# Astronomy 142 — Recitation #9

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## Formulas to remember

### Hubble's Law

Galaxies in the uniform Universal expansion

$$v_r = H_0 d \qquad H_0 = 73.04 \text{ km s}^{-1} \text{ Mpc}^{-1} = 22.4 \text{ km s}^{-1} \text{ Mly}^{-1} \tag{1}$$

### Redshift

$$z = \frac{\lambda - \lambda_0}{\lambda_0} \tag{2}$$

### SN Ia magnitude

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$$m_V^0 = M_V^0 + 5 \log\left(\frac{d}{1 \text{ Mpc}}\right) + 25 \qquad M_V^0 = -19.14 \tag{3}$$

### Black hole accretion

$$L = \frac{dE}{dt} = \eta c^2 \frac{dm}{dt} \qquad \eta \approx 0.1 \tag{4}$$

### Eddington luminosity

$$L < L_E = \frac{3GMm_p m_e^2 c^5}{2e^4} \qquad M_E > \frac{2e^4 L}{3Gm_p m_e^2 c^5} \tag{5}$$

### Superluminal motion in AGNs

$$v_{\perp,\text{apparent}} = \frac{v \sin\theta}{1 - \frac{v}{c}\cos\theta} \qquad (v_{\perp,\text{apparent}})_{\text{max}} = \frac{v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma v \tag{6}$$

## Workshop problems

**Remember!** The workshop problems that you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in ASTR 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem in some sort of bound notebook.

1. **(Group discussion)** A type Ia supernova happens when the mass of a white dwarf, accreting material from a close-by normal or giant stellar companion, approaches the Stoner-Anderson-Chandrasekhar mass, $M = 1.44 M_\odot$.

(a) If mass is added to a white dwarf, does its radius get larger, smaller, or stay the same? Is this different from what happens when mass is added to an ordinary nondegenerate star?

(b) If *heat* is suddenly added to a white dwarf — for example, along with the accreted mass — does its radius get larger, smaller, or stay the same? Is this different from what happens when heat is suddenly added to an ordinary nondegenerate star?

(c) In white dwarfs, electrons are degenerate, but nuclei behave as an ideal gas. What happens to the temperature and density of the gas of nuclei as the white dwarf accretes mass and (because of energy conservation) heat?

(d) Suppose that during this process, *additional* heat (besides that accreted) is generated within the star — for instance, by thermonuclear fusion of the nuclei. What happens to the temperature and density of the gas of nuclei? Is this different from what happens to the temperature and density when heat is created within an ordinary nondegenerate star?

(e) As a result: why does mass accretion by white dwarfs near the Chandrasekhar limit result in runaway thermonuclear deflagration and explosion, rather than stable, slow fusion power generation as in ordinary stars?

2. Consider the geometry described in class for superluminal motion. A clump within a quasar jet moves at speed $v$ along a straight trajectory at an angle $\theta$ with respect to an Earth-bound observer's line of sight. Said observer records the blob's position on the sky at two times, $t = 0$ and $t_0$.

   (a) Derive the expression for the apparent speed $v_{\perp,\mathrm{apparent}}$ of this blob in the plane of the sky, in terms of $v$ and $\theta$.

   (b) From the resulting expression, show that there is a maximum value of $v_{\perp,\mathrm{apparent}}$ for a given jet speed $v$.

3. Return to the galaxy that we studied in Question 3 on Problem Set 8. Suppose that the dark matter halo follows its given functional form out to a radius $R_1$ that is equal to ten times the scale radius $R_0$, after which it drops more rapidly and becomes negligible. Integrate the expressions for mass and light per unit area from the center out to $R = R_1 = 10R_0$, use the expression derived for $\rho_0/\mathcal{L}(0)$, and obtain a value for $M/L$ that applies to the entire galaxy. What is this value in solar units?

**Learn your way around the sky** (A feature *exclusive* of ASTR 142 recitations.) You may find the lab's celestial globes and the program Stellarium useful in answering these questions about the celestial sphere and the constellations.

4. **Secant ZA.** If you want to measure stellar magnitudes accurately while the stars rise from low altitude to high and then set back to low altitude again, you must correct for atmospheric extinction. This correction will have to be made in the RR Lyrae observing project for this class. Fortunately, this is easier than correcting for interstellar extinction:

   Suppose that the atmosphere is an infinite plane-parallel which we can assume to be uniform in density. Suppose further that the flux we would measure from a star at the zenith is $f_0$ in the absence of the atmosphere. Then the flux with the atmosphere in place is $f = f_0 e^{-\tau_0}$, where $\tau_0$ is the extinction of the atmosphere toward the zenith (expressed as an optical depth).

   (a) Suppose that the star has moved over to a zenith angle of $ZA$. If the thickness of the atmosphere is $d$, what is the length of the path through the atmosphere for the line of sight toward the star?

   (b) If the optical depth is proportional to path length through the atmosphere, what is the atmospheric optical depth toward the star with zenith angle $ZA$?

   (c) If the sky is clear and dry, the extinction must be fairly small, since we can see the stars well as they rise and set, not just when they are close to zenith. Make a first-order approximation and derive a simple formula for the flux $f$ received from a star at zenith angle $ZA$ in terms of the unextinguished flux $f_0$ and the minimum atmospheric optical depth $\tau_0$.

(d) Describe how you could, therefore, measure $f_0$ and $\tau_0$, assuming these quantities to be constant throughout the night.

**Intro to Python** (A feature *exclusive* of ASTR 142 recitations.)

5. Two stars — call them $A$ and $B$ — with declinations 28.0° and 20.0° respectively, are observed every half hour from an observatory with latitude 43.0°. The signals in "data numbers" and the zenith angles of the stars at each time are given in the data file `StarsAB.txt` on the course website. Star $A$ is a well-known, well-behaved star with time-independent magnitude 8.0. The properties of star $B$ are unknown and for you to figure out.

   (a) The atmosphere can be assumed to be a plane-parallel layer of gas, set between our location on Earth's surface and outer space. Like all gases, our atmosphere is very good at absorbing and/or scattering light. Therefore, any object that we observe will be fainter than what would be measured if we were above the atmosphere. The fraction of the light that is "removed" from the observed signal due to the atmosphere is known as the atmospheric extinction.

   Determine the minimum value of atmospheric extinction, $\tau_0$, for the provided data, assuming that it was constant throughout the night. Then, determine the signal of star $A$ corrected for atmospheric extinction (that is, the signal of star $A$ if there was no atmosphere).

   (b) Using the atmospheric extinction as a function of $ZA$ determined for star $A$, correct the observations for star $B$ for atmospheric extinction. Then, determine the magnitude of each star as seen in each observation, and plot the magnitudes as a function of time throughout the night. What have you learned about star $B$?