Astronomy 142 — Recitation #7

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Formulas to remember

"Pressure" from random motions of stars

$$P = \rho v_r^2 \tag{1}$$

where ρ is the mass density of stars (mass of stars per unit Galactic volume including the space between the stars, not the mass density of the stars themselves) and v_r is the random component of motion of a typical star, separate from any systematic (e.g. orbital) motion.

1D hydrostatic equilibrium

$$\frac{dP}{dz} = -\rho g_z \tag{2}$$

Mass per unit area in a hydrostatically-supported galactic disk

$$\mu = \frac{g_z}{2\pi G} = \frac{v_z^2}{2\pi G H} \tag{3}$$

Relaxation time

$$t_c = \frac{v^3}{4\pi G^2 m^2 n} \frac{1}{\ln\left(\frac{2R}{r}\right)} = \left(\frac{2R}{v}\right) \frac{N}{24\ln\left(\frac{N}{2}\right)} \tag{4}$$

Virial theorem

In thermal equilibrium or steady state,

$$\frac{d^2I}{dt^2} = 2K + U = K + E \tag{5}$$

where I, K, U, and E are the moment of inertia, total kinetic energy, total potential energy, and total mechanical energy of a closed system of particles whose interactions can be characterized by a scalar potential. It is often the case that $\frac{d^2I}{dt^2} = 0$, for which the virial theorem simplifies to $K = -\frac{U}{2}$.

Rotation curves

$$\begin{split} v(r) &= \sqrt{\frac{GM}{r}} & \text{Point mass: Keplerian rotation} \\ v(r) &= r\sqrt{\frac{4\pi G\rho_0}{3}} & \text{Constant density: solid-body rotation} \\ v(r) &= \sqrt{4\pi G\rho_0 r_0^2} = \text{constant} & \frac{1}{r^2} \text{ density distribution: flat rotation curve} \end{split}$$

Abbreviations

- $\text{kpc} = 10^3 \text{ pc}$
- Myr = 10^6 yr
- Gyr = 10^9 yr
- 1 km/s = 1.022 pc/Myr

Workshop problems

Remember! The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in ASTR 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem in some sort of bound notebook.

- 1. In the Sun's neighborhood of the Galactic disk, suppose that the density can be thought of as an infinite planar distribution with a plane surface density μ (mass per unit disk area) and volume density ρ (mass per unit volume) that decreases sharply with increasing elevation z above or below z = 0.
 - (a) Under these assumptions, explain why the equation of hydrostatic equilibrium can be written to good approximation as

$$\frac{dP}{dz} = -\pi G \rho \mu \tag{6}$$

- (b) Solve this equation for the density as a function of z, and show that the density scale height obtained is the same as what we found in class.
- 2. The number of pairs among N objects. This result appears in so many contexts in physics that it is worth it to work through the derivation.
 - (a) In how many distinctly different ways can N distinguishable objects be arranged? (If you prefer a concrete situation: Suppose that you have N books to put on a shelf. In how many different ways can you place them on the shelf?)
 - (b) Now suppose that the N objects are indistinguishable from one another. In how many distinctly different ways can one arrange these objects? (For example, N identical books on a shelf.)
 - (c) In how many distinctly different ways can one arrange N objects, of which n and N n are of two distinguishable types, but of which the n objects of the first type are indistinguishable from one another, and similarly the N n are indistinguishable from one another? (For example, n identical blue books and N n identical green books on a shelf.)
 - (d) So how many different pairs are there among the N identical objects?
- 3. What is the total gravitational potential energy of a cluster of N stars that have typical mass m and typical separation r?
- 4. Use the virial theorem to show that a cluster of N stars with typical random velocity v and typical separation r has total mass $M = \frac{2rv^2}{G}$.

Intro to Python (A feature *exclusive* of ASTR 142 recitations.)

5. Plot $\Delta \theta_{\varepsilon}$ that you solved for in question 4 of last week's recitation as a function of time through its one-year period of oscillation. Express $\Delta \theta_{\varepsilon}$ in minutes of RA and time in days.

Comment on the plotted result. What does it mean for the real Sun to reach the meridian at a different time than the mean Sun? (Hint: Where is the mean Sun when the clocks read noon?) What does it mean for time-telling by sundials as compared to (good) mechanical clocks?