

# Astronomy 142 — Recitation 9

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## Formulas to remember

### Galactic cloud position from longitude and radial velocity measurements

$$v(r) = r\Omega(r) = v_{r,\max} + r_{\odot}\Omega(r_{\odot}) \sin \ell \qquad r = r_{\odot} \frac{v \sin \ell}{v_r + v \sin \ell} \qquad (1)$$

### Distribution of light in normal galaxies

Elliptical galaxies	$\mathcal{L}(r) = \mathcal{L}(0)e^{-(r/r_0)^{1/4}}$
Spiral galaxy bulges	$\mathcal{L}(r) = \mathcal{L}_B(0)e^{-(r/r_{B0})^{1/4}}$
Spiral galaxy disks	$\mathcal{L}(r) = \mathcal{L}_D(0)e^{-(r/r_{D0})}$

### Distribution of mass in normal galaxies

Elliptical galaxies	$M(r) = \frac{6rv_r^2}{G}$	$\mu(r) = \frac{1}{2\pi r} \frac{d}{dr} M(r)$
Spiral galaxies	$M(r) = \frac{v^2 r}{G}$	$\mu(r) = \frac{v^2}{2\pi Gr} + \frac{v}{\pi G} \frac{dv}{dr}$

## Leavitt's Law

Classical Cepheid variables

$$\overline{M_V} = -2.77 \log \Pi - 1.69$$
$$\overline{m_V} - \overline{M_V} = 5 \log \left( \frac{d}{10 \text{ pc}} \right)$$

## Workshop problems

**Remember!** The workshop problems that you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in ASTR 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem in some sort of bound notebook.

1. *Spiral structure in the Milky Way.* Figure 1 contains reproduced H I 21 cm line observations obtained with the Dwingeloo radio telescope when successively directed to points along the galactic equator at intervals of  $5^\circ$  in galactic longitude (van de Hulst et al. 1954). The value of Galactic longitude (in degrees) corresponding to each spectrum is obtained by adding 33 to the number with which it is labeled in Figure 1, subtracting 360 if the result exceeds 360. From the horizontal scale in Figure 1, the frequency shift and the radial velocity with respect to the local standard of rest is directly obtained. (Recall that

a small interval in frequency  $\Delta\nu$  corresponds to a small interval in wavelength given by  $\Delta\lambda = -c\Delta\nu/\nu^2$ , and that the wavelength shift due to motion of a light source with nonrelativistic radial velocity  $v$  is  $\Delta\lambda/\lambda = v/c$ .

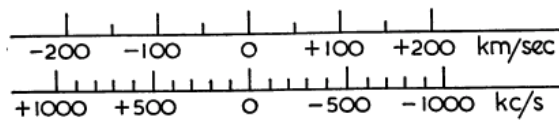
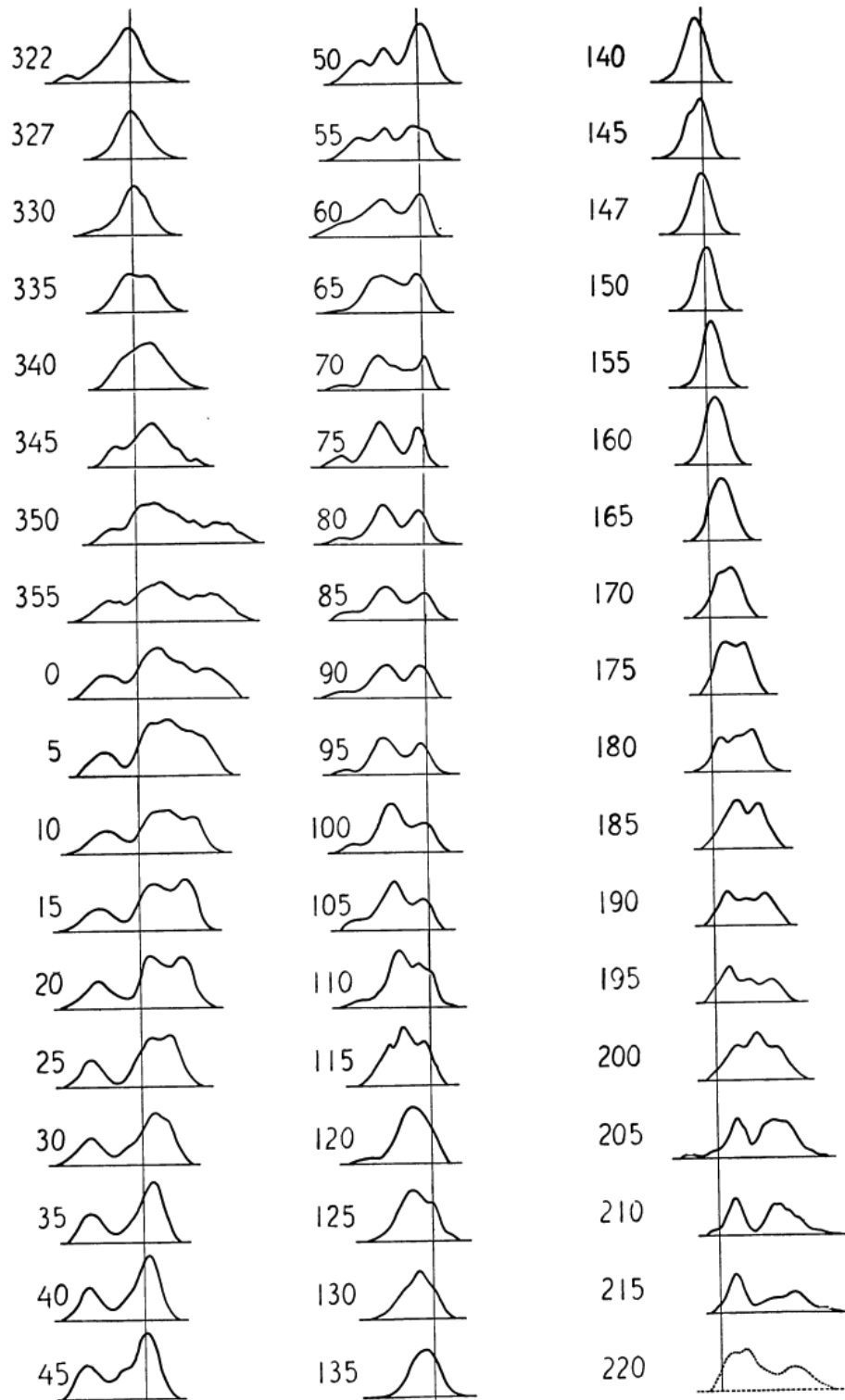
Determine the Galactocentric distances of the diffuse clouds represented by peaks in the 21 cm line spectra (Figure 1) and plot the distances as a function of angular position to reveal the spiral structure of atomic gas in the Milky Way. Assume that  $r_\odot = 8.4$  kpc and  $\Omega(r_\odot) = 9.8 \times 10^{-16}$  rad/s = 30 km/s/kpc.

1. For  $\ell = 8^\circ - 63^\circ$ , find the HI peak with the largest redshift; assume that this peak represents hydrogen at the tangent point, deduce  $v_r$ ,  $r$ ,  $v_r/r = \Omega(r) - \Omega(r_\odot)$ , and then  $\Omega(r)$ . The radial velocities can be measured on the figure with a ruler.
  2. For the other peaks in each (or every other) longitude interval, you are now in a position to consecutively calculate  $v_r$ ,  $\sin \ell$ ,  $\Omega(r) - \Omega(r_\odot)$ ,  $r$ , and then, using the law of sines,  $\theta$  (see Figure 2). Begin with the first (left) maximum of the profile at  $\ell = 88^\circ$ . This can be done rather conveniently in Excel; remember that the Excel trig functions expect the arguments to be in radians.
  3. Plot each cloud in a polar diagram representing the galactic plane. (In such plots, the Sun is usually drawn above the Galactic center so that the  $\ell = 0$  axis points down.) Remember that the distance  $r$  is measured *from the Galactic center*. You may find it helpful to do this plot in Cartesian coordinates, which in the layout of Figure 2 would be  $x = r \sin \theta$  and  $y = r \cos \theta$ .
  4. When  $r$  is found to be smaller than  $r_\odot$ , the distance ambiguity applies. This ambiguity cannot be solved without additional data and consideration. Therefore, limit your investigation to the region outside the solar orbit, with the exception of the tangent points. (You have already completed one side.) This means that for  $0 < \ell < 180^\circ$ , use only the peaks for which  $v_r < 0$ ; and for  $180^\circ < \ell < 360^\circ$ , use only the peaks for which  $v_r > 0$ .
  5. Connect the plotted positions of the hydrogen clouds by smooth lines, in so far as continuity is suggested by the successive profiles. Compare your results to the cartoon of the Galaxy's structure as is currently known (from the lecture notes). Which spiral features do you detect?
2. Look up the Two-Micron All-Sky Survey (2MASS) image of the two galaxies NGC 5194 and NGC 5195. What are their Hubble types? Explain.

**Learn your way around the sky, lesson 9.** (A feature *exclusive* of ASTR 142 recitations.) You may find the lab's celestial globes and the program Stellarium useful in answering these questions about the celestial sphere and the constellations.

3. Because of the obliquity of the ecliptic (i.e. the tilt of the Earth's equatorial plane with respect to the plane of the Solar System), the declination of the Sun changes throughout the year. As mentioned last week, the ecliptic is tilted by  $\psi = 23.44^\circ$ ; the Sun appears lowest in the sky — to a northern observer — at Winter solstice (3:27 UTC on December 22, 2023) and highest in the sky half a year later. Make an educated guess: what is the functional form of the Sun's declination as a function of time,  $\delta_\odot(t)$ ?
4. Over the past few weeks, we have learned that sidereal time does not advance at a uniform rate with respect to the mean Sun (or time measured on a well-regulated clock) owing to the eccentricity of Earth's orbit around the Sun and to the obliquity of the ecliptic. If all has gone well, you have derived the following formulas for the corrections to the sidereal time from these two effects, known together as the Equation of Time:

$$\Delta\theta_\varepsilon(t) = \frac{\Delta\omega_0}{\omega} \sin(\omega(t - t_0)) \quad \Delta\theta_o(t) = \frac{\Delta\omega_0(VE)}{2\omega} \sin(2\omega(t - t_1)) \quad (2)$$



Survey of line profiles at various longitudes.

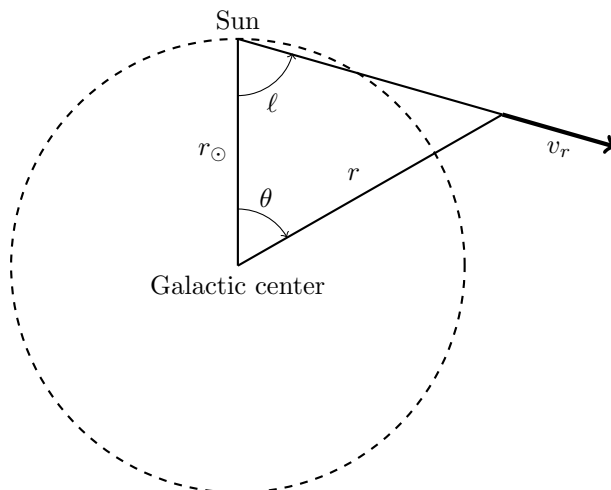


Figure 2: Geometry of longitude and LSR radial velocity measurements. The dotted circle is the orbit of the LSR about the Galactic center.

where the angles  $\Delta\theta_\varepsilon$  and  $\Delta\theta_o$  come out in radians as these equations are written, where

$$\omega = \sqrt{\frac{GM_\odot}{AU^3}} = 1.99 \times 10^{-7} \text{ rad/s}$$

$$\Delta\omega_0 = -\frac{\omega_p - \omega_a}{2} \quad \omega_p = \frac{1}{a(1 \pm \varepsilon)} \sqrt{GM_\odot \left( \frac{2}{a(1 \pm \varepsilon)} - \frac{1}{a} \right)}$$

$$\Delta\omega_0(VE) = \omega - \omega \cos \psi$$

$$t_0 = \text{Jan 4, 2023 at 16:17} = \text{day 3.6785 (perihelion)}$$

$$t_1 = \text{Mar 20, 2023 at 21:25} = \text{day 78.8924 (Vernal equinox)}$$

and where the times are given in UTC (coordinated universal time, i.e. time at 0 longitude). Plot the declination of the Sun against the total orbital correction to the sidereal time,  $\Delta\theta_\varepsilon(t) + \Delta\theta_o(t)$ , for times ranging through 2024. Arrange your plot so that it is square and so that the angular scale is the same on both axes.

5. Google “analemma,” and compare the images you find to the plot you just made. From this comparison, explain the meaning of the axes of your plot.
6. Why do copies of your plot appear on the faces of sundials and on old-fashioned globes?