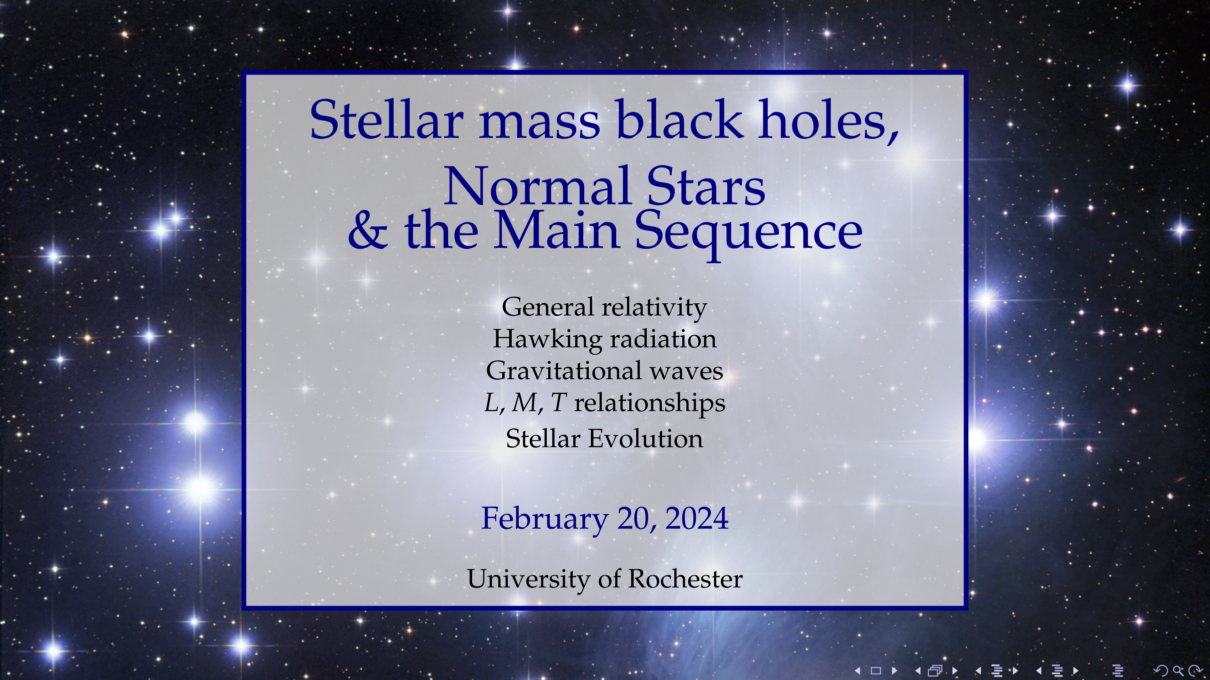

Stellar mass black holes, Normal Stars & the Main Sequence

General relativity
Hawking radiation
Gravitational waves
L, M, T relationships
Stellar Evolution

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Stellar Mass Black Holes, Normal Stars & the Main Sequence

- General relativity & its prediction of black holes
- Hawking radiation
- Gravitational radiation
- Relationships among luminosity, mass, and effective temperature.
- Stellar evolution
- Changes on the main sequence
- Shell hydrogen fusion and subgiants
- Late stellar evolution: the giant branch, horizontal branch, and asymptotic giant branch

Reading: Kutner Sec. 3.5 & 10.1, Ryden Sec. 14.4, Shu Ch. 8

The Pleiades (M45), a 120 Myr-old stellar cluster consisting entirely of main sequence stars.

Orbits around black holes

- Orbits outside the BH's horizon, further away than $1.5R_{\text{Sch}}$ (in the coordinate system of a distant observer), still turn out to be ellipses.
- The resulting **coordinate speed** in orbit (for the coordinate system of a distant observer) is the same as that obtained in Newtonian gravity:

$$v = r\frac{d\phi}{dt} = \sqrt{\frac{GM}{r}}$$

- At the horizon, the radial component of the coordinate speed of light is zero: **light cannot escape**. Thus, no information can reach a distant observer from, or from within, the BH horizon.
- For non-spinning black holes, orbits with coordinate radius $< 3R_{\text{Sch}}$ are unstable to small perturbations.

Orbits around black holes

- There are no orbits with coordinate radius $< 1.5R_{\mathrm{Sch}}$ for a non-spinning black hole. At this radius, the local orbital speed is the speed of light, and smaller orbits would require impossibly higher speeds.
- *You* cannot orbit at this close distance because your rest mass is nonzero; if you could, you could train your binoculars straight ahead (in the ϕ direction) and see the back of your head.
- To get closer to the horizon, the descent must be radial while balancing gravity with thrust, as in a rocket launch.
- If the black hole spins, the innermost stable orbit and the photon orbit are smaller than $3R_{\mathrm{Sch}}$ and $1.5R_{\mathrm{Sch}}$ if the particle orbits in the same direction as the spin, and larger if it orbits in the opposite direction.

Light and black holes

- Within $r = 1.5R_{\text{Sch}}$, all geodesics (possible paths for light) terminate at the BH horizon.
- Thus, from near the horizon, the sky appears to be compressed into a small range of angles directly overhead. The range of angles is smaller the closer one is to the horizon, and vanishes at the horizon.
- Objects in the sky will appear bluer than their natural colors as well due to the gravitational Doppler shift.
- Space itself is stuck to the horizon, since one end of all the geodesics are there.
- If the horizon began to rotate, the ends of the geodesics would rotate with it. (This harmonizes with time stopping there.)

Gravitational forces around black holes

- Gravitational acceleration turns out to be

$$a = \frac{GM}{r^2}\sqrt{\frac{1}{1-\frac{R_{\mathrm{Sch}}}{r}}}$$

  This has the familiar Newtonian form at large r but blows up at $r = R_{\mathrm{Sch}}$.
  - Thus, in a vertical descent to a hovering position just above the horizon, very large gravitational accelerations would be encountered.
- **Tidal forces** are the same near a black hole as in Newtonian gravity, and are finite at the horizon. For an object of length Δr in the radial direction and Δx in the crosswise directions,

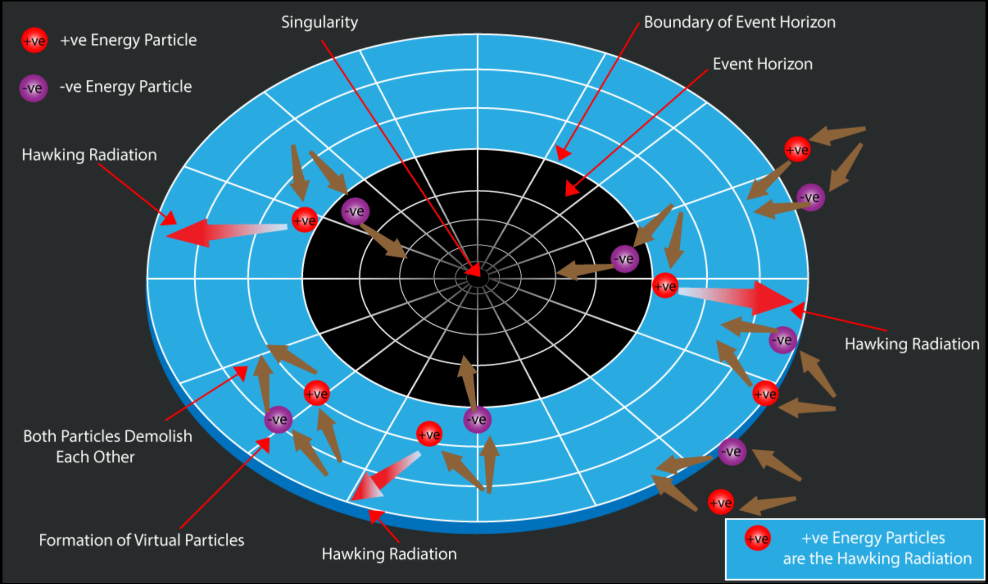
$$\Delta a_r = \frac{2GM}{r^3}\Delta r \qquad \Delta a_\phi = -\frac{GM}{r^3}\Delta x$$

Example

For a 2 m person and a $10M_\odot$ BH, the radial tidal acceleration, Δa_r, at the event horizon is 2×10^{10} cm/s^2, or $2\times10^7 g$.

$\Delta a_r = 1g$ for a $4.6\times10^4 M_\odot$ BH. Thus, if you want to fall freely past the horizon of a BH to see what happens, choose a large one so as not to be torn apart before you get there.

Hawking radiation



Hawking radiation: Black holes emit light!

- Virtual particle-antiparticle pairs created by vacuum fluctuations can be split by the strong gravity near a horizon.
- Both of the particles can fall into the horizon, but it is also possible for one to fall in while the other escapes.
- If one particle escapes it looks to a distant observer like the BH is “emitting” the particle.

Doesn’t this violate energy conservation?

No. The energy conservation debt created by the un-recombined vacuum fluctuation is paid back by the BH itself; its mass decreases by E/c^2, where E is the energy of the escaping particle.

Black hole evaporation

- Hawking radiation is emitted more efficiently if the tides at the horizon are stronger. You will show in recitation that the tides at the horizon are **larger** for smaller-mass black holes.
- The emission is the same as a **blackbody** of temperature

$$T = \frac{hc^3}{16\pi^2 kGM} \qquad \left(\propto \sqrt{\Delta a_r}\right)$$

- Thus an isolated BH will eventually **evaporate**. Some calculated evaporation times:

BH Mass	**Evaporation Time**
$10^9 M_\odot$	10^{94} yr
$2M_\odot$	10^{67} yr
10^8 g	1 sec

No-hair theorem

After a star has collapsed into a BH, the horizon is smooth. Nothing protrudes from it, and almost everything about the star that gave rise to the BH has lost its identity during the formation of the BH.

Metaphor: no “hair” (information) is left to “stick out” of the horizon.

- Any protrusion, prominence, or other departure from spherical smoothness gets turned into gravitational radiation, i.e., it is radiated away during the collapse.
- Any magnetic field lines emanating from the star close up and get radiated away in the form of light during the collapse.
- The identity of the matter that made up the star is lost. Nothing about its previous configuration can be reconstructed.
- Even the matter/antimatter distinction is lost. Two stars of identical mass — one of matter and one of antimatter — would produce identical black holes.

No-hair theorem

The black hole has only three quantities in common with the start that collapsed to create it: **mass**, **spin**, and **electric charge**.

- That is, in common with the star as it was *immediately before the formation of the horizon*.
- Only very tiny black holes can have much electric charge. Stars are electrically neutral.
- Spin makes the black hole depart from a spherical shape, but it is still smooth.

From an observational standpoint, the bare BH is a lot like a giant elementary particle with only a few measurable properties.

GRO J1655-40: A real stellar mass black hole

We know of ∼70 good candidates for stellar-mass black holes, of which at least 50 are rock-solid cases. **GRO J1655-40**, an X-ray binary in Scorpius, is one example of rock-solid evidence for a black hole.

- GRO J1655-40 is a bright, soft X-ray transient (flaring) object discovered by the *Compton* γ-ray Observatory in 1994.
- It appeared as a nova (Nova Scorpii 1994) in visible light during the X-ray burst.
- It produced another optical/X-ray outburst in 2005.
- Normal stars, unless very young, and ordinary novae do not emit much light at X-ray wavelengths.
- To get electric charges to emit X-rays, they need to be accelerated close to c. If done with gravity, this requires a neutron star or a black hole.
- When bursting, GRO J1655-40 exhibits rapidly variable X-ray emission: the brightness changes by a factor of 2 in $\Delta t \approx 0.3$ ms. Implications:
  1. The object is at most 100 km across, which would give a light transit time of 0.3 ms.
  2. 100 km is far too small to be a star or a white dwarf.

GRO J1655-40: A real stellar mass black hole

- When it is not bursting, the source looks like a normal star, rather similar to the Sun (V1033 Sco, $m_1 = 1.1 M_\odot$).
  - The star's brightness indicates a distance of 3.2 kpc.

The star's spectral lines show it to be a **single-line spectroscopic binary** system: star and invisible companion in orbit.

- So the X-ray bright object is the invisible companion.
- Its period $P = 2.62$ days and $v_{1r} = 216$ km/s.
- Thus, the mass function (recall PS2) is

$$f(m_1, m_2) = \frac{P v_{1r}^3}{2\pi G} = 2.7 M_\odot < m_2$$

Companion exceeds max NS mass?

The star is eclipsed when the system is in outburst but not when it is quiescent, so we view the orbit not exactly edge-on (70°). Thus we know the mass of the X-ray bright companion precisely (Shahbaz 2003):

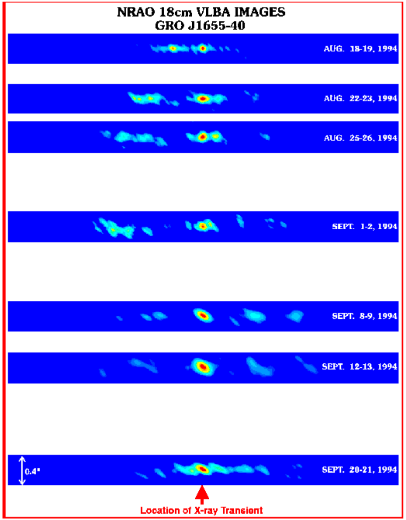
$$m_2 = 5.99 \pm 0.42 M_\odot$$

Companion definitely exceeds max NS mass

GRO J1655-40: A real stellar mass black hole

- GRO J1655-40 emits relativistic bipolar “jets” of material, and the speed can be measured from the proper motion of the clumps in the jets (Hjellming 1995).
- The outflow speed is $0.92c$, orientation is 85° from the line of sight, and 15° from the system rotation axis.
- The shapes of the clumps indicate that the jet is precessing with a period similar to the orbit.

Ejection speeds tend to be similar to escape speeds.
Nothing but a BH would eject material at $0.92c$.



GRO J1655-40: A real stellar mass black hole

A $6M_\odot$ non-spinning black hole has a horizon circumference of 111 km and an innermost stable orbit of 333 km. Material in this orbit will circle the black hole at 367 Hz.

- However, the X-ray brightness of GRO J1655-40 is often seen at 450 Hz, not 367 Hz, for long stretches of time (Strohmayer 2001).
- This behavior is called **quasiperiodic oscillation**.
- Nothing besides very hot material in a stable orbit can do this so reproducibly at this frequency.
- Thus there are stable orbits closer to the black hole than they can be if it does not spin.
- The maximum spin rate of a BH corresponds to a coordinate speed at the horizon of c. GRO J1655-40 spins at 21% of this rate.

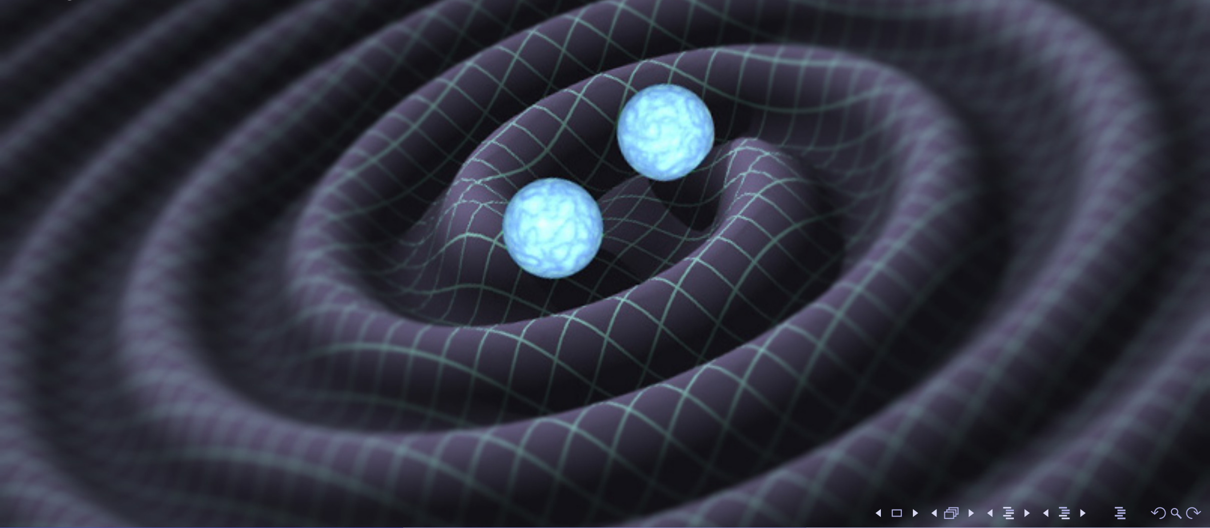
GRO J1655-40: A real stellar mass black hole

Thus we have many lines of evidence that GRO J1655-40 is a binary system consisting of a $1.1M_\odot$ main-sequence star and a $6.0M_\odot$ black hole. Until 2016, this was as solid as such a case gets. Let us review the evidence:

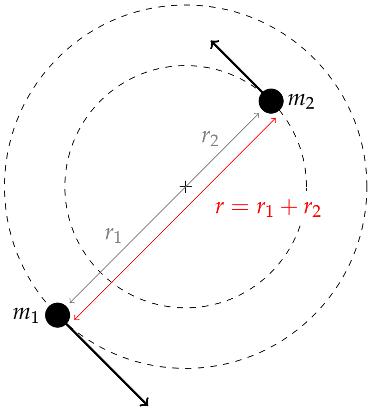
- High-energy radiation (X-rays)
- "Variability size": X-ray object too small to be a star
- Orbital dynamics: mass of dark companion is precisely determined and far too large to be a neutron star or white dwarf.
- It is too faint to be an ordinary $6M_\odot$ star at 3.2 kpc.
- Quasiperiodic X-ray oscillations are easily explained as due to spin in a BH.
- Relativistic jets are ejected from the system at nearly light speed.

Gravitational radiation from black hole mergers

Cartoon of the spacetime distortion created by two compact objects (neutron stars) in a close binary orbit. Image from LIGO.



Gravitational radiation from a binary system



GW luminosity of a binary system:

$$L_{\mathrm{GW}} = \frac{32}{5} \frac{G^4}{c^5} \frac{(m_1 m_2)^2 (m_1 + m_2)}{r^5}$$

Example

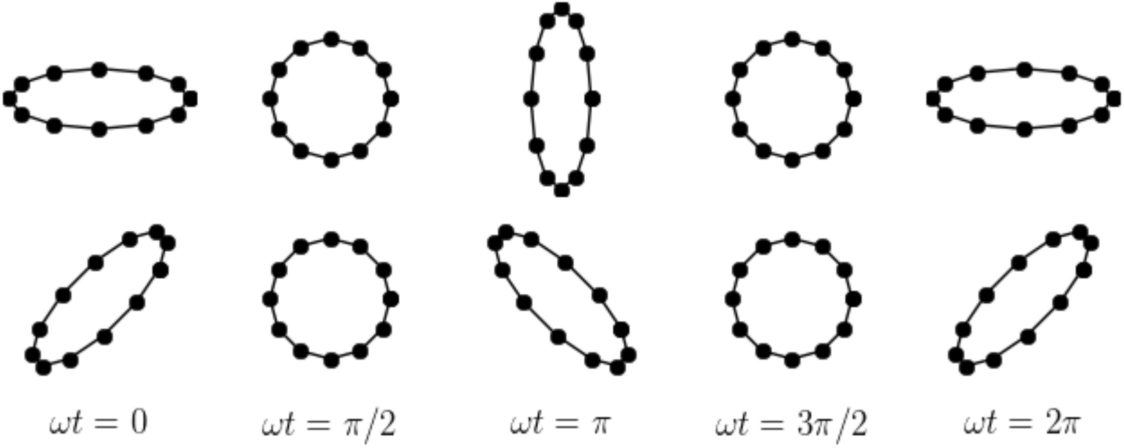
$m_1 = m_2 = 5M_\odot$ and $r = 1$ AU:

$$L_{\mathrm{GW}} = 1.4 \times 10^{24} \mathrm{erg\ s}^{-1} \approx 3 \times 10^{-10} L_\odot$$

But if $r = R_\odot$, then

$$L_{\mathrm{GW}} = 6.4 \times 10^{35}\ \mathrm{erg\ s}^{-1} = 165 L_\odot$$

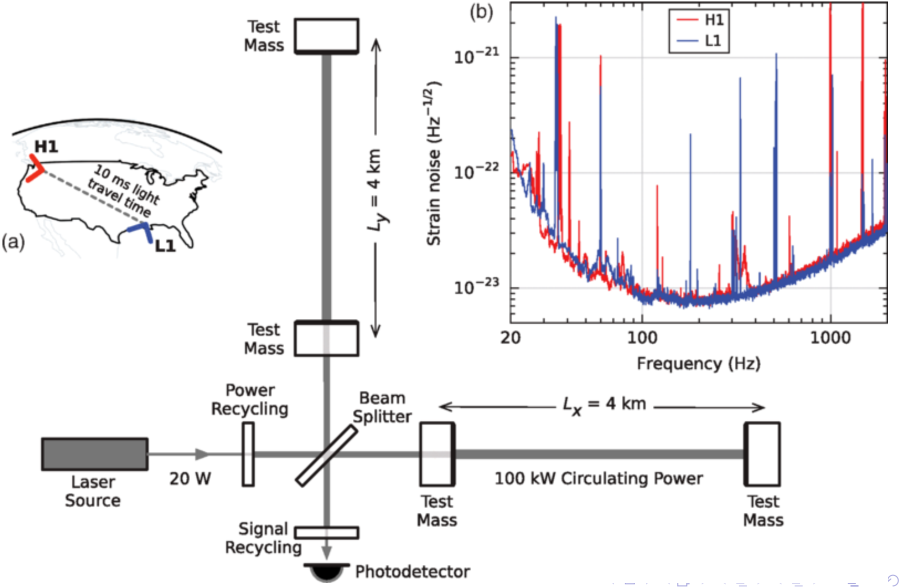
Properties of gravitational waves (GWs)



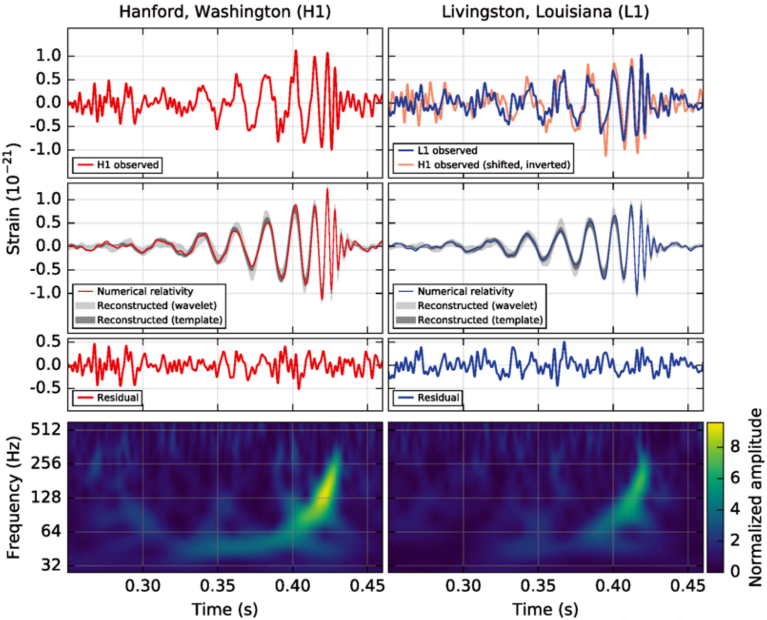
Effect of GWs on a ring of test particles (Li 2014).

Detecting gravitational waves

Diagram of a LIGO detector (Abbott et al. 2016).



Black hole merger signal



Observation of a BH-BH merger at the two LIGO sites, September 14, 2015 (Abbott et al. 2016).

The rise in frequency as a function of time (the "chirp") is characteristic of a binary inspiral.

Black hole merger signal

The estimated strain of the GW signal, the BH-BH separation, and the relative velocity of the merger estimated from numerical GR (Abbott et al. 2016).

Note the **close separation** and **highly relativistic velocities** of the BHs before the merger and "ringdown."

Simulation of a BH-BH merger