Stellar Mass Black Holes, Stars, Stellar Clusters & Stellar Evolution

> General Relativity Gravitational Waves L, M, T Relationships & Stellar Evolution Open and Globular Stellar Clusters

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Stellar Mass Black Holes, Stars, Stellar Clusters & Stellar Evolution

- General Relativity and black holes
- Gravitational radiation
- Relationships among luminosity, mass, and effective temperature
- Stellar evolution
- Changes on the main sequence
- Shell hydrogen fusion and subgiants
- Late stellar evolution: the giant branch, horizontal branch, and asymptotic giant branch
- Evolution of high mass stars: the iron catastrophe
- Type II (core-collapse) supernovae
- Open and globular clusters as stellar clocks

Reading: Kutner Ch. 11.1 & 13; Ryden Sec. 14.2–14.3, 17.2, & 18.4; Shu Ch. 8 & 9



Supernova progenitor simulation (Mosta et al. 2014). Colors indicate entropy.

Escape velocities from stars

Neglecting relativity, an increasingly bad approximation:

$$E = \frac{1}{2}mv_{esc}^2 - \frac{GMm}{R} = 0$$

$$\therefore v_{esc} = \sqrt{\frac{2GM}{R}}$$

$$= 619 \text{ km/s} = 0.002c \qquad \qquad \text{Sun}$$

$$= 6970 \text{ km/s} = 0.023c \qquad \qquad 1M_{\odot} \text{ white dwarf}$$

$$= 154000 \text{ km/s} = 0.514c \qquad \qquad 1M_{\odot} \text{ neutron star}$$

And note that $v_{\rm esc} = c \approx 3 \times 10^5 \, \rm km/s$ when

$$R = \frac{2GM}{c^2}$$

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Beyond the NS maximum mass: Black holes

- The maximum mass of a neutron star is ~ 2.2M_o. There is no known physical process that can support a heavier object without internal energy generation.
- A non-spinning heavier object will collapse past neutron-star dimensions and soon thereafter become a black hole, an object from which even light cannot escape if emitted within a distance

$$R_{\rm Sch} = \frac{2GM}{c^2}$$
 Schwarzschild radius

of the object as measured by a distant observer.

- This spherical surface is called the event horizon or simply the "horizon" of the black hole.
- The nonrelativistic result $R = 2GM/c^2$ is, by accident, the same as R_{Sch} derived with the general theory of relativity (GR).

Black holes & General Relativity

First published by Albert Einstein in 1916, GR is a description of the effect of gravity at any strength, even handling large amounts of mass shrunk to small dimensions. It involves new mathematical concepts beyond the scope of this course.

The theory provides a set of **field equations** to describe gravity:

spacetime curvature
$$\rightarrow G_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu} \leftarrow \text{mass and energy}$$

In plain English (*Gravitation* by Misner, 1973):

- Mass causes space and time to be **curved**, or **warped**.
- ► The resulting curvature of space determines **how masses will move**.

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Gravitational time dilation

Time and space are substantially warped near black holes.

- To distant observers, time intervals on clocks near black holes appear to be slow compared to their own local identical clocks.
- > This effect is known as **gravitational time dilation** or **gravitational redshift**:

time measured
by a distant
$$\rightarrow \Delta t = \frac{\Delta \tau}{\sqrt{1 - \frac{R_{\text{Sch}}}{r}}} > \Delta \tau \leftarrow \text{proper time near}$$

observer

Thus, to a distant observer, time appears to stop at the event horizon:

$$\Delta t \to \infty \text{ as } r \to R_{\mathrm{Sch}}$$

This behavior gave the horizon its original name: the "Schwarzschild singularity."

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Gravitational length dilation

Near a black hole, a *small* length $\Delta \mathcal{L}$ measured simultaneously (as with a measuring tape) between two points on a radial line is greater than the distance Δr between the points, measured by a distant observer:

proper length here
$$\Delta \mathcal{L} = \frac{\Delta r}{\sqrt{1 - \frac{R_{\text{Sch}}}{r}}} > \Delta r \leftarrow \frac{\text{length measured by}}{\text{a distant observer}}$$

- Neither Δr nor the radius *r* measured by a distant observer of a point near the black hole has meaning as a physical distance to an observer near the black hole.
 - r and Δr are called **coordinate distances**.
- ► However, the **circumference** *C* of a circle through that point and centered on the black hole **turns out to have the same value in all frames**. Think of *r* only as $r = \frac{C}{2\pi}$.

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Circular orbits and their radii in GR

Circles in flat spacetime: $C = 2\pi r = \pi d$. That, of course, is the very definition of π .



Circular orbits in flat space (all in same plane)

- Concentric circular orbits in flat space.
- The distance between each orbit is 1.
- Local and distant observers report the same distances between concentric orbits.



Circular orbits in space warped by a black hole

- Imagine a BH whose horizon has circumference 2π.
- The distant observer still reports distances of 1 between each orbit.
- The local observer reports warped distances.
- Both observers measure the same circumferences!



Visualization of warped space: "Hyperspace"

- ► To connect these circles with segments of the "too long" lengths, it is helpful to consider them to be offset from each other along an **imaginary** dimension ⊥ to *x* and *y* but which is not *z*.
- If the additional dimension were z, then the circles would not appear to lie on a plane.
- Such additional dimensions comprise hyperspace.



Embedding diagrams

- This is why you often see the equatorial plane of a black hole represented as a funnel-shaped surface, as if made from a stretched rubber sheet.
- It is important to note that the direction of stretch is in hyperspace.
- The scene would not look like a funnel to an observer of three spatial dimensions.



No-hair theorem

After a star has collapsed into a BH, the horizon is smooth. Nothing protrudes from it, and almost everything about the star that gave rise to the BH has lost its identity during the formation of the BH.

Metaphor: no "hair" (information) is left to "stick out" of the horizon.

- Any protrusion, prominence, or other departure from spherical smoothness gets turned into gravitational radiation, i.e., it is radiated away during the collapse.
- Any magnetic field lines emanating from the star close up and get radiated away in the form of light during the collapse.
- The identity of the matter that made up the star is lost. Nothing about its previous configuration can be reconstructed.
- Even the matter/antimatter distinction is lost. Two stars of identical mass one of matter and one of antimatter would produce identical black holes.

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No-hair theorem

The black hole has only three quantities in common with the start that collapsed to create it: **mass**, **spin**, and **electric charge**.

- ▶ That is, in common with the star as it was *immediately before the formation of the horizon*.
- Only very tiny black holes can have much electric charge. Stars are electrically neutral.
- Spin makes the black hole depart from a spherical shape, but it is still smooth. From an observational standpoint, the bare BH is a lot like a giant elementary particle with only a few measurable properties.

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Gravitational radiation from black hole mergers

Cartoon of the spacetime distortion created by two compact objects (neutron stars) in a close binary orbit. Image from LICO.

Properties of gravitational waves (GWs)



Effect of GWs on a ring of test particles (Li 2014).

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Detecting gravitational waves



Black hole merger signal

The estimated strain of the GW signal, the BH-BH separation, and the relative velocity of the merger estimated from numerical GR (Abbott et al. 2016).

Note the **close separation** and **highly relativistic velocities** of the BHs before the merger and "ringdown."



Details of the first observed BH-BH merger

From numerical simulations of binary BH mergers (Abbott et al. 2016), the September 14, 2015 event corresponded to:

- ▶ $m_1 = 35.6^{+4.8}_{-3.0} M_{\odot}$
- ▶ $m_2 = 30.6^{+3.0}_{-4.4} M_{\odot}$
- $L_{\text{peak}} = 3.6^{+0.5}_{-0.4} \times 10^{56} \text{ erg/s} \approx 200^{+30}_{-20} M_{\odot} c^2 \text{ s}^{-1}!$

How do we know that these were not neutron stars?

- ▶ No evidence for corresponding electromagnetic radiation.
- Best-fit masses from numerical GR of the binary merger far exceed maximum allowed neutron star mass.

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Gravitational wave detections to date

Since 2015, we have successfully detected 89 BH-BH merger events, two NS-NS merger events, and five BH-NS merger events, thanks to LIGO and VIRGO. It is rare in physics for data to be so unambiguous. These provide extremely the existence of black holes of tens of solar



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GW170817: a NS-NS merger

The first GW detection to be confirmed by non-gravitational means





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