Failed Stars, Stellar Remnants, Normal Stars & the Main Sequence

Brown dwarfs & Giant planets General relativity Hawking radiation Gravitational waves *L, M, T* relationships Stellar Evolution

February 20, 2025

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Failed Stars, Stellar Remnants, Normal Stars & the Main Sequence

- Brown dwarfs & giant planets
- General relativity & its prediction of black holes
- Hawking radiation
- Gravitational radiation
- Relationships among luminosity, mass, and effective temperature.
- Stellar evolution
- Changes on the main sequence
- Shell hydrogen fusion and subgiants
- Late stellar evolution: the giant branch, horizontal branch, and asymptotic giant branch



The Pleiades (M45), a 120 Myr-old stellar cluster consisting entirely of main sequence stars.

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Reading: Kutner Sec. 3.5 & 10.1, Ryden Sec. 14.4, Shu Ch. 8

Neutron star masses

- The maximum mass of a neutron star is not easy to calculate, as it requires both GR and a strong-interaction equation of state.
- The maximum mass turns out to be about $2.2M_{\odot}$ (Lattimer 2012).
- ► The maximum cannot be > 3M_☉ or the equation of state would imply a sound speed v_s > c.
- The theoretical calculation of the NS *R*–*M* relation is beyond the scope of an undergraduate course.



Far below *M*_{SAC}: Brown dwarfs and giant planets

When stars form, they **contract** until they are hot enough in the center (about 3×10^6 K) to ignite the pp chain fusion reactions. Recall that

$$egin{split} \Gamma_c &= rac{P_c \mu_c}{
ho_c k} pprox rac{\mu_c}{k} \left(rac{GM^2}{R^4}
ight) \left(rac{1}{150}rac{R^3}{M}
ight) \ &= 15.7 imes 10^6 \left(rac{M}{M_\odot}
ight) \left(rac{R_\odot}{R}
ight) \, \mathrm{K} \end{split}$$

for solar-type stars, if gravity is supported by gas pressure.

- For small masses this involves gas pressures that become smaller than the electron degeneracy pressure, so degeneracy pressure can stop the contraction and prevent the object from reaching fusion temperatures. This imposes a lower mass limit on what can become a star.
- The **H-burning limit** turns out to be $0.08M_{\odot}$.

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Far below *M*_{SAC}: Brown dwarfs and giant planets

Depending upon how they are formed and what their mass is, such objects are called either **brown dwarfs** or **giant planets**.

- Because they cannot replace the energy that leaks away in the form of radiation, they simply remain at the size determined by degeneracy pressure and cool off forever.
- Thus, if they are very old, they are very faint. This prevented their detection until 1995.
- Today, thousands are known from deep near-IR surveys and from Spitzer Space Telescope observations.
- Once it was thought that these objects could be numerous enough to comprise a significant (and invisible) component of the mass in the Galaxy.
- We will come back to this idea when we discuss **Dark Matter**.

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Degenerate star *R*–*M* relations

Gray line: causality cutoff A star smaller than $5.8\pi GM/c^2$ turns out to require an EOS with

$$\frac{\partial P}{\partial \rho} > c^2$$

which in turn implies a sound speed $v_s > c$, violating the order of cause and effect.



Degenerate star *R*–*M* relations

Below these lines, the interior of a stable star would have

- Gray line: $v_s > c$
- Dark gray line: $P > \infty$
- Black line: event horizon (would be a black hole)

In all these systems, the very large masses and small distances require a General Relativistic treatment.



Beyond the NS maximum mass: Black holes

- The maximum mass of a neutron star is ~ 2.2M_o. There is no known physical process that can support a heavier object without internal energy generation.
- A non-spinning heavier object will collapse past neutron-star dimensions and soon thereafter become a black hole, an object from which even light cannot escape if emitted within a distance

$$R_{\rm Sch} = \frac{2GM}{c^2}$$
 Schwarzschild radius

of the object as measured by a distant observer.

- This spherical surface is called the event horizon or simply the "horizon" of the black hole.
- The nonrelativistic result $R = 2GM/c^2$ is, by accident, the same as R_{Sch} derived with the general theory of relativity (GR).

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Embedding diagrams

- This is why you often see the equatorial plane of a black hole represented as a funnel-shaped surface, as if made from a stretched rubber sheet.
- It is important to note that the direction of stretch is in hyperspace.
- The scene would not look like a funnel to an observer of three spatial dimensions.



Orbits around black holes

- Orbits outside the BH's horizon, further away than 1.5R_{Sch} (in the coordinate system of a distant observer), still turn out to be ellipses.
- The resulting coordinate speed in orbit (for the coordinate system of a distant observer) is the same as that obtained in Newtonian gravity:

$$v = r \frac{d\phi}{dt} = \sqrt{\frac{GM}{r}}$$

- At the horizon, the radial component of the coordinate speed of light is zero: light cannot escape. Thus, no information can reach a distant observer from, or from within, the BH horizon.
- For non-spinning black holes, orbits with coordinate radius < 3R_{Sch} are unstable to small perturbations.

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Orbits around black holes

- There are no orbits with coordinate radius $< 1.5R_{Sch}$ for a non-spinning black hole. At this radius, the local orbital speed is the speed of light, and smaller orbits would require impossibly higher speeds.
- *You* cannot orbit at this close distance because your rest mass is nonzero; if you could, you could train your binoculars straight ahead (in the ϕ direction) and see the back of your head.
- To get closer to the horizon, the descent must be radial while balancing gravity with thrust, as in a rocket launch.
- If the black hole spins, the innermost stable orbit and the photon orbit are smaller than $3R_{\text{Sch}}$ and $1.5R_{\text{Sch}}$ if the particle orbits in the same direction as the spin, and larger if it orbits in the opposite direction.

Light and black holes

- Within $r = 1.5R_{Sch}$, all geodesics (possible paths for light) terminate at the BH horizon.
- Thus, from near the horizon, the sky appears to be compressed into a small range of angles directly overhead. The range of angles is smaller the closer one is to the horizon, and vanishes at the horizon.
- Objects in the sky will appear bluer than their natural colors as well due to the gravitational Doppler shift.
- Space itself is stuck to the horizon, since one end of all the geodesics are there.
- If the horizon began to rotate, the ends of the geodesics would rotate with it. (This harmonizes with time stopping there.)

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Gravitational forces around black holes

Gravitational acceleration turns out to be

$$a = \frac{GM}{r^2} \sqrt{rac{1}{1 - rac{R_{
m Sch}}{r}}}$$

This has the familiar Newtonian form at large *r* but blows up at $r = R_{Sch}$.

- Thus, in a vertical descent to a hovering position just above the horizon, very large gravitational accelerations would be encountered.
- Tidal forces are the same near a black hole as in Newtonian gravity, and are finite at the horizon. For an object of length Δ*r* in the radial direction and Δ*x* in the crosswise directions,

$$\Delta a_r = \frac{2GM}{r^3} \Delta r \qquad \qquad \Delta a_\phi = -\frac{GM}{r^3} \Delta x$$

February 20, 2025 (UR)

Example

For a 2 m person and a $10M_{\odot}$ BH, the radial tidal acceleration, Δa_r , at the event horizon is 2×10^{10} cm/s², or $2 \times 10^7 g$.

 $\Delta a_r = 1g$ for a $4.6 \times 10^4 M_{\odot}$ BH. Thus, if you want to fall freely past the horizon of a BH to see what happens, choose a large one so as not to be torn apart before you get there.

Hawking radiation



Hawking radiation: Black holes emit light!

- Virtual particle-antiparticle pairs created by vacuum fluctuations can be split by the strong gravity near a horizon.
- Both of the particles can fall into the horizon, but it is also possible for one to fall in while the other escapes.
- If one particle escapes it looks to a distant observer like the BH is "emitting" the particle.

Doesn't this violate energy conservation?

No. The energy conservation debt created by the un-recombined vacuum fluctuation is paid back by the BH itself; its mass decreases by E/c^2 , where *E* is the energy of the escaping particle.

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Black hole evaporation

- Hawking radiation is emitted more efficiently if the tides at the horizon are stronger. You will show in recitation that the tides at the horizon are **larger** for smaller-mass black holes.
- > The emission is the same as a **blackbody** of temperature

$$T = \frac{hc^3}{16\pi^2 kGM} \qquad \left(\propto \sqrt{\Delta a_r}\right)$$

> Thus an isolated BH will eventually **evaporate**. Some calculated evaporation times:

BH Mass	Evaporation Time
$10^9 M_{\odot}$	10 ⁹⁴ yr
$2 M_{\odot}$	10 ⁶⁷ yr
10 ⁸ g	1 sec

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