Astronomy 142 — Recitation #10

Prof. Kelly Douglass

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Formulas to remember

Galaxy distribution in clusters

Number per unit area on the sky

$$\mathcal{N}(r) = \mathcal{N}_0 e^{-(r/r_0)^{-1/4}} \tag{1}$$

Relaxation time

$$t_c \cong \left(\frac{2r_0}{v}\right) \frac{N}{24\ln\left(\frac{N}{2}\right)} \tag{2}$$

Thermal and escape speeds

$$v_{\rm th} = \sqrt{\frac{3kT}{m_{\rm H}}}$$
 $v_{\rm esc} = \sqrt{\frac{2GM}{r_0}}$ (3)

Virial mass

Of a thermalized cluster

$$M = \frac{2r_0\overline{v^2}}{G} = \frac{6r_0\overline{v_r^2}}{G} \tag{4}$$

Mass from Keplerian orbits

$$M = \frac{rv^2}{G} \tag{5}$$

Atmospheric extinction correction

$$f = f_0 e^{-\tau} = f_0 e^{-\tau_0 \sec ZA}$$

\$\approx f_0(1 - \tau_0 \sec ZA) = f_0 - f_0 \tau_0 \sec ZA\$

Observed temperature of a distant blackbody

$$T_0 = T \frac{R(t)}{R(t_0)} = \frac{T}{1+z}$$
(6)

Robertson-Walker absolute interval

$$ds^{2} = c^{2}dt^{2} - R^{2}(t) \left[\frac{dr_{*}^{2}}{1 - kr_{*}^{2}} + r_{*}^{2}d\theta^{2} + r_{*}^{2}\sin^{2}\theta d\phi^{2} \right]$$
(7)

Scale factor

$$Rr_* = r$$
 $R_0 = R(t_0)$ (today) $\frac{R(t)}{R_0} = \frac{1}{1+z}$ (8)

Friedmann equation

Neglecting radiation, so only good to $z \sim 3600...$

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G}{3}\rho_m - \frac{\Lambda}{3} = -k\frac{c^2}{R^2} \tag{9}$$

Critical density

(present value)

$$u_{c_0} = \frac{3c^2 H_0^2}{8\pi G} = 9.3 \times 10^{-9} \text{ erg/cm}^3$$
(10)

Normalized densities, today

Neglecting radiation, to good approximation

$$\Omega_{M_0} = \frac{8\pi G \rho_{M_0}}{3H_0^2} \qquad \Omega_{\Lambda_0} = \frac{\Lambda}{3H_0^2} \qquad k = \frac{H_0^2 R_0^2}{c^2} \left(\Omega_{M_0} + \Omega_{\Lambda_0} + 1\right) \tag{11}$$

Workshop problems

Remember! The workshop problems that you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in ASTR 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem in some sort of bound notebook.

- 1. The Coma Cluster contains about 10,000 galaxies, has a core radius r_0 of about 3 Mpc, and has a radialvelocity dispersion $\sqrt{\overline{v_r^2}}$ of 977 km/s. Typically, the luminosity of the cluster galaxies is $L_g = 5 \times 10^8 L_{\odot}$. The cluster also contains $M_{\text{X-ray}} = 3 \times 10^{14} M_{\odot}$ in gas at temperature $T = 10^8$ K.
 - (a) Calculate the mass in galaxies (using the results of Problem ?? and the virial mass for the Coma Cluster. Compare these results to the mass of X-ray-emitting gas. What fraction of the cluster's mass is dark matter?
 - (b) Calculate the crossing time and the relaxation time for the cluster, and compare these with the age of the Universe (14 billion years). Has the cluster had time to come to virial equilibrium in the usual manner, by numerous elastic collisions of the galaxies?
 - (c) Calculate the thermal speed of hydrogen in the hot X-ray-emitting gas, the escape speed from the sum of the galaxy and X-ray masses, and the escape speed from the virial mass. Is the presence of hot X-ray-emitting gas a strong argument for the presence of substantial dark matter on its own?
- 2. So far, we know of three different forms of the absolute interval between events: that for flat spacetime (special relativity), which in Cartesian form is

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

For the surroundings of the horizon of a nonspinning black hole of mass M, the absolute interval in the spherical coordinates of a distant observer is

$$ds^{2} = \left(1 - \frac{2GM}{rc^{2}}\right)c^{2}dt^{2} - \frac{dr^{2}}{1 - \frac{2GM}{rc^{2}}} - r^{2}d\theta^{2} - r^{2}\sin^{2}\theta d\phi^{2}$$

The absolute interval that corresponds to the Robertson-Walker metric in stretchy-spherical-grid coordinates is

$$ds^{2} = c^{2}dt^{2} - R^{2}\left(\frac{dr_{*}^{2}}{1 - kr_{*}^{2}} + r_{*}^{2}d\theta^{2} + r_{*}^{2}\sin^{2}\theta d\phi^{2}\right)$$

- (a) Consider two events consisting of ticks on a single given clock. What is the algebraic sign of ds^2 between these two events in each of the absolute intervals given above? (This would be called a timelike interval.)
- (b) Consider two events consisting of positions in space with separation measured by a ruler. What is the algebraic sign of ds^2 between these two events in each of the absolute intervals given above? (This would be called a spacelike interval.)
- (c) The form of the absolute interval in the vicinity of a mass-density singularity is very complicated, but it has one particularly striking property: $ds^2 < 0$ under all conditions. (Make sure that you see this demonstrated some day, in a GR class, or by reading the works of Belinskii, Khalatnikov, and Lifshitz, for example in *Adv. Phys* 31, 639–667 (1982).) Is there such a thing as a timelike interval near a singularity? Is there such a thing as time (as we know it) near a mass-density singularity?
- (d) What happened before the Big Bang?
- 3. Using special relativity, demonstrate that light moves along paths with $ds^2 = 0$. (Such paths are called **null geodesics**.)