Inside Living Stars, Failed Stars & Stellar Remnants

Pulsating stars & the Instability strip Helioseismology & the standard solar model Degeneracy pressure White dwarfs & Neutron stars Brown dwarfs & Giant planets General relativity Hawking radiation Gravitational waves

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University of Rochester

Stellar-mass sized black holes

- Radial and nonradial pulsations in stars
- Pulsating stars and the instability strip
- Helioseismology and the standard solar model
- Degeneracy pressure of electrons and neutrons
- White dwarfs & neutron stars
- Brown dwarfs & giant planets
- General relativity & its prediction of black holes
- Hyperspace
- Hawking radiation
- Gravitational radiation

Reading: Kutner Sec. 8.4 & 11.5, Ryden Sec. 18.3



Right: HST image of the Andromeda galaxy with an X-ray image inset of an ultra-luminous X-ray source (a stellar-mass black hole). From the MPE.

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Acoustic waves in stars

Now consider the "pipe" to be represented by the surface and center of a star. The "pipe" is *open* on the surface and *closed* at the center, since the compression wave cannot move to r < 0. Hence, the period of the longest standing wave is

$$\Pi = 4 \int_0^R \frac{dr}{v_s} = \frac{4}{\sqrt{\frac{2\pi}{3}\gamma G\rho}} \int_0^R \frac{dr}{\sqrt{R^2 - r^2}} = 4\sqrt{\frac{3}{2\pi\gamma G\rho}} \int_0^1 \frac{dx}{\sqrt{1 - x^2}}$$
$$= 4\sqrt{\frac{3}{2\pi\gamma G\rho}} \int_0^{\pi/2} \frac{\cos u \, du}{\sqrt{1 - \sin^2 u}}$$
$$= 4\sqrt{\frac{3}{2\pi\gamma G\rho}} \int_0^{\pi/2} du = 4\sqrt{\frac{3}{2\pi\gamma G\rho}} \frac{\pi}{2}$$
$$\Pi = \sqrt{\frac{6\pi}{\gamma G\rho}}$$

Acoustic waves in stars

For a uniform density star that is the mass and size of the Sun, we get

$$\Pi = \sqrt{\frac{6\pi}{\gamma G \rho}} = 11,700 \text{ s} = 183 \text{ min} = 3 \text{ hr}$$

and the period increases if the density of the star decreases.

Note that:

- Pressure waves manifest themselves as oscillations of surface temperature and radius of the star, which in turn cause oscillations of the magnitude of the star.
- The details of the periods and amplitudes of the oscillations are sensitive to the density, its variations, and the equation of state (*P ρ* relation) **inside** the star.
- ▶ There is good agreement with the "loudest" pulsating stars.

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Loud sound waves in stars: Pulsators

Since the discovery of the pulsation of Mira (*o* Ceti) around 1600 and δ Cephei in 1784, eight types of pulsating stars have been found:

Туре	Period	Ampl. (Δm)	Notes
Long-period (Mira) variables	100–700 days	2–7 mag	red giants
Classical (Pop I) Cepheids	1–50 days	0.5–1.5 mag	supergiants
W Virginis stars (Pop II Cepheids)	2–45 days	0.5–1.5 mag	giants
RR Lyrae stars	1–48 hr	1–1.5 mag	giants
δ Scuti stars	1–3 hr	0.1–0.6 mag	near the main sequence
β Cephei stars	3–7 hr	0.05–0.2 mag	near the main sequence
SX Phoenicis stars (Pop II δ Scus)	1–12 hr	0.01–0.15 mag	near the main sequence
ZZ Ceti stars	100–1000 s	0.1–0.3 mag	white dwarfs

More information: General Catalog of Variable Stars.

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Patterns of pulsators: The instability strip (IS)



Pulsating stars are not spread randomly in the luminosity-temperature space of the H-R diagram, nor are they all of a certain composition.

- Mira variables are supergiants on the asymptotic giant branch
- β Cep stars are all B stars lying just above the main sequence (MS).
- All the others lie along a long, narrow, nearly vertical patch of the H-R diagram at effective temperatures ~ 10⁴ K; this is the instability strip (IS).
- δ Scu stars are close to the main sequence but most IS inhabitants are not there long; they develop rapidly after leaving the MS.

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Origin of the instability strip: the κ -mechanism

In a typical star,

- An increase in the compression of a layer of the star's interior causes an increase in the temperature and density of the material.
- While opacity increases with increasing density, it *decreases* with increasing temperature. Opacity is more strongly dependent on the temperature than the density, so an increase in compression results in a decrease in the opacity.
- A decrease in opacity allows more radiation to escape, thus cooling down the interior.
- This maintains an equilibrium condition in the star's interior, where its temperature and pressure are kept constant.

Origin of the instability strip: the κ -mechanism

Variable stars have a layer of helium towards their surface.

- ▶ When this layer is heated beyond 40,000 K, the He atoms become fully ionized.
- This increases the number of free particles, so the increase in density supersedes the increase in temperature, causing the opacity to *increase*.
- ► This results in a build-up of radiation pressure behind the layer.
- Eventually, the layer is pushed outward, cools, and the He ions recombine with the free electrons.
- This decrease in density causes the opacity to decrease, reducing the outward pressure and allowing the layer to fall back towards the center of the star.

This is called the κ -mechanism, named after the symbol most astrophysicists use for opacity. The period of this cycle perfectly matches the period for acoustic oscillations in these stars.

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Nonradial stellar oscillations

- Some pressure waves may have transverse components as well as radial components.
- Such waves can be trapped between the surface and a layer of certain density in the interior as they propagate.
- At the surface, the waves reflect due to the density drop-off.
- In the interior, the waves refract, bending up due to the increase of v_s with depth.



Propagation of acoustic waves corresponding to modes with $\ell = 30$ and $\nu = 3$ mHz (deeply penetrating rays) and $\ell = 100$, $\nu = 3$ mHz (shallowly penetrating rays) (Christiansen 2003).

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Solar oscillations & helioseismology

The Sun oscillates in many different modes with periods of \sim 5 min. This was discovered in 1962 (Leighton et al. 1962) and later explained as nonradial oscillations (Ulrich 1970).

- It took a long time to notice these oscillations because they are so small. The velocity amplitudes are typically 100 cm/s with displacements of tens of meters.
- Many **thousands** of modes have now been identified.
- The period of each mode gives the integral of

$$rac{1}{v_s} = \sqrt{rac{
ho}{\gamma P}}$$

along a different path through the Sun's interior.

Different modes penetrate more deeply into the interior, providing good coverage for the outer 90% of the volume.

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Precise knowledge of the solar interior

To summarize, we have:

- A direct measure of pp fusion per unit time at the very center of the Sun using measurements of solar neutrinos, corrected for neutrino oscillation.
- Thousands of measurements of integrals of

$$rac{1}{v_s} = \sqrt{rac{
ho}{\gamma P}}$$

from which the density, pressure, temperature, abundances, etc. can be determined over most of the solar volume.

- Confidence that we know precisely and accurately the thermodynamic parameters of all parts of the solar interior without being able to see the interior directly.
- The result is the Standard Solar Model (see, e.g., Bahcall et al. 2004, Vinyoles et al., 2017).

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The Standard Solar Model



P(r), T(r), $\rho(r)$, and M(r) from Carroll and Ostlie, An Introduction to Modern Astrophysics

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The Standard Solar Model



Fusion power generation (left) and radiation and convection zones (right), showing the dominant energy transport mechanism for each region, from Carroll and Ostlie, An Introduction to Modern Astrophysics

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Electron degeneracy pressure

Consider a gas of electrons, produced by ionization from atoms with nuclear charge *Ze* and baryon number *A*. The electron and nuclear densities are related by

$$n_e = Zn_+$$

and the mass density is

$$\begin{aligned}
\rho &= m_e n_e + A m_p n_+ & (m_p \approx m_n) \\
\approx A m_p n_+ & (m_p \gg m_e)
\end{aligned}$$

Therefore, $n_e = Z\rho / Am_p$ and the **equation of state** for electron degeneracy pressure is

$$P_e = 0.0485 \frac{h^2}{m_e} \left(\frac{Z}{A}\right)^{5/3} \left(\frac{\rho}{m_p}\right)^{5/3}$$

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Polytropic equation of state

Note that the equation of state is polytropic:

$$P \propto \rho^{\gamma}$$

where $\gamma = (n + 1)/n$ is called the polytropic exponent and *n* is called the polytropic index. This is one of the few functional forms for which the equations of stellar structure yield simple solutions. Example models:

Index n	Notes
0.5 - 1	Neutron stars
1.5	Fully convective cores, brown dwarfs, gaseous planets
3	MS stars (Sun) and relativistic degenerate cores (WDs)

As *n* increases, the density distribution is more heavily weighted toward r = 0.

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Electron degeneracy in stars

Suppose we use the new EOS (equation of state) instead of the ideal gas law to balance gravity and support a star. Our former scaling relationships derived from the ideal gas law and gravity were

$$P_c \propto \frac{GM^2}{R^4} \qquad \qquad \rho_c \propto \frac{M}{R^3}$$

Precise calculations for the polytropic EOS $P \propto \rho^{5/3}$ and gravity turn out to give

$$P_c = 0.77 rac{GM^2}{R^4}$$
 $ho_c = 1.43 rac{M}{R^3}$

We can now use this to estimate the typical size of a star balanced by electron degeneracy pressure.

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Size of an electron degenerate star

$$P_{c} = P_{e}$$

$$0.77 \frac{GM^{2}}{R^{4}} = 0.0485 \frac{h^{2}}{m_{e}} \left(\frac{Z}{A}\right)^{5/3} \frac{\rho^{5/3}}{m_{p}^{5/3}}$$

$$= 0.0485 \frac{h^{2}}{m_{e}} \left(\frac{Z}{A}\right)^{5/3} \left(\frac{1.43M}{m_{p}}\right)^{5/3} \frac{1}{R^{5}}$$

$$\therefore R = 0.114 \frac{h^{2}}{Gm_{e}m_{p}^{5/3}} \left(\frac{Z}{A}\right)^{5/3} M^{-1/3}$$

A couple of points:

- There is no temperature dependence here! This is much more simple than a normal star.
- ▶ *R* is much smaller than a normal star of equal mass.

White dwarfs

Numerical example

For $M = 1M_{\odot}$ and Z/A = 0.5, $R = 9 \times 10^8$ cm $\approx 1.4R_{\oplus}$. This object, a **white dwarf**, has the mass of a star and the size of a planet!

- Remarkable feature of *R*–*M* relation: *R* decreases with increasing *M* (Stoner 1930).
- Cause: larger mass *M* requires larger supporting *P_e*
- Larger *P_e* implies larger electron momenta *p* confined to a smaller "box" Δ*x*. The entire object is smaller!



Earth compared to the white dwarf star Sirius B From NASA+ESA.

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Special Relativity & the white dwarf mass limit

- To support higher mass (smaller) white dwarfs, larger electron momenta (and speeds) are required.
- Special Relativity: electron velocities cannot exceed $c \approx 3 \times 10^{10}$ cm/s.
- Therefore, there is an upper bound (a maximum) to the mass that can be achieved by a white dwarf (Anderson 1929, Stoner 1930, Chandrasekhar 1931).
- Note: when $v \sim c$, p is not simply mv. In this extreme relativistic limit,

$$P_e = 0.123 hcn_e^{4/3}$$

▶ Note that the relativistic and nonrelativistic expressions for P_e are equal at $n_e = 10^{30} \text{ cm}^{-3}$, about the density of the core of a $0.3M_{\odot}$ white dwarf (Stoner 1930).

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Stoner-Anderson-Chandrasekhar mass ("Chandrasekhar limit")

For $P \propto \rho^{4/3}$ and gravity, the central pressure and density of a star are

$$P_c = 11 \frac{GM^2}{R^4} \qquad \qquad \rho_c = 12.9 \frac{M}{R^3}$$

Balance P_c with the relativistic electron degeneracy pressure P_e and the radius disappears from the equation (you will show this in your homework). The result is a maximum mass known as the **Stoner-Anderson-Chandrasekhar mass** or the **Chandrasekhar limit**:

$$M_{\rm SAC} = 0.2 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p = \boxed{1.44M_{\odot}}$$

for Z/A = 0.5 (carbon).

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Neutron stars

What happens in a dead star with $M > M_{SAC}$?

- Such a star simply cannot be supported by electron degeneracy pressure. Add a little too much mass and it will either collapse gravitationally or explode.
- During the collapse, the extra energy liberated from gravity, plus the high density, can help drive some endothermic nuclear reactions, notably

 $\Delta E + e^- + p \rightarrow n + \nu_e$

- But neutrons are also fermions, and neutron degeneracy pressure can also balance gravity. A neutron star is formed (Tolman 1939, Oppenheimer & Volkov 1939).
- The nonrelativistic *R*-*M* relation for a NS is

$$R = 0.0685 \frac{h^2}{Gm_p^{8/3}} M^{-1/3}$$

This is the mass of a star and the size of a **city**.

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Size of a $1M_{\odot}$ neutron star

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Pulsars

- Most of the neutron stars we observe appear to us as pulars.
- A pulsar is a rapidly rotating NS (or WD) that emits a beam of electromagnetic radiation.
- Neutron stars can also be observed as X-ray flaring members of *low-mass binary systems* (LMXBs).
- PS B1919+21 was the first pulsar to be discovered in 1967 (Hewish et al. 1968)
- Right: the center of the Crab Nebula (M1), the remnant of SN 1054.



Optical/X-ray overlay from the Hubble and Chandra

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telescopes.