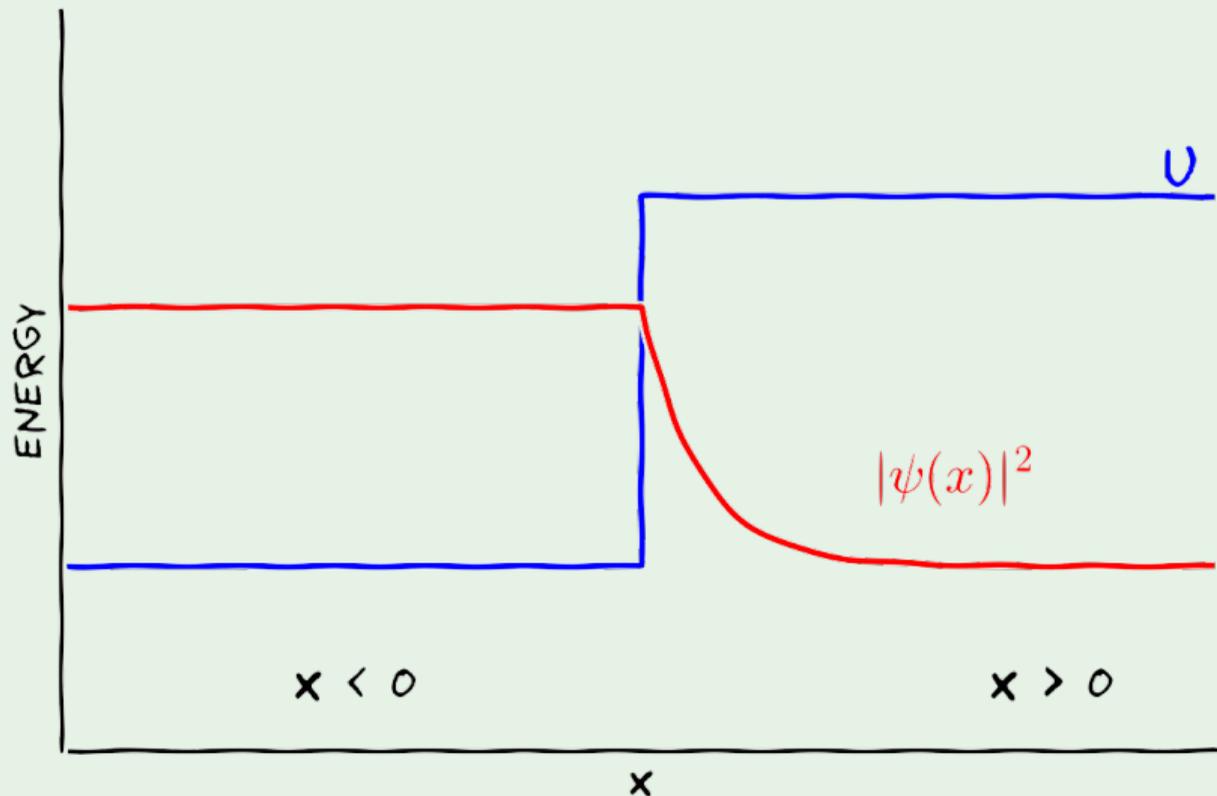
$$m = \frac{m_1 m_2}{m_1 + m_2}, \quad v = v_1 - v_2$$

Classically, they cannot get closer together than r_{\min} , where

$$W = \frac{1}{2} m v^2 = \frac{q_1 q_2}{r_{\min}} \implies r_{\min} = \frac{2 q_1 q_2}{m v^2}$$

Temperature dependence of proton fusion rate

Next, consider tunneling through a 1D infinite potential energy “step” barrier of height U .



Start with the Schrödinger Equation:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U\psi = E\psi, \quad E < U$$

$$\psi(x > 0) = \frac{2k_1}{k_1 + k_2} e^{ik_2x}$$

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}} \quad \text{and} \quad k_2 = i\kappa_2 = \sqrt{\frac{2m(E - U)}{\hbar^2}}$$

The probability density that the particle is in the non-classical region $x > 0$ is nonzero:

$$p(x) = |\psi|^2 = \psi^* \psi = \frac{4k_1^2}{k_1^2 - \kappa_2^2} e^{-2\kappa_2x}$$