Stars, Stellar Clusters & Stellar Evolution

Stellar Evolution
Open and globular stellar clusters

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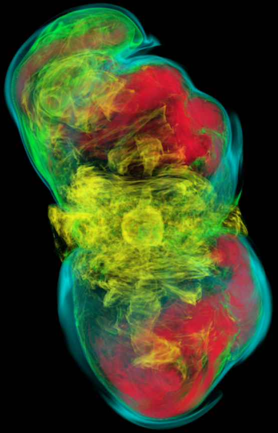
University of Rochester

Stars, Stellar Clusters & Stellar Evolution

- Stellar evolution
- Changes on the main sequence
- Shell hydrogen fusion and subgiants
- Late stellar evolution: the giant branch, horizontal branch, and asymptotic giant branch
- Evolution of high mass stars: the iron catastrophe
- Type II (core-collapse) supernovae
- Open and globular clusters as stellar clocks

Reading: Kutner Ch. 11.1 & 13; Ryden Sec. 14.2–14.3, 17.2, & 18.4; Shu Ch. 8 & 9



Supernova progenitor simulation (Mosta et al. 2014). Colors indicate entropy.

Back to “live” stars

Let us scale our results on stellar structure and luminosity to normal stars of all masses.

As usual, we will work in terms of scaling relations, e.g.,

$$P_c \propto \frac{M^2}{R^4} \qquad \text{instead of} \qquad P_c = 19\frac{GM^2}{R^4}$$

because in the end we will express everything in terms of solar parameters and all the constants will cancel out. For example:

$$\frac{P_c(M)}{P_{c,\odot}} = \frac{19GM^2/R^4}{19GM_\odot^2/R_\odot^4} = \left(\frac{M}{M_\odot}\right)^2\left(\frac{R}{R_\odot}\right)^{-4}$$

$$\implies P_c = P_{c,\odot}\left(\frac{M}{M_\odot}\right)^2\left(\frac{R}{R_\odot}\right)^{-4}$$

The theoretical luminosity–mass relation

Start from the mean free path ℓ of a photon produced at the center of a star. As we have seen, in terms of the number of steps N and the time t,

$$N = \frac{3R^2}{\ell^2} \qquad t = \frac{N\ell}{c} = \frac{3R^2}{\ell c}$$

The average photon mean free path in the Sun is $\ell = 0.5$ cm. How does ℓ scale with average temperature and density?

It turns out this is quite complex (ASTR 453) to show. Skipping to the answer:

$$\ell \propto \begin{cases} T^{3.5}\overline{\rho}^{-2} & \text{if } M \lesssim 1M_\odot \\ \overline{\rho}^{-1} & \text{if } M \gtrsim 1M_\odot \end{cases}$$

where $\overline{\rho} = \frac{3M}{4\pi R^3}$

The theoretical luminosity–mass relation

This result tells us how luminosity scales with temperature (recall u_r from a previous lecture):

$$\begin{aligned} L &= \frac{u_r V}{t} \qquad (u_r = \text{radiation energy density}) \\ &\approx \left(\frac{4}{c}\sigma T^4\right)\left(\frac{4}{3}\pi R^3\right)\left(\frac{\ell c}{3R^2}\right) \\ &\propto \ell R T^4 \end{aligned}$$

Hydrostatic equilibrium and ideal-gas pressure support the star against its own weight, implying that

$$\begin{aligned} P &\propto \frac{GM^2}{R^4} \text{ and } P \propto \overline{\rho} T \\ T &\propto \frac{P}{\overline{\rho}} \propto \frac{GM^2}{R^4}\frac{R^3}{M} \propto \frac{M}{R} \end{aligned}$$

The theoretical luminosity–mass relation

Thus, for low-mass stars ($M \lesssim 1 M_\odot$):

$$L \propto \ell R T^4 \propto \frac{T^{3.5}}{\overline{\rho}^2} R T^4 \propto \left(\frac{M}{R}\right)^{3.5} \left(\frac{R^3}{M}\right)^2 R \left(\frac{M}{R}\right)^4$$

$$\propto \frac{M^{5.5}}{R^{0.5}}$$

For higher-mass stars ($M \gtrsim 1 M_\odot$),

$$L \propto \ell R T^4 \propto \frac{1}{\overline{\rho}} R T^4 \propto \left(\frac{R^3}{M}\right) R \left(\frac{M}{R}\right)^4$$

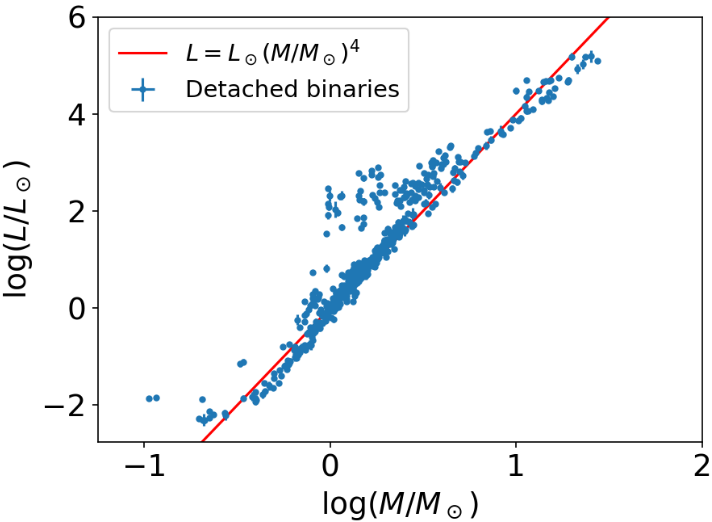
$$\propto M^3$$

Compromise:

$$L \propto M^4 \Longrightarrow \frac{L}{L_\odot} = \frac{M^4}{M_\odot^4} \Longrightarrow \boxed{L = L_\odot \left(\frac{M}{M_\odot}\right)^4}$$

Comparison to experiment: L vs. M

From Malkov 2007



The lifetimes of stars

Note how fast luminosity increases with increasing mass. Because of this, the more massive the star, the shorter its lifetime.

The pp-chain fusion energy supply is $\Delta E \approx 0.03Mc^2 \propto M$.

$$L \propto M^4$$

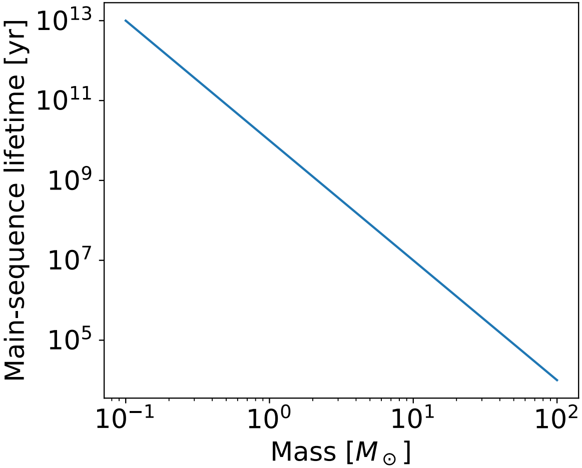
Thus

$$\tau = \frac{\Delta E}{L} \propto M^{-3}$$

or

$$\frac{\tau}{\tau_\odot} = \frac{M_\odot^3}{M^3}$$

$$\boxed{\tau \approx 10^{10}\ \text{yr}\left(\frac{M}{M_\odot}\right)^{-3}}$$



Theoretical $R - M$ and $T - M$ relations

Fusion reactions comprise a sort of thermostat; the temperature in the interior slowly varies with M and R in a main sequence star, as we have previously seen.

Take T to be approximately constant within a given star. Since $T \propto M/R$,

$$R \propto M \implies \boxed{R = R_\odot \left(\frac{M}{M_\odot}\right)}$$

For temperature, recalling that $L = 4\pi R^2 \sigma T_e^4$ and $L \propto M^4$ (our "compromise"),

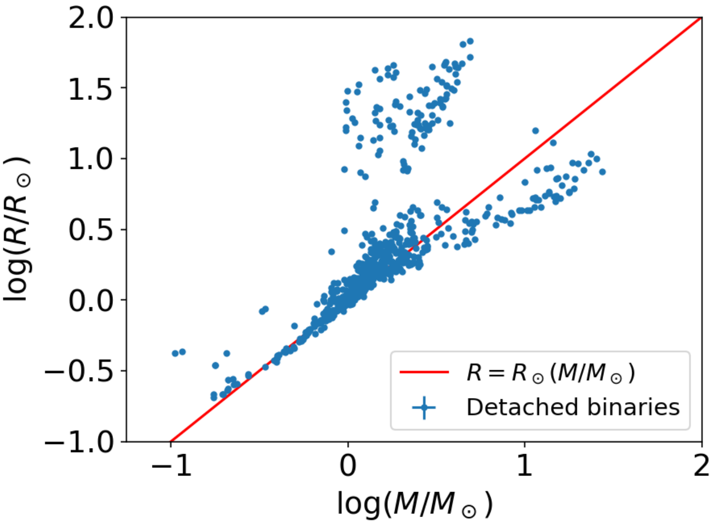
$$4\pi R^2 \sigma T_e^4 \propto M^4$$

$$M^2 T_e^4 \propto M^4 \implies T_e \propto M^{1/2}$$

$$\boxed{T_e = T_{e,\odot} \left(\frac{M}{M_\odot}\right)^{1/2}}$$

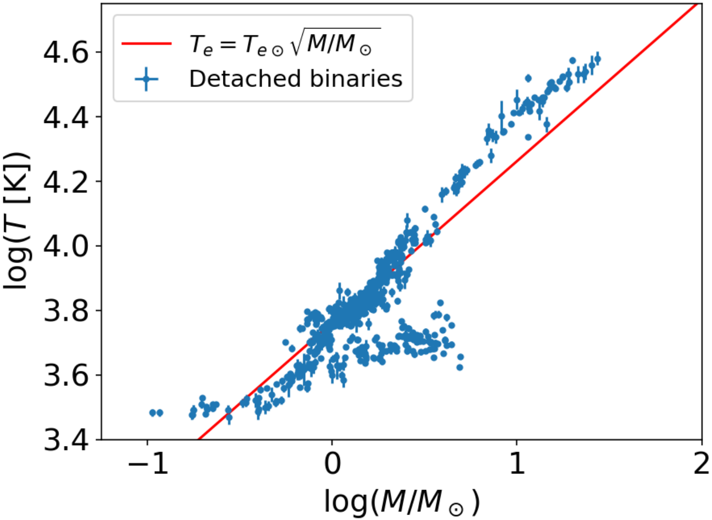
Comparison to experiment: R vs. M

From Malkov 2007



Comparison to experiment: T_e vs. M

From Malkov 2007



Theoretical main sequence in the $L - T_e$ relation (H-R diagram)

Combine the $T - M$ relation with the scaling rule $L \propto M^4$:

$$L \propto M^4 \propto T_e^8$$

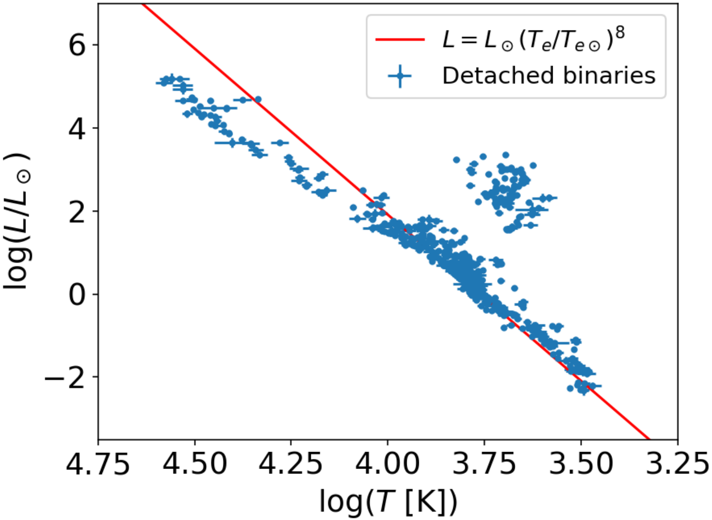
$$\boxed{L = L_\odot \left(\frac{T_e}{T_{e,\odot}} \right)^8}$$ The main sequence

All of these results are in reasonable agreement with data, which indicates that we have included most of the relevant physics in our discussions.

In fact, very detailed models need to be built to do better. This is covered in ASTR 453, a graduate-level course, not ASTR 241. But, you can see a realistic 1D stellar evolution code at docs.mesastar.org.

Comparison to experiment: L vs. T_e

From Malkov 2007



Mean molecular weight

For pure ionized hydrogen,

$$\mu = \frac{m_p + m_e}{2} \approx 0.5 m_p$$

For pure ionized helium,

$$\mu = \frac{3.97 m_p + 2 m_e}{3} \approx 1.32 m_p$$

In general we express the molecular weight in terms of the mass fraction X of hydrogen, the mass fraction Y of helium, and the mass fraction Z of everything else ("metals"). For a fully ionized gas,

$$\frac{m_p}{\mu} \approx 2X + \frac{3}{4}Y + \frac{1}{2}Z$$

For example, the mean molecular weight for ionized gas with $X = 0.70$, $Y = 0.28$, and $Z = 0.02$ (the abundances found on the Solar surface) is

$$\mu = 0.62 m_p$$

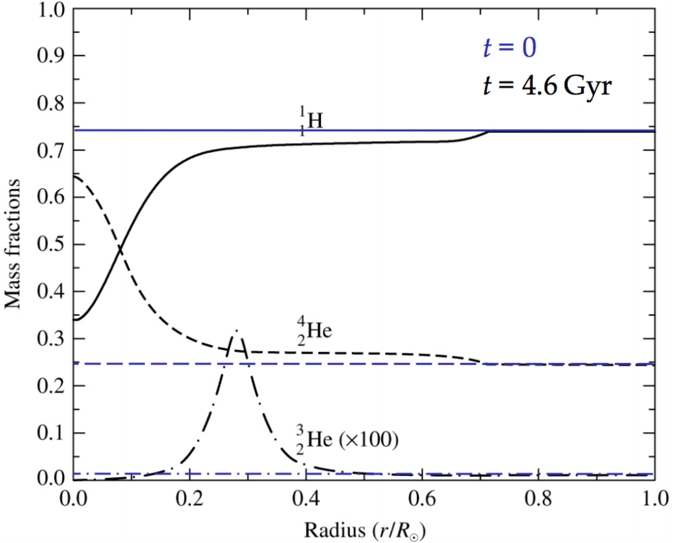
Stellar evolution on the Main Sequence

As hydrogen burns in the stellar core, fusing into heavier elements, the mean molecular weight of a star slowly increases.
In the center of the Sun today,

$$\mu = 1.17 m_p$$

At a given temperature, the ideal gas law says this would result in a lower gas pressure and less support for the star's weight:

$$P = nkT = \frac{\rho k T}{\mu}$$

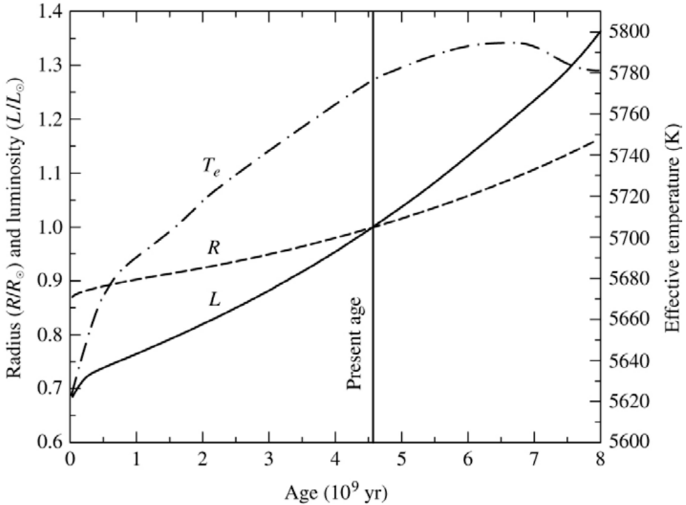


From "An Introduction to Modern Astrophysics" by Carroll and Ostlie.

Stellar evolution on the Main Sequence

Therefore, as time goes on:

- The core of the star slowly contracts and heats up.
- The radius and effective temperature of the star slowly increase in response to the new internal temperature and density distribution.
- The luminosity slowly and slightly increases in response to the increase in radius and effective temperature.



From "An Introduction to Modern Astrophysics" by Carroll and Ostlie.

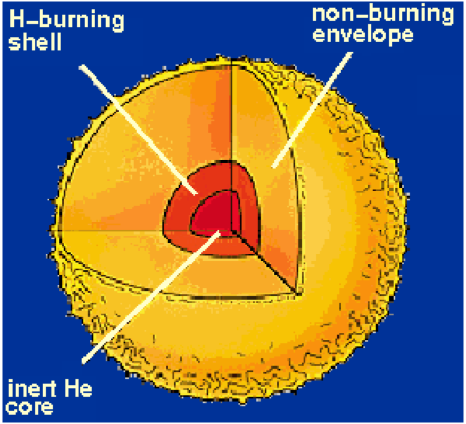
Shell hydrogen burning & the subgiant phase

Eventually, a star will exhaust the hydrogen at the very center.

The temperature is insufficient to ignite helium fusion but is high enough just outside the center for a shell of hydrogen fusion to provide support for the star. Thus,

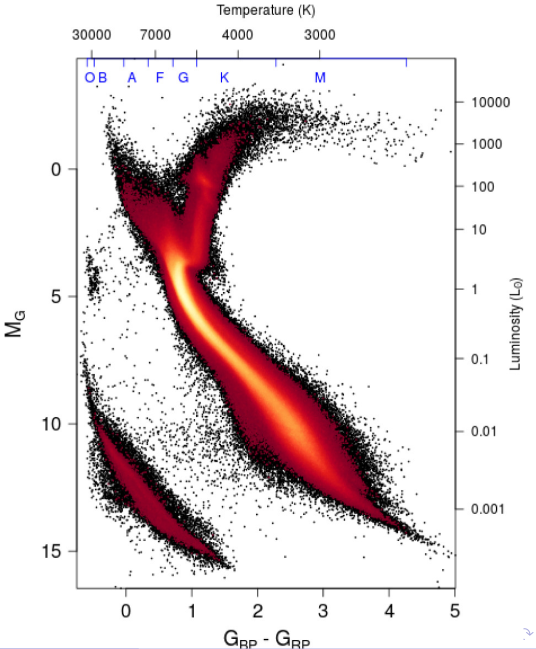
- T is nearly constant in the core (**isothermal helium core**), which keeps increasing in mass due to hydrogen depletion.
- There is increased luminosity and further expansion of the envelope of the star.
- There is a decrease in effective temperature.

This is called the **subgiant phase**. The star moves off the main sequence, upwards and to the right on the H-R diagram.



Observational H-R diagram

- Observational H-R diagram showing absolute magnitude vs. color index.
- Made using distances from the Gaia mission



Degeneracy in the isothermal core

The subgiant phase ends when the mass of the isothermal He core becomes too great for support of the star.

- Reason for a maximum weight that can be supported by pressure in the core: **electron degeneracy pressure**.
- The core is like a white dwarf, except with additional external pressure.
- The maximum mass in the core (Schonberg & Chandrasekhar 1942) is

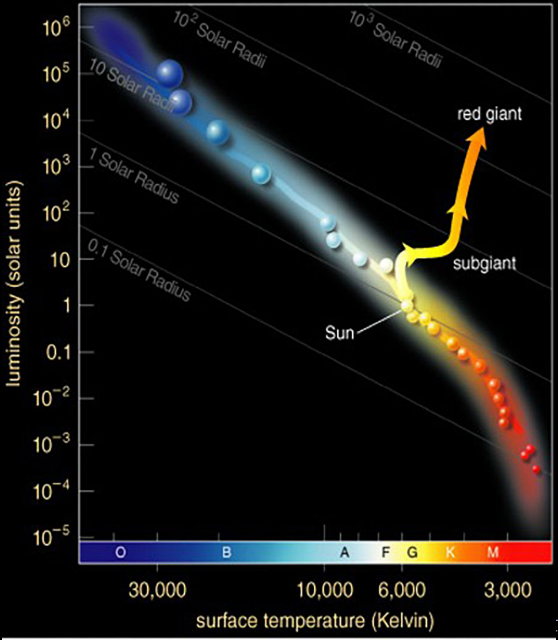
$$\begin{aligned}\frac{M_{\text{iso. core}}}{M_{\text{total}}} &= 0.37\left(\frac{\mu_{\text{envelope}}}{\mu_{\text{iso. core}}}\right)^2 \\ &\approx 0.37\left(\frac{0.62}{1.32}\right)^2 \\ &= 0.08 \text{ for the Sun}\end{aligned}$$

At this point comes the **red giant phase**.

The red-giant phase

After the maximum mass of the core is exceeded, the star moves up and to the right in the H-R diagram.

- The core collapses, causing a rise in ρ_c and T_c.
- The convection zone extends inward ("dredge-up").
- Stellar radius dramatically increases due to the increase in radiation pressure from the interior, which now dominates support against gravitational collapse.



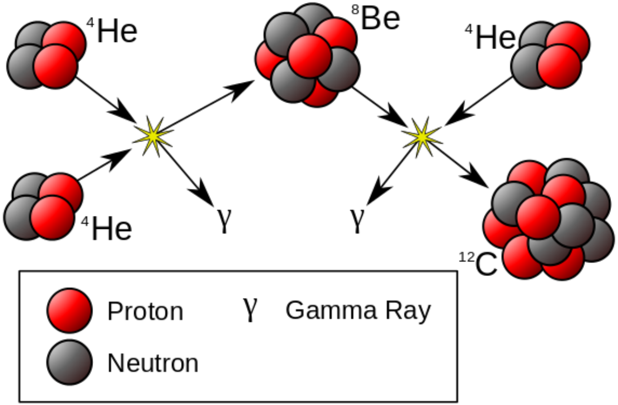
The triple-α process

The core temperature reaches 10^8K and the triple-α process

$$3\,{}^4_2\mathrm{He} \rightarrow {}^8_4\mathrm{Be}^* + {}^0_0\gamma + {}^4_2\mathrm{He} \rightarrow {}^1 2_6\mathrm{C} + 2\,{}^0_0\gamma$$

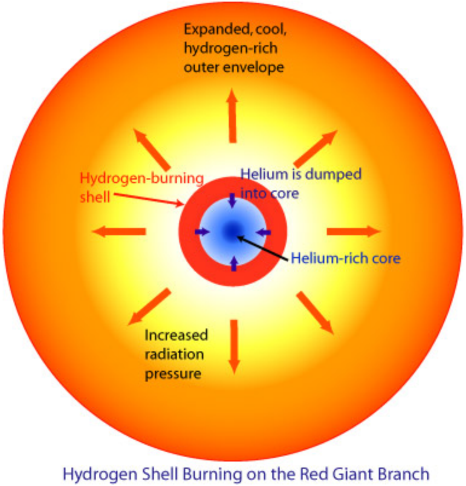
begins helium fusion. The onset is very rapid in stars with $M \geq 2M_\odot$, leading to a phenomenon called the helium flash.

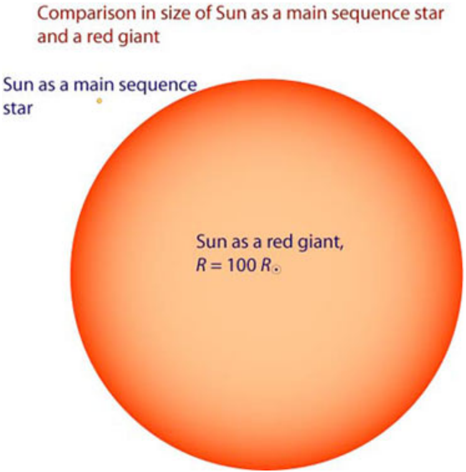
The half-life of ^{8}Be is 10^{-16} s, at which point it decays back into two ^{4}He nuclei unless it interacts with a third ^{4}He nucleus to form ^{12}C. Some of the ^{12}C will then fuse with another ^{4}He to form ^{16}O.



A $1M_\odot$ star as a Red Giant

Red giant structure: extremely dense He core and extremely tenuous and cool H envelope ($\rho = 10^{-4}$ g/cm^3).



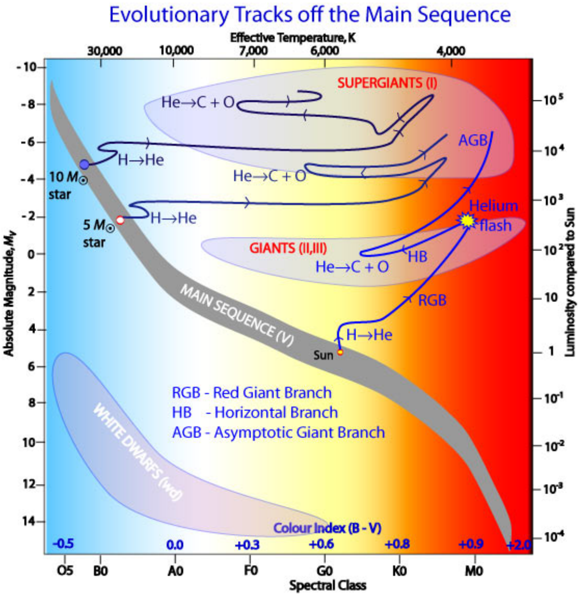


Images from ATNF.

Late stages of stellar evolution

- The **horizontal branch** is the phase after triple-α onset.
- Interior: core He burning, shell H burning. The core is on the **He main sequence**.
- Note the differences and similarities between a $1M_{\odot}$, $5M_{\odot}$, and $10M_{\odot}$ star.

Diagram from ATNF.



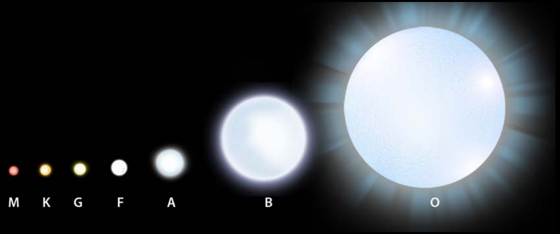
Nomenclature: Spectral type

The “Harvard” spectral type of a star is a classification based on the strength of absorption lines of several molecules, atoms, and ions (Cannon & Pickering 1901).

- Generally, types A–M correspond to steady decreases in the strength of hydrogen lines. Type O is weaker still.
- Type P for planetary nebulae and Q for miscellaneous. N not used.
- In 1925, Cecilia Payne showed that the spectral type sequence OBAFGKM is a sequence of **decreasing temperature** from roughly 35,000K to 3,500K.
- Numbers 0–9 add a further refinement of temperature within the spectral classes (0 = hottest, 9 = coolest).
- L, T, and Y recently added for cooler brown dwarfs.

Examples: Vega is an A0 star; the Sun is a G2 star; Pollux is a K2 star; Betelgeuse is an M2 star.

Harvard spectral classification



Class	T_e [K]	M [$M_\odot$]	R [$R_\odot$]	L [$L_\odot$]	Frac. MS Pop. [%]
O	≥ 30000	≥ 16	≥ 6.6	≥ 30000	0.00003
B	10000 – 30000	2.1 – 16	1.8 – 6.6	25 – 30000	0.13
A	7500 – 10000	1.4 – 2.1	1.4 – 1.8	5 – 25	0.6
F	6000 – 7500	1.04 – 1.4	1.15 – 1.4	1.5 – 5	3
G	5200 – 6000	0.8 – 1.04	0.96 – 1.15	0.6 – 1.5	7.6
K	3700 – 5200	0.45 – 0.8	0.7 – 0.96	0.08 – 0.6	12.1
M	2400 – 3700	0.08 – 0.45	≤ 0.7	≤ 0.08	76.45

Table of spectral classes O through M for the Main Sequence. From wikipedia. See also Habets & Heintze (1981) and Ledrew (2001).

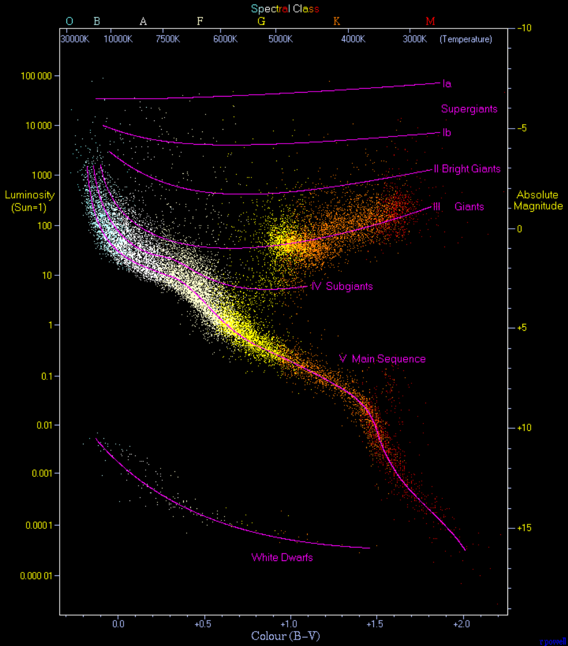
Nomenclature: Luminosity class

At Yerkes Observatory in the 1940s, Morgan, Keenan, and Kellman added another dimension to classification with luminosity classes (Morgan et al. 1943).

- **V** main sequence or "**dwarf**" stars (the Sun)
- **IV** "**subgiants**", brighter in V by 1 or 2 magnitudes for the same spectral type
- **III** normal "**giants**", another few magnitudes brighter
- **II, I** "**bright giants**" and "**supergiants**," even brighter (Antares, Betelgeuse)
- **VI, VII** "**subdwarfs**" and "**white dwarfs**" (Sirius B). Subdwarfs are metal-poor MS stars. White dwarfs are degenerate stellar remnants.

Examples: Vega is an A0V star; the Sun is G2V, Pollux is K2III, and Betelgeuse is M2I.

H-R diagram

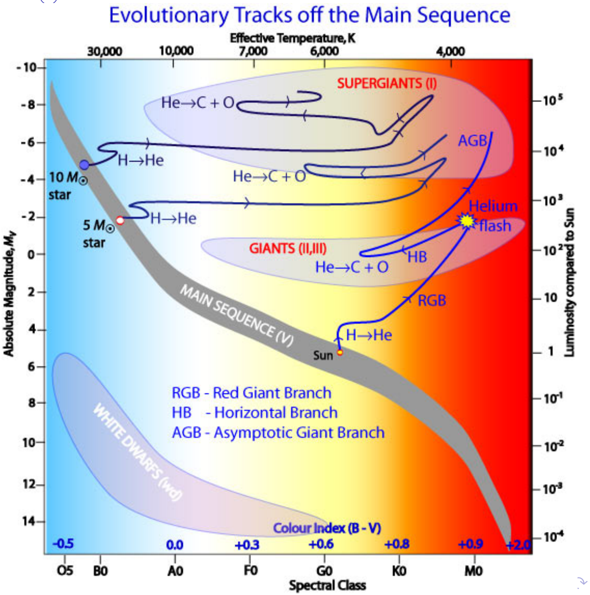


Spectral Class
O B A F G K M
30000K 10000K 7500K 6000K 5000K 4000K 3000K (Temperature)
Luminosity (Sun=1)
100 000
10 000
1000
100
10
1
0.1
0.01
0.001
0.0001
0.000 01
Absolute Magnitude
-10
-5
0
+5
+10
+15
Ia
Supergiants
Ib
II Bright Giants
III Giants
IV Subgiants
V Main Sequence
White Dwarfs
0.0
+0.5
+1.0
+1.5
+2.0
Colour (B–V)

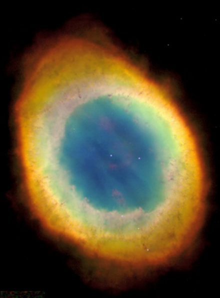
After the horizontal branch: $M < 2M_\odot$

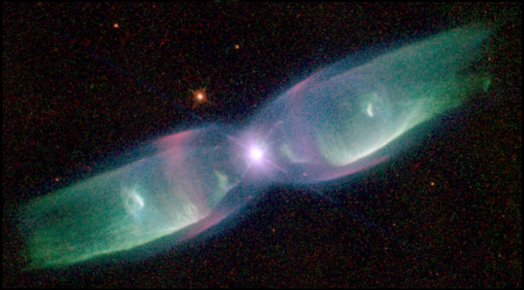
- Isothermal C-O core forms as He is exhausted.
- Not enough weight to overcome degeneracy pressure.
- Core cannot collapse and ignite C-O fusion (requires 500 MK!)
- H/He burning in outer core, ejection of rest of stellar envelope, forming a **planetary nebula** (few 1000 yr timescale).

The core becomes a $0.6M_\odot$ C-O white dwarf with $T_0 \sim 10^8$ K. It lasts forever unless it has a close stellar companion.



Examples of planetary nebulae





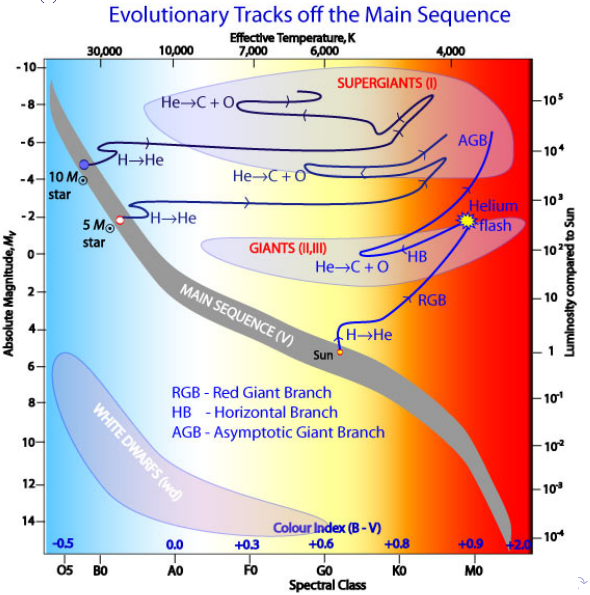
Clockwise from top left:

- NGC 6543 (Cat's Eye)
- M57: Ring Nebula
- Abell 39
- M2-9

After the horizontal branch: $M > 2M_\odot$

- **Asymptotic giant branch** (ABG/supergiant) evolution
- Repeated core collapse after fuel exhaustion, up to Si fusion to produce Fe-peak elements.
- R and L steadily increase while T_e decreases.
- Each successive fuel is exhausted faster than the last. For a $15M_\odot$ star (Woosley & Janka 2006):

Fuel	H	He	C	Ne
Time	10^7 yr	10^6 yr	10^3 yr	0.7 yr
	O, Mg	Si, ...	Fe ...	
	2.6 yr	18 dy	1 s	



What happens after the nuclear fuel is exhausted?

For most $M > 2M_\odot$ stars:

- During burning of the heavier elements and radiative support of the stellar envelope, stars tend to be hydrodynamically unstable
- This leads to the loss of large fractions of the stellar mass.
- Oscillations: note that evolution takes stars across the instability strip, which is nearly vertical at $T_e \sim 5000$ K.
- **Stellar winds** can also remove significant amounts of material.

These processes can keep a star's core mass below the SAC limit, so the final states of the star are like those of lower-mass objects: planetary nebula phase and white dwarf remnant.