

Astronomy 142 — Recitation #11

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Formulas to remember

Normalized scale factor

$$a(t) = \frac{R(t)}{R_0} = \frac{1}{1+z} \quad (1)$$

Friedmann equation

Neglecting radiation, so only good to $z \sim 3600 \dots$

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G}{3}\rho_m - \frac{\Lambda}{3} = -k\frac{c^2}{R^2} \quad (2)$$

$$\dot{a}^2 = H_0^2 \left[1 + \Omega_{M_0} \left(\frac{1}{a} - 1 \right) + \Omega_{\Lambda_0} (a^2 - 1) \right] \quad (3)$$

Flat universe

For a flat universe, $\Omega_{M_0} = \Omega$, $\Omega_{\Lambda_0} = 1 - \Omega$, $0 < \Omega < 1$

$$t(a) = \frac{2}{3H_0\sqrt{1-\Omega}} \ln \left(\sqrt{\frac{1-\Omega}{\Omega}} a^3 + \sqrt{1 + \frac{1-\Omega}{\Omega}} a^3 \right) \quad (4)$$

Acoustic horizon

$$\ell_d \sim \frac{ct_d}{a_d} \quad (5)$$

Useful power-series expansions

$$\begin{aligned} \sin^{-1} x &= 1 + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \dots & \sinh^{-1} x &= 1 - \frac{1}{6}x^3 + \frac{3}{40}x^5 + \dots \\ (1+x)^n &= 1 + nx + \frac{n(n-1)}{2}x^2 + \dots & \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} + \dots \end{aligned}$$

Workshop problems

Remember! The workshop problems that you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in ASTR 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem in some sort of bound notebook.

1. The highest redshift observed to date for a quasar is $z = 10.1$.
 - (a) How long ago, in years, did the quasar emit the light that we detect today? So how many light years distant is the quasar? Also express your answer in Mpc. This distance is called the **light-travel-time distance**.
 - (b) How long, in Mpc, is the null geodesic connecting here and now with the quasar at the time that the light was emitted? This distance is generally known as the **comoving radial distance**. We calculated the comoving radial distance to the decoupling surface, Δr , yesterday in lecture, in the course of measuring the curvature of the universe, k .
 - (c) We know of galaxies more distant than this, currently out to a redshift $z = 13.27$. Why is it so special to find such a high redshift quasar?
2. Consider a flat, matter-dominated universe, i.e., one in which $\Lambda = 0$ and $k = 0$. Solve the Friedmann equation for this universe, obtaining a relation between the scale factor and time since the Big Bang, and an expression for the present age of the universe. Compare your results to those from the universes discussed in class.
3.
 - (a) Solve the Friedmann equation to obtain the t - a relation for an empty universe.
 - (b) At what redshift do the corresponding look-back times (that is, times before the present) of the flat matter-dominated universe of Problem 2 and the flat empty universe differ by 10%? That is, at what redshift would the Hubble diagram for a flat matter-dominated universe depart by 10% from a straight-line Hubble relation? (*Hint*: It is permitted to let Mathematica, Wolfram Alpha, etc. do the algebra here; consider solving graphically rather than algebraically.)
4. Our Universe is well described by $H_0 = 74.03 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_{M_0} = 0.31$, and $\Omega_{\Lambda_0} = 0.69$. The temperature of the cosmic microwave background indicates that decoupling took place at redshift $z = 1089.9$.
 - (a) What is the age of the Universe with these parameters?
 - (b) How long after the Big Bang did decoupling occur, according to these parameters?

Intro to Python (A feature *exclusive* of ASTR 142 recitations.)

5. **A closed universe:** Consider a universe with $\Omega_{M_0} = \Omega > 1$ and $\Omega_{\Lambda} = 0$.
 - (a) In a procedure similar to that followed in class for the $\Omega < 1$ flat universe, solve the Friedmann equation to obtain a relation between the normalized scale factor a and time since the Big Bang t .
 - (b) Plot a against t and calculate the age of this universe in Gyr for $\Omega_{M_0} = 2$ and our usual value of H_0 .
 - (c) This is a closed universe, so the expansion eventually halts, reverses, and collapses to a point (generally called a “Big Crunch”). The collapse and Big Crunch are not evident in the formula or plot in parts a and b. Take a guess as to why not.