Interstellar Gas & Star Formation

Molecular Clouds
Gravitational Collapse
Free-Fall Timescale
Pre-MS stars and the Hayashi Track

March 7, 2024

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Star formation

Topics:

- Molecular clouds and gravitational stability
- Collapse of a spherical cloud: free-fall time
- Synopsis of star formation
- The initial mass function
- Formation of protostellar disks and jets
- Pre-MS stars and the Hayashi track

Reading: Kutner Ch. 15, Ryden Sec. 17.1

Pillar and Jets HH 901/902
Hubble Space Telescope • WFC3/UVIS

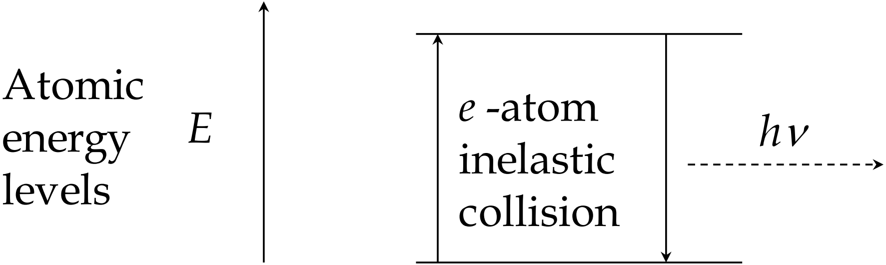
NASA, ESA, and M. Livio and the Hubble 20th Anniversary Team (STScI) STScI-PRC10-13a

Heating and cooling of the ISM

Warm ionized medium, H II regions, and planetary nebulae

Heating: **Photoionization** by starlight; UV light with $E > 13.6$ eV ionizes hydrogen, imparting kinetic energy to the electrons produced.

Cooling: **Recombination** of H or **collisional excitation** by free electrons, followed by emission of forbidden lines by ions of C, N, O, and S (mostly). The more metals, the more efficiently the cooling.



Atomic energy levels

E

e -atom inelastic collision

$h\nu$

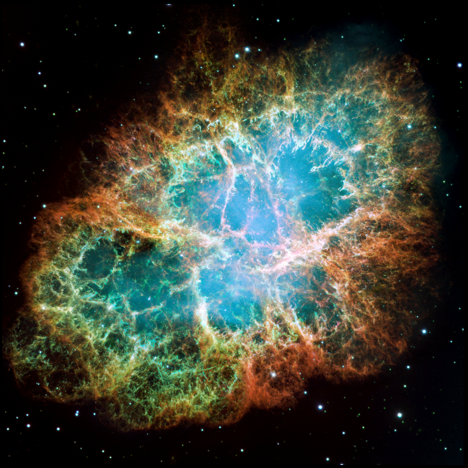
Heating and cooling of the ISM

Supernova remnants

Heating Viscous dissipation in the deceleration of the supersonic blast wave, i.e., shock heating.

Cooling Same as for photoionized regions, except

- There are fewer high-ionization species in SNRs than in the other ionized media.
- There are additional abundant, low-ionization species, like Si and Fe, that are liberated into the gas phase if the blast wave is fast enough to destroy dust grains.



The Crab Nebula, the archetype SN remnant, in the visible (HST)

Heating and cooling of the ISM

Warm neutral medium and cold neutral medium

Heating UV light ($h\nu = 5 - 13$ eV) in background starlight, known as the interstellar radiation field (ISRF).

- Heating occurs via the photoelectric effect on dust grains and photoionization of C.
- Carbon can be ionized by $E = h\nu = 11$ eV photons and thus is usually singly ionized (C II) in the diffuse ISM even when hydrogen is neutral and atomic.

Cooling Excitation of excited carbon (C II) by collisions with H atoms and electrons, followed by radiation in the forbidden 157.7 μm line.

Heating and cooling of the ISM

Molecular Clouds

Heating :

- **Outer layers** Photoelectrons produced by starlight on dust grains and low-ionization elements.
- **Interior** Cosmic rays: ions accelerated to high energies ($\sim 10^{15}$ eV) in supernova remnants.

Cooling Collisional excitation by H_2 followed by radiation in the rotational lines of the more abundant molecules like CO, OH, H_2O.

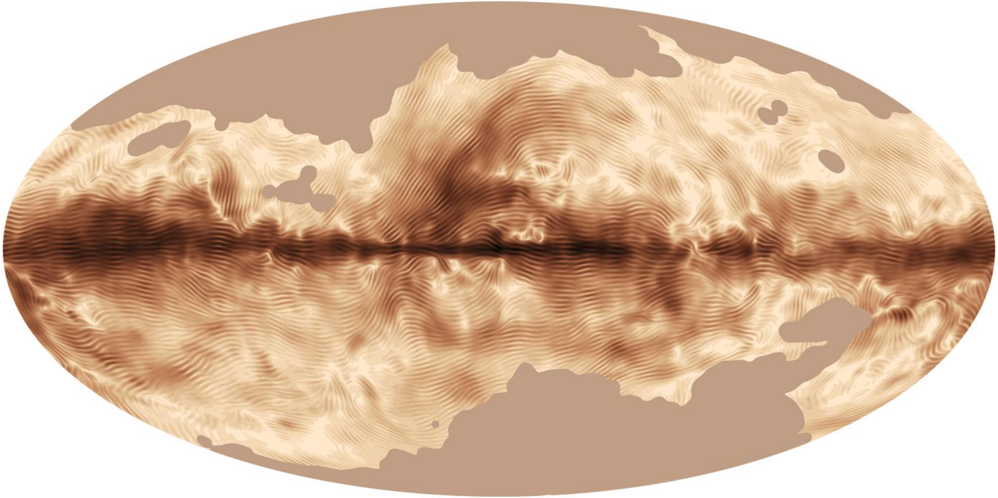
- Molecular clouds are frequently dense and cold enough to be unstable and collapse under their own weight.
- As we will see, the gravitational instability of molecular clouds is the principal means by which stars are formed in the galaxy.

Molecular clouds: Internal structure

- Molecular clouds are **clumpy**; they tend to consist of denser clumps in a range of sizes and masses, blending into a less-dense background.
- Molecular clouds are **cold**; they tend to have $T < 20$ K.
- Molecular clouds are **turbulent**: the random internal velocities are typically ~ 1 km/s, much larger than the average molecular speeds in a quiescent gas in equilibrium at the same temperature. It has therefore been mechanically stirred up, by supernova blasts, stellar winds, and the galaxy's differential rotation.
- Molecular clouds (and clumps) generally **rotate**, or tumble, slowly.
- Molecular clouds are **magnetized**: they are threaded by the same magnetic flux present when the material was diffuse and atomic. Now it has been compressed to a much smaller size, and the fields are correspondingly larger. The field lines are often sheared and twisted by motion of the ionized material in which it is threaded.

Magnetic field of the ISM

The whole sky, with direction of B shown by polarization of the emission by magnetically-aligned dust grains, by the Planck satellite



Molecular clouds & star formation

As a result:

- Clumps get massive enough and cold enough that the gas pressure cannot hold up their weight, and they collapse.
  - As clumps collapse they heat in their cores.
  - If they are sufficiently massive and collapse to a small enough scale, temperatures can reach the fusion ignition point and a star is formed.
- The clumps are constantly being rearranged, compressed, or distended by turbulence.
- Collapse does not happen with spherical symmetry; often collapse is easier along the axis of rotation because of centrifugal forces.

Collapse of a clump: the Jeans mass

Consider a uniform-density clump and its central pressure:

$$\begin{aligned} P_c &\approx \frac{GM^2}{R^4} && \text{from weight} \\ &= \frac{\rho k T}{\mu} && \text{from ideal gas law} \end{aligned}$$

Balancing the pressure gives

$$\frac{GM^2}{R^4} = \rho \frac{GM}{R} = \frac{\rho k T}{\mu} \implies \frac{M}{R} = \frac{kT}{\mu G}$$

and assuming the mass is uniformly distributed gives

$$\rho = \frac{3M}{4\pi R^3} \implies R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$

$$\therefore M = \frac{kT}{\mu G}\left(\frac{3}{4\pi\rho}\right)^{1/3} M^{1/3} \implies \boxed{M = \left(\frac{kT}{\mu G}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2}}$$

Collapse of a clump: the Jeans mass

Define the **Jeans mass**, M_J, as the critical mass for collapse:

$$M_J = \left(\frac{kT}{\mu G}\right)^{3/2} \left(\frac{3}{4\pi\rho}\right)^{1/2}$$

If a clump's mass exceeds the Jeans mass for its density, temperature, and composition, it will collapse under its weight.

Example: A pure molecular Hydrogen cloud

For a cloud of H_2:

$$\begin{aligned} \mu &= 3.3 \times 10^{-24}\ \mathrm{g} \\ T &= 20\ \mathrm{K} \\ n &= 1.4 \times 10^{5}\ \mathrm{cm}^{-3} \\ \therefore M_J &= 1 M_\odot \end{aligned}$$

Spherical free-fall time: Derivation

Follow a test particle that starts from rest at distance r_0 from the center of the cloud at time $t = 0$. How long until it reaches the center, i.e., until the clump collapses to a point?

$$F = ma$$

$$-\frac{GMm}{r^2} = m\frac{d^2r}{dt^2}$$

This is a **1D, nonlinear 2nd-order differential equation**. M is the mass interior to r and is constant. Multiplying both sides by $\frac{dr}{dt}$ and integrating over t:

$$\int_0^t dt' \frac{dr}{dt'} \frac{d^2r}{dt'^2} = -GM \int_0^t \frac{1}{r^2} \frac{dr}{dt'} dt'$$

Note that $v = \frac{dr}{dt}$ and $dv = \frac{d^2r}{dt^2} dt$ and start solving.

Spherical free-fall time: Derivation

$$\int_0^t \frac{dr}{dt'}\frac{d^2r}{dt'^2}dt' = -GM\int_0^t \frac{1}{r^2}\frac{dr}{dt'}dt'$$

$$\int_0^v v'\,dv' = -GM\int_{r_0}^r \frac{dr'}{r'^2}$$

$$\frac{1}{2}v^2 = \frac{GM}{r} - \frac{GM}{r_0}$$

Substitute back $v = \frac{dr}{dt}$:

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{GM}{r} - \frac{GM}{r_0}$$

$$\frac{dr}{dt} = -\sqrt{\frac{2GM}{r} - \frac{2GM}{r_0}}$$

choosing the negative root so that the particle moves to smaller r.

Spherical free-fall time: Derivation

Now insert $M = \frac{4}{3}\pi r_0^3 \rho_0$:

$$\frac{dr}{dt} = -r_0 \sqrt{\frac{8\pi G \rho_0}{3} \left(\frac{r_0}{r} - 1\right)}$$

Make a geometric u substitution $r = r_0 \cos^2 u$:

$$\begin{aligned}
\frac{dr}{dt} = r_0 \frac{d}{dt} \cos^2 u &= -2r_0 \cos u \sin u \frac{du}{dt} \\
\implies \frac{du}{dt} &= \frac{1}{2\cos u \sin u} \sqrt{\frac{8\pi G \rho_0}{3} \left(\frac{1}{\cos^2 u} - 1\right)} \\
&= \frac{1}{2\cos^2 u \sin u} \sqrt{\frac{8\pi G \rho_0}{3} (1 - \cos^2 u)} \\
&= \frac{1}{2\cos^2 u} \sqrt{\frac{8\pi G \rho_0}{3}}
\end{aligned}$$

Spherical free-fall time: Derivation

Separate variables and integrate:

$$\int_{u_0}^{u} \cos^2 u'\,du' = \frac{1}{2}\sqrt{\frac{8\pi G\rho_0}{3}} \int_0^{t_{\mathrm{ff}}} dt \quad \text{where}$$

$$\begin{array}{llll} t = 0 : & r = r_0 & r_0 = r_0 \cos^2 u_0 & \implies u_0 = 0 \\ t = t_{\mathrm{ff}} : & r = 0 & 0 = r_0 \cos^2 u_0 & \implies u_0 = \dfrac{\pi}{2} \end{array}$$

$$\implies \int_0^{\pi/2} \cos^2 u'\,du' = \frac{1}{2}\int_0^{\pi/2} \left[1 + \cos 2u'\right] du' = \frac{1}{2}\sqrt{\frac{8\pi G\rho_0}{3}} \int_0^{t_{\mathrm{ff}}} dt$$

$$\left[\frac{u}{2} + \frac{\sin 2u}{4}\right]_0^{\pi/2} = \frac{\pi}{4} = \frac{t_{\mathrm{ff}}}{2}\sqrt{\frac{8\pi G\rho_0}{3}}$$

$$\boxed{t_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32 G\rho_0}}}$$

Time scale of spherical gravitational collapse

For spherically symmetric collapse in free fall, we see that

$$t_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32G\rho_0}} \qquad\qquad \rho_0 = \mu n = \text{initial mass density}$$

$$= 10^{12}\ \mathrm{s} = 30\ \mathrm{kyr}$$

for $\rho_0 = 4.6 \times 10^{-19}\ \mathrm{g/cm^3}$.

- This is very fast by astrophysical standards. We should therefore have to get very lucky to catch a star in the act of formation.

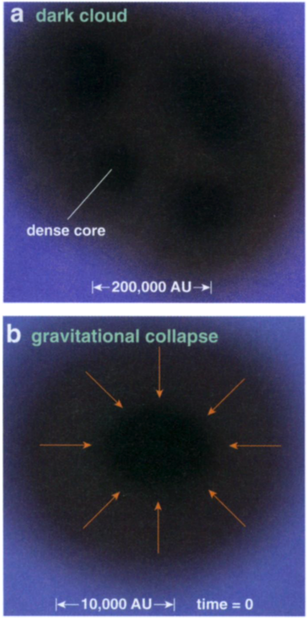
- The result applies to a spherically symmetric collapse, and we know the collapse cannot really be very spherical (as angular momentum is conserved) so this is offered only as a crude estimate.

Star formation

1. A fragment of an interstellar molecular cloud (a) becomes unstable and begins collapsing (b), either because the material has cooled or because it has become compressed (**triggered**) from the outside.

2. The collapse proceeds from the **inside out** and **anisotropically**. The central denser region collapses faster:

$$t_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32G\rho_0}}$$



Star formation

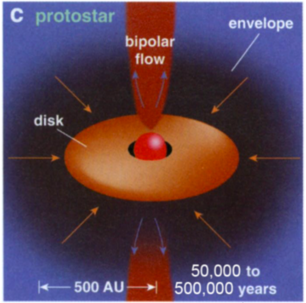
3. Because of conservation of angular momentum, and to a lesser degree from magnetic forces, collapse can always proceed much faster in one dimension than the other two.

   Soon a **disk configuration** is established with a well-defined and very dense central condensation: a **protostar** (c).

   - Protostars have larger luminosities than the stars that they will become, but their heat does not come from fusion.
   - Instead, it is largely **accretion power**: gas falling from large radii onto the protostar's surface.
   - The core of a protostar is about 10 times larger in radius than the star that it will become, and it accretes gas from the disk at a rate

     $$\frac{dm}{dt} = 10^{-5} - 10^{-7} M_\odot/\text{yr}$$

     decreasing with core mass and as time goes on (Watson et al. 2016).



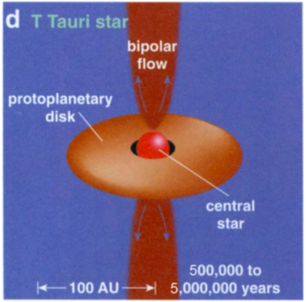
Star formation

4. The disk continues to **accrete** further material from the surrounding molecular cloud, and the protostar accretes material from the disk (d). Due to disk "viscosity," mass moves inward and angular momentum outward.

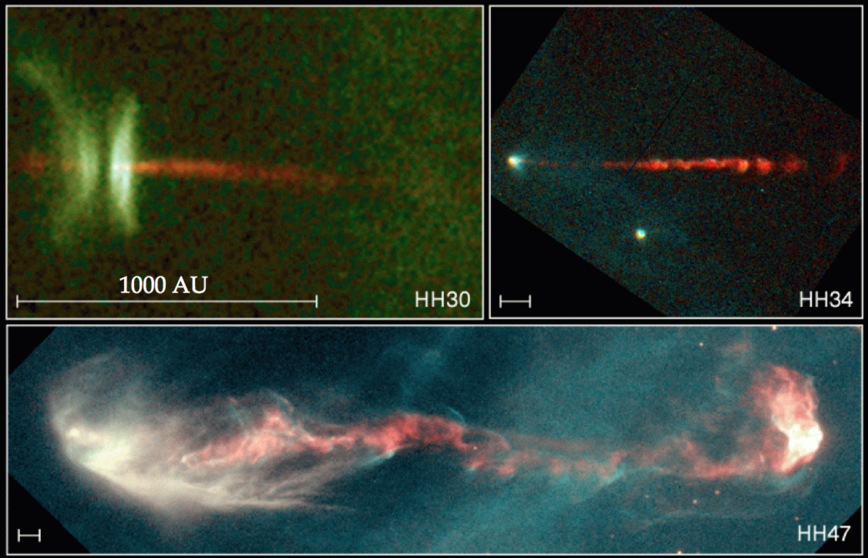
   As the disk and central object accretes gas and dust, they drive a **bipolar outflow** into their surroundings, which is thought to carry off the last of the accreted material's angular momentum.

   By the time that the clump outside the disk is used up or driven away by the outflow, the accretion rate is much less, and most of the luminosity is from **slow contraction of the core**.

   For low-mass stars, this core is called a **T Tauri star** when in this phase.



Bipolar jets and disks associated with young stars



More jets: Carina Nebula

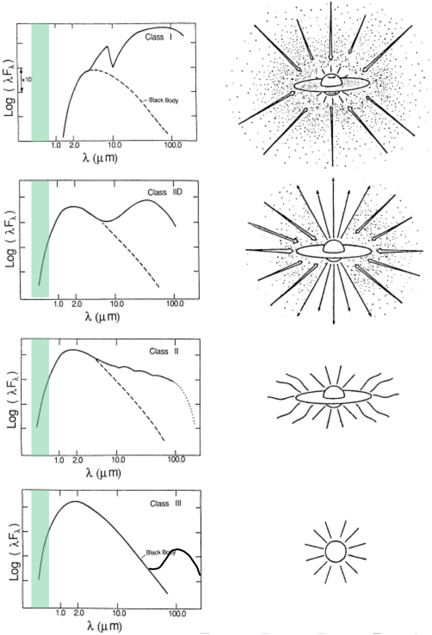
Star formation

5. The central object becomes a star and its wind and radiation eventually stop the accretion and dissipate the gas in the disk.

   Class 0 or I protostar: class 0 seen at mm/radio; class I: far-IR

   Class II protostar: “T Tauri stars,” opaque disk, envelope almost gone, accretion onto star from disk at rate that decreases with age, dropping below $10^{-10} M_\odot\ \mathrm{yr}^{-1}$ by 5 Myr.

   Class III protostar: very little dust and gas left in disk. Accretion has stopped.

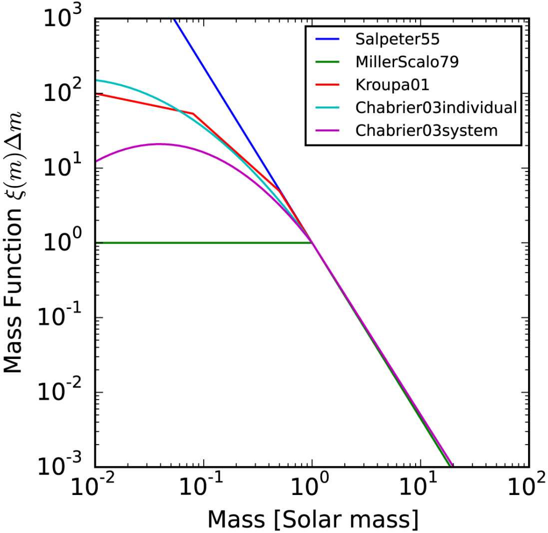


Left: spectral energy distributions of protostar classes (Wilking 1989). Green band shows visible light.

The initial mass function

The **initial mass distribution** is an empirical function describing the distribution of stellar masses that will form from a single molecular cloud.

Observations of numerous young stellar clusters reveal that their stellar mass distributions are nearly identical (the primary difference due to the cluster's age).



Pre-Main Sequence stars

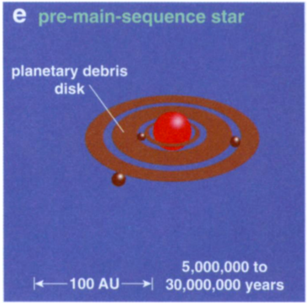
- It takes lower-mass young stars millions of years to settle down to their final sizes and get fusion running.
- During this slow gravitational collapse, the conversion of gravitational potential energy dominates the luminosity of young stars. Recall that gravitationally-driven luminosity is
$$L = -\frac{3}{5}\frac{GM^2}{R^2}\frac{dR}{dt}$$
- The gravitational collapse luminosity decreases with time and the star's effective temperature changes little while this is going on. So the young star descends almost vertically at first through the H-R diagram. This path is called the Hayashi track.

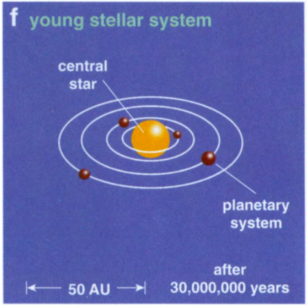
Pre-Main Sequence stars

6. After 5 Myr or so, the core is no longer accreting mass, and the gas in the disk is either used up or driven away.

   It still lies well above its eventual resting place on the main sequence, in which condition we call it a pre-main sequence star (e).

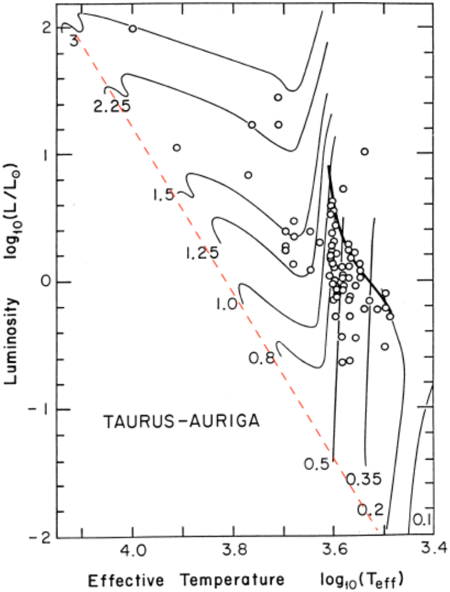
7. Finally, the core reaches pp-chain temperatures, fusion begins, and the star heats up, moving **horizontally** in the HR diagram, towards bluer colors and the main sequence.
   - This part of its HR path is called the Henyey track.





Pre-Main Sequence stars

- The Henyey and Hayashi tracks happen to be the reverse of the path each star takes at the end of its life as it becomes a subgiant and a red giant.
- For a given age, the redder (lower mass, later spectral type) stars lie further above the main sequence than the bluer ones.



Right: T Tauri tracks (Stahler 1988). The Main Sequence is shown by the diagonal line.