The Milky Way

Stellar Relaxation Time
Differential Rotation of the Disk
Local Standard of Rest
Galactic Rotation Curves

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University of Rochester

The Milky Way

- More on stars as a gas: stellar relaxation time, and equilibrium
- Differential rotation of the stars in the disk
- The local standard of rest
- Rotation curves and the distribution of mass
- Spiral structure in the Galaxy

Reading: Kutner Sec. 16.2–16.3, Ryden Sec. 19.4–19.6, Shu Ch. 12

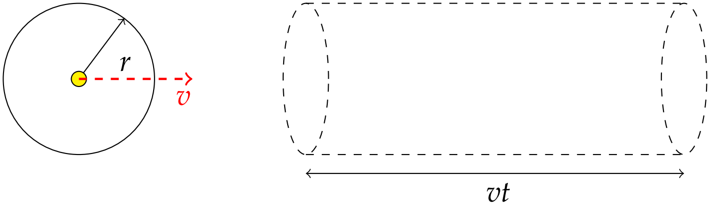
NGC 3351, a spiral galaxy resembling the shape of the Milky Way. Image from Adam Block, Mt. Lemmon Sky Center, U. Az.

Relaxation time of a stellar cluster

In order to behave as a gas, stars need to elastically collide enough times for their random kinetic energy to be shared in a thermal fashion.

- However, stellar encounters, even distant ones, are rare on human time scales.
- **Relaxation time**: The time between elastic stellar encounters, based on the density and velocity distribution, is one way to quantify how long it takes for a cluster of stars to thermalize
- If a gravitationally bound cluster is much older than its relaxation time, the stars can be described as a gas (the system has a relatively uniform temperature, pressure, etc.).

Relaxation time of a stellar cluster



Suppose that a star has a gravitational "sphere of influence" of radius $r \gg R_*$ and moves at speed v between encounters. In time t, its sphere of influence sweeps out a cylinder of volume $V = \pi r^2 v t$.

If the number density of stars is n, there will be exactly one star in the cylinder if

$$nV = n\pi r^2 v t_c = 1$$

$$\implies t_c = \frac{1}{n\pi r^2 v} \qquad \textbf{Relaxation time}$$

Relaxation time of a stellar cluster

What is the appropriate radius r? Choose r such that the gravitational potential energy is equal to the average stellar kinetic energy:

$$\frac{Gm^2}{r} = \frac{1}{2}mv^2 \qquad \Longrightarrow \qquad t_c = \frac{v^3}{4\pi G^2 m^2 n}$$

Done in more detail (ASTR 232): for a spherical cluster with a **"core" radius** R, it can be shown that

$$t_c = \frac{v^3}{4\pi G^2 m^2 n} \frac{1}{\ln\left(2R/r\right)}$$

Not far from our rough estimate, as the logarithm is a very slow function.

Relaxation time of a stellar cluster

In your homework, you will show that, for such a cluster of N stars with core radius R and typical stellar mass m,

$$v^2 = \frac{G(N-1)m}{2R}$$

Assuming $N \gg 1$ and substituting these into the expression for relaxation time t_c gives

$$t_c \approx \left(\frac{2R}{v}\right) \frac{N}{24 \ln\left(N/2\right)}$$

The time $t_x = 2R/v$ is called the crossing time. It is the time it takes a star moving at mean speed v to traverse the core of the cluster of diameter $2R$ if it does not collide.

Thermal equilibrium: The virial theorem

A handy way for bookkeeping random motions in thermal equilibrium is the virial theorem:

In an isolated system of particles that exert forces on each other described by scalar potentials (gravity, Coulomb force, etc.), the system's moment of inertia I, total kinetic energy K, total potential energy U, and total mechanical energy E are related by

$$\frac{d^2I}{dt^2} = 2K + U = K + E$$

In many cases $d^2I/dt^2 = 0$, in which case K, U, and E are related by

$$K = -\frac{1}{2}U = -E$$

It is often easy to calculate U and the systematic-motion part of K; thus we can get the random-motion part of K via the virial theorem.

Thermal equilibrium

Uniform Density Cluster

Suppose a uniform-density star cluster ($N \gg 1$ stars of mass m, total mass $M = Nm$) has radius R and rotates like a solid body at angular speed Ω. What is the random speed v of a typical star in this cluster?

Since the cluster structure is in equilibrium, $d^2I/dt^2 = 0$, so

$$K = \frac{1}{2}\sum_i m_i v_i^2 + \frac{1}{2}I\Omega^2 = -\frac{1}{2}U$$

$$\frac{1}{2}Nmv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\Omega^2 = \frac{1}{2}\left(\frac{3}{5}\frac{GM^2}{R}\right)$$

$$v = \sqrt{\frac{3}{5}\frac{GM}{R} - \frac{2}{5}R^2\Omega^2}$$

Note that we assumed a constant density sphere to compute I.

Thermal equilibrium

The proof of the virial theorem is not difficult but it is long, so we will skip it for now.

- The proof will be given in PHYS 235 and ASTR 232.
- You can also see Chapter 3 of Ryden.

Caveats related to the use of the virial theorem:

- The theorem only applies to thermal equilibrium or steady-state motion.
- Thus, before every use, the system should be checked to see that it is old enough to be in thermal equilibrium.
- "Old enough" means that the system's age is much longer than the relaxation time.

Rotation curves

Last class, we discussed how to weigh the Milky Way's disk in the solar neighborhood. How can we measure the mass elsewhere in the galaxy?

This would be straightforward, if we could stand in a nonrevolving reference frame outside of the Galaxy — say, in another galaxy, from which we could view the Milky Way's disk edge-on:

- A measurement of the radial velocity of the stars or gas at any radius allows a determination of the mass within that radius. This is called a **rotation curve**.
- The largest radial velocity at a given position, $v_{r,\max}$, is from motion tangent to the line of sight; thus, material in orbit with radius r, if the orbits are circular.
- The mass contained within the radius r is given simply by Newton's second law:

$$F = -\frac{GMm}{r^2} = ma = -m\frac{v_{r,\max}^2}{r} \implies M(r) = \frac{r v_{r,\max}^2}{G} = \frac{r^3\Omega^2}{G}$$

Rotation curves

Point mass M

Consider a test mass m orbiting a point mass M. The rotation curve of the point mass is given by

$$F = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\boxed{v(r) = \sqrt{\frac{GM}{r}}}$$

This is **Keplerian motion**; v decreases with increasing r.

Rotation curves

Constant density, spherically symmetric mass M

In this case, the test mass orbits a constant density spherically symmetric mass:

$$M(r) = \frac{4\pi}{3}\rho_0 r^3$$

$$F = \frac{GM(r)m}{r^2} = \frac{mv^2}{r}$$

$$\frac{Gm}{r^2}\frac{4\pi}{3}\rho_0 r^3 = \frac{mv^2}{r}$$

$$\boxed{v(r) = r\sqrt{\frac{4\pi G\rho_0}{3}}}$$

This is **solid body rotation**; v increases linearly with increasing r.

Rotation curves

Spherical symmetry with $1/r^2$ density distribution

In this case, the test mass orbits a spherically symmetric mass with $\rho(r) = \rho_0(r_0/r)^2$, where r_0 is the core radius of the Galaxy.

$$M(r) = \int_0^r \rho(r') 4\pi r'^2 \, dr' = 4\pi\rho_0 r_0^2 \int_0^r dr' = 4\pi\rho_0 r_0^2 r \;\propto\; r$$

$$F = \frac{GM(r)m}{r^2} = \frac{mv^2}{r}$$

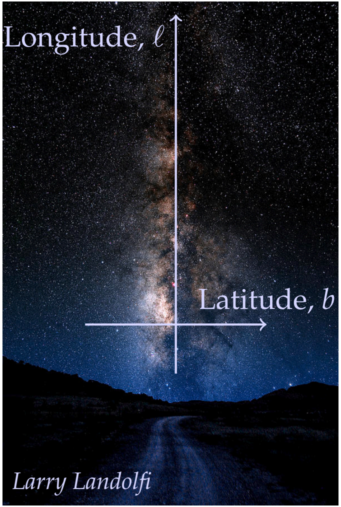
$$\frac{Gm}{r^2} 4\pi\rho_0 r_0^2 r = \frac{mv^2}{r}$$

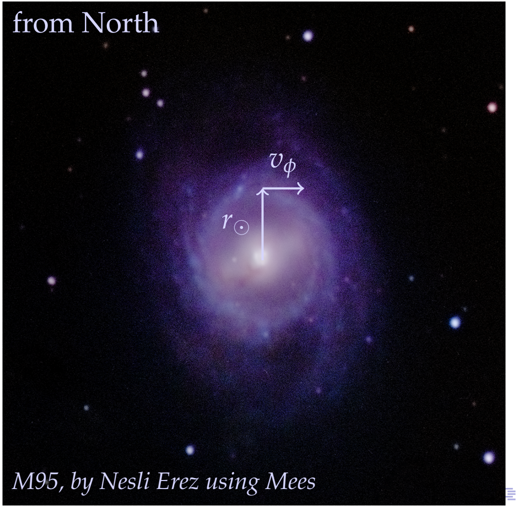
$$\boxed{v(r) = \sqrt{4\pi G\rho_0 r_0^2} = \text{constant}}$$

This is a **flat rotation curve** and is observed in disk galaxies, including ours.

Mass as a function of radius in the Milky Way

It is more complicated than this to measure the mass of the Milky Way, because we live in the disk in a reference frame that revolves around the Galactic center.





Rotation of the stellar population

Averaging over the random motions, the radial velocities of nearby stars ($d \lesssim 500$ pc) reveal **differential rotation** in the disk of the Galaxy.

- The rotation is differential in the sense that different radii have different angular velocities, decreasing monotonically as a function of galactocentric radius.
- The radial and transverse motions are related to the differential rotation by the Oort constants.
  - The Oort constants are the first-order coefficients in a Taylor series expansion of the stellar velocity field, with respect to the distance d from the Sun.
  - There are four of these: A, B, C, and K. We only need to worry about the first two for now.

Rotation of the stellar population & Oort's constants

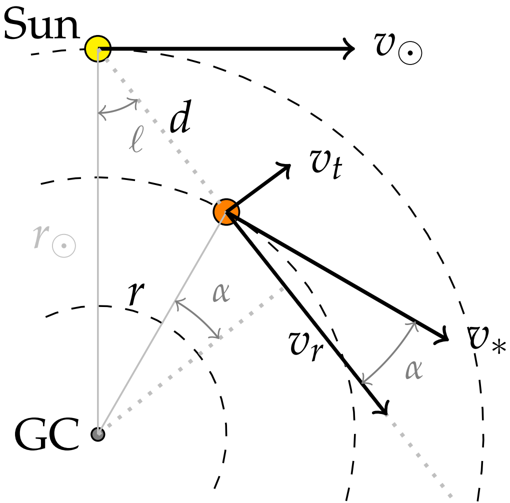
The Oort constants combine four observables: radial velocity, v_r, proper motion, v_t, Galactic longitude, ℓ, and distance, d:

$$v_r = Ad \sin 2\ell \qquad v_t = Ad \cos 2\ell + Bd$$

where Oort's constants are defined as

$$A = -\frac{r}{2}\frac{d\Omega}{dr} \qquad B = -\frac{1}{2r}\frac{d}{dr}(r^2\Omega)$$

with $\Omega = A - B$.



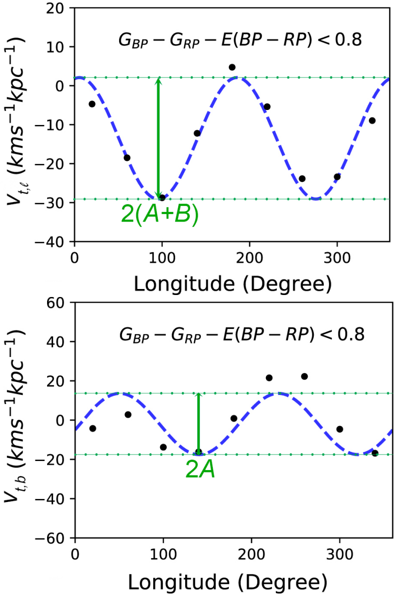
Oort's constants

Measuring the Oort constants requires accurate radial velocity measurements v_r, proper motions v_t, and distances d over a wide range of distance.

The *Gaia* mission is substantially improving these measurements.

The best determinations (Bovy 2017, Li et al. 2019) do not even use v_r, opting instead for the use of the ℓ and b components of the proper motion:

$$A = 15.1 \pm 0.1 \text{ km/s/kpc}$$
$$B = -13.4 \pm 0.1 \text{ km/s/kpc}$$



The local standard of rest

From A and B, we get the average rotational motion of the Sun's orbit, called the **local standard of rest (LSR)**. For the values above, and $r_\odot = 8.15 \pm 0.15$ pc (Reid et al. 2019),

$$\Omega = A - B = (9.24 \pm 0.06) \times 10^{-16} \text{ rad/s}$$
$$v_\phi = r_\odot \Omega = 232 \pm 5 \text{ km/s}$$
$$P = \frac{2\pi}{\Omega} = (216 \pm 6) \times 10^6 \text{ yr}$$

The Solar System actually moves slightly with respect to the LSR (average circular motion) at about 7 km/s.

From the motion of the LSR, the Galaxy within $r_\odot = 8.15$ kpc can be weighed:

$$M = \frac{v_\phi^2 r_\odot}{G} = (1.02 \pm 0.04) \times 10^{11} M_\odot$$

Rotation curves from H I and CO lines

Unlike stars, diffuse and molecular clouds can be detected throughout the disk of the Galaxy, notably via the H I 21 cm line and the CO $J = 1$–0 2.6 mm line.

These clouds are too fuzzy and distant to generally measure their proper motions, but their radial velocities (velocity along the line of sight) using the **Doppler shifts** of spectral lines can be measured with exquisite accuracy:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} = -\frac{\Delta v}{v_0} = \frac{v_r}{c}$$

Along a given line of sight through the plane:

- Maximum radial velocity must come from an orbit tangent to the line of sight. Motion parallel to the line of sight has $\cos\theta = 1$.
- Thus, distance and rotational motion of **tangent points** is very well determined: $r = r_\odot \tan \ell$.
- Elsewhere, there is a **distance ambiguity**: for lines of sight toward the inner Galaxy (first and fourth quadrant), there are two locations with the same radial velocity.

Interpretation of HI line profiles