The Big Bang, GR & the Universe

Homogeneity and Isotropy of the Universe
Big-Bang and Steady State Cosmology
Detection of the Big Bang
Robertson-Walker Metric
Friedmann Equation
Ages and Fates of Flat Universes
The Cosmological Constant

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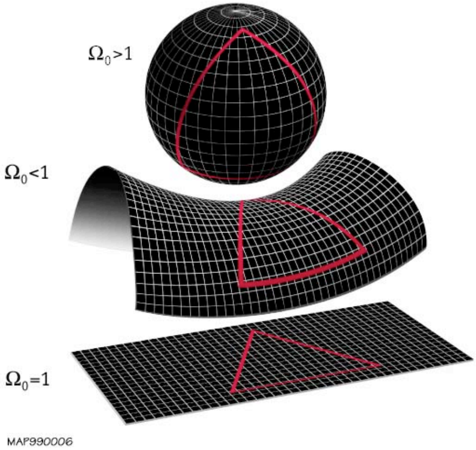
University of Rochester

The Big Bang, General Relativity & the Universe

- The expanding, isotropic, and homogeneous Universe
- Big-Bang and steady-state cosmology
- Alpher, Herman, and decoupling
- Penzias and Wilson's detection of the Big Bang's blast
- The Robertson-Walker metric and its use
- The Friedmann equation and its solutions
- The ages and fates of flat universes
- The cosmological constant

Reading: Kutner Sec. 20.4, Ryden Sec. 23.2–23.5



Possible spatial geometries of the Universe: from Wikimedia Commons and G. Hinshaw (NASA).

Hubble & the structure of the Universe

Recall that Hubble showed the Universe expands according to

$$v_r = H_0 d$$

Two more conclusions Hubble made based on a large collection of worldwide observations:

1. The Universe is **isotropic** on large scales, looking the same in all directions.
2. The Universe is **homogeneous** on large scales, looking the same at all locations.

Hubble at the 100″ reflector on Mt. Wilson (Life).

Isotropy and homogeneity

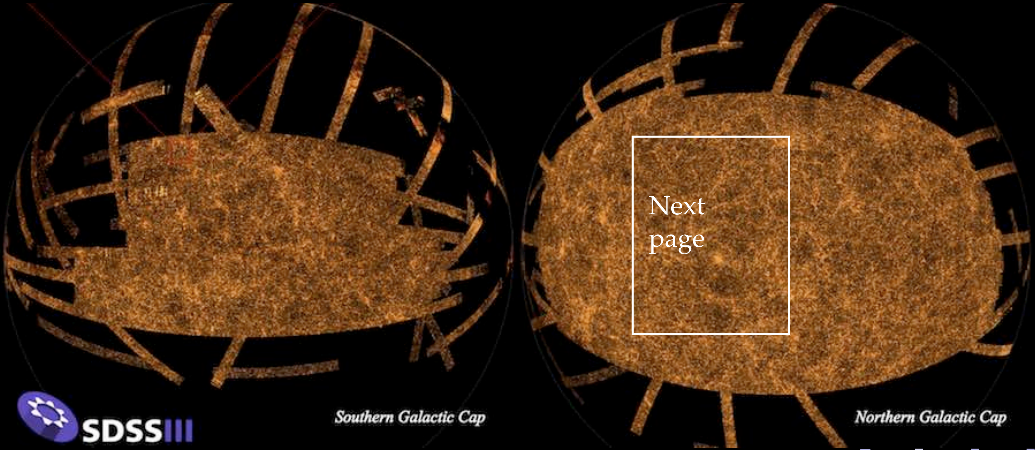
Note that by small scale we mean similar to or smaller than the typical distance between galaxy clusters.

By large scale we mean much greater than the typical distance between clusters, but still small compared to the observed size of the Universe.

- Recall: Galaxies are typically 1 Mpc apart. Cluster cores are typically less than $\sim$3 Mpc in radius. Clusters are typically 10 Mpc apart.
- The Universe is obviously not uniform on small scales: the sky is full of stars, galaxies, and galaxy clusters, all containing mass of relatively high density with virtually empty space between.
- By this definition of “large” and “small” scale, we will turn out to be OK with the assumption of Euclidean geometry, despite what comes next.

Isotropy on large scales

Galaxies in the Sloan Digital Sky Survey (SDSS) are shown below. The survey covers $3.55\ \mathrm{sr} = 11{,}700\ \mathrm{deg}^2$, or 28% of the sky, centered on the Galactic poles, and identified all galaxies brighter than $R = 22.2$.

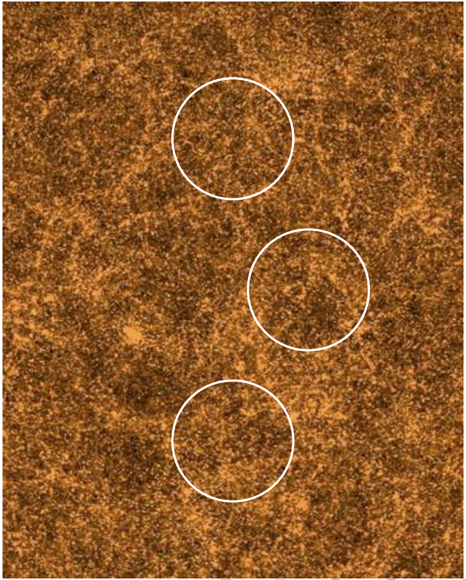


Isotropy on large scales

Each dot in the image is a galaxy. Each "smudge" or filament is a cluster or supercluster.

Isotropy on the scale represented by these circles means that approximately the same number of galaxies are contained within them, no matter where on the sky they are placed.

I.e., the Universe basically looks the same in every direction.



Homogeneity on large scales

The **homogeneity** of the Universe means that the number density of galaxies is uniform on **large scales**. In other words, the Universe looks the same from any viewpoint.

Hubble found this by observing great numbers of galaxies in selected parts of the sky and plotting the number per solid angle brighter than flux f_0 as a function of f_0, and seeing that

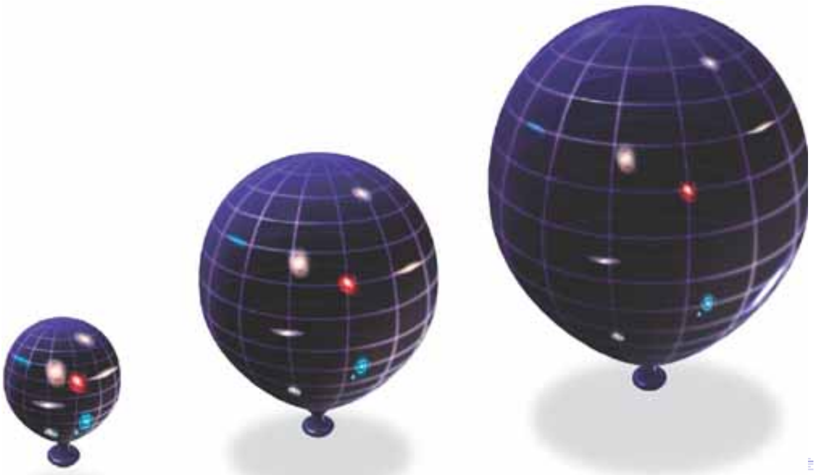
$$N(f > f_0) \propto f_0^{-3/2}$$

Recall our discussion of the distribution of stars in the Milky Way; $f_0^{-3/2}$ is expected in a uniform distribution.

Since the Universe is homogeneous and expanding, we would observe the same recession velocities no matter our location. That is, there is no unique center in space for the expansion, as in an ordinary explosion and blast wave.

Understanding the expansion

A classic metaphor: Imagine that we are on the surface of an expanding balloon. At any location, all points around us will appear to be receding, with no point being the "center" of the expansion.

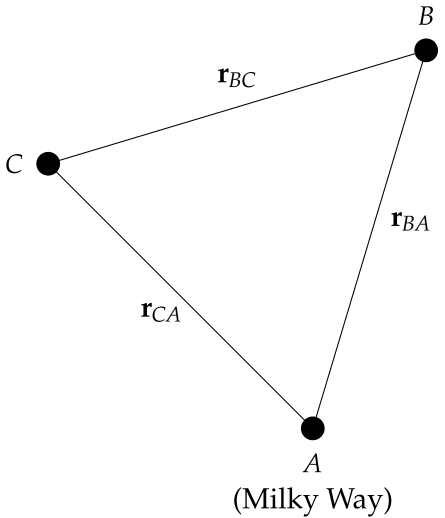


Understanding the expansion

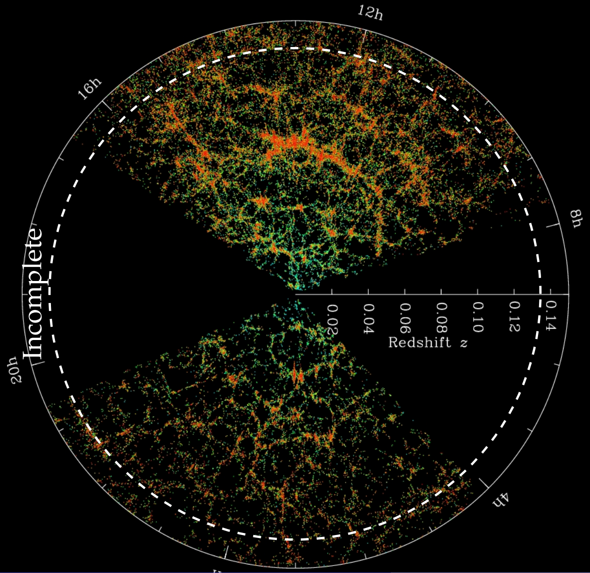
In Euclidean (flat) space, $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ is along $\mathbf{r}$, so

$$\begin{aligned} \mathbf{v}_{BC} &= \mathbf{v}_{BA} - \mathbf{v}_{CA} \\ &= H_0(\mathbf{r}_{BA} - \mathbf{r}_{CA}) \\ &= H_0\mathbf{r}_{BC} \end{aligned}$$

Thus, observers in galaxies B and C see the same Hubble Law that we do: the expansion looks the same from points A, B, and C.



Homogeneity on large scales

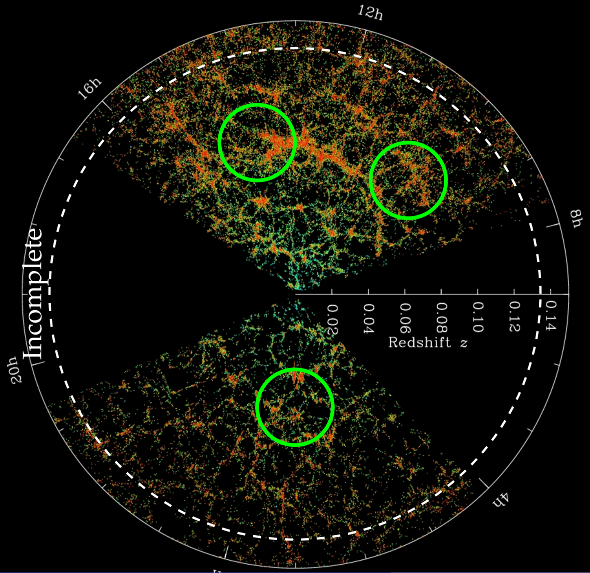


This is a 2.5°-thick slice in declination of the SDSS survey along the celestial equator ($\delta = 0$).

Redshifts are indicated by the radial axis, and galaxy luminosities are indicated by color (red = bright, blue = faint).

Note that the Galactic Plane has been cut out.

Homogeneity on large scales



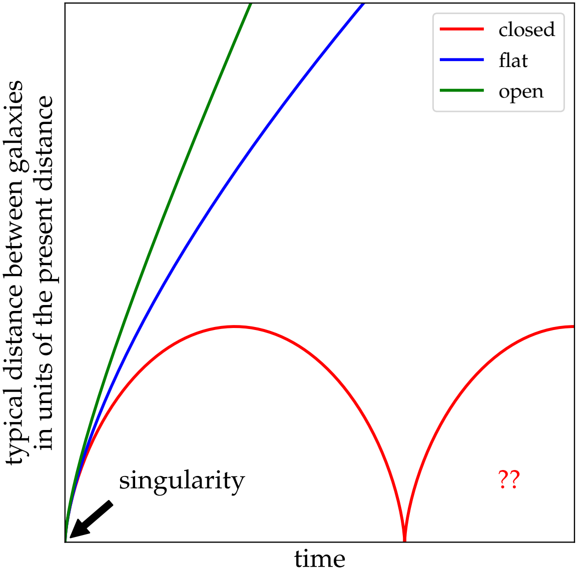
Homogeneity on the scale of these circles' diameters means that approximately the same numbers of galaxies are contained within them, no matter where we put them within the volume of the Universe.

The map is complete out to $z = 0.13$ ($\sim$ 550 Mpc) and it is self-evidently homogeneous.

Early post-Hubble cosmology

Theorists immediately applied Einstein's general theory of relativity to isotropic, homogeneous, expanding universes and revealed a problem:

- All such model universes displayed a **mass-density singularity** at the origin.
- In fact, it can be proven quite generally that this is so; see Hawking & Penrose (1970).
- Such singularities — unphysical at first glance — arouse the suspicion of theoretical physicists.



Early post-Hubble cosmology

Accordingly, many leading cosmologists such as Einstein, de Sitter, Hoyle, Gold, Burbidge, and Arp promulgated steady-state models of the Universe, in which singularities are **not realized**.

- Example singularity that is not realized: At $r = 0$, the force between two finite-size masses in Newtonian gravity is infinite.

The steady-state models also maintained the Universe at constant density on (time) average despite the Hubble expansion.

- This was accomplished by positing the steady creation of matter out of nothing; they considered this violation of energy conservation less serious than mass-density singularities!

"**Big Bang**" was Sir Fred Hoyle's pejorative term for models with mass-density singularities.

Early post-Hubble cosmology

Unintimidated, those who worried less about singularities than energy conservation — notably George Gamow — adopted "Big Bang" as a short description of their class of models.

Observational tests gradually started going in favor of Big Bang cosmology in the 1950s:

- E.g., the number counts of faint radio galaxies, which indicated a peak in the volume density of these objects in the early Universe ($\therefore$ higher density at earlier times and galaxy evolution).
- If we were in a steady-state universe, then there would be no expected evolution in the density of galaxies, i.e., Hubble's $f_0^{-3/2}$ observation should be true everywhere. But this is not the case…

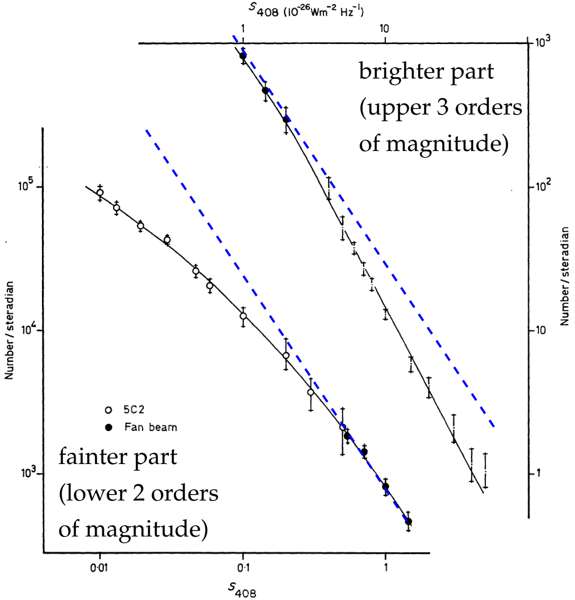
Early post-Hubble cosmology

The count of radio sources as a function of brightness (Pooley & Ryle 1968).

If the Universe were uniform and constant in density and galaxies never changed, then

$$N(f > f_0) = A f_0^{-3/2}$$

shown by the dashed line with slope $-3/2$ in the figure.



Alpher, Herman, & decoupling

Proposed in the late 1940s by Gamow's students Ralph Alpher and Bob Herman (Gamow 1948, Alpher & Herman 1948), the Universe started off very hot and dense and cooled as it expanded, down to the very low temperature range corresponding to the darkness of the night sky.

Then, they identified an event undergone at the expansion at a specific temperature: decoupling.

- The early Universe was so hot and dense that matter and photons were in thermal equilibrium (coupled), and all atoms were ionized.
- The photons eventually cooled (redshifted) to the point that they were incapable of ionizing atoms.
- In the laboratory, this takes place at about $T = 3000$ K for hydrogen.

Alpher, Herman, & decoupling

This event is called decoupling, because matter is opaque to radiation at the same blackbody temperature at temperatures above 3000 K; the average photon will be absorbed by ionization of the first neutral atom it finds.

- Thus matter and radiation are tightly coupled by ionization and recombination in the early Universe.

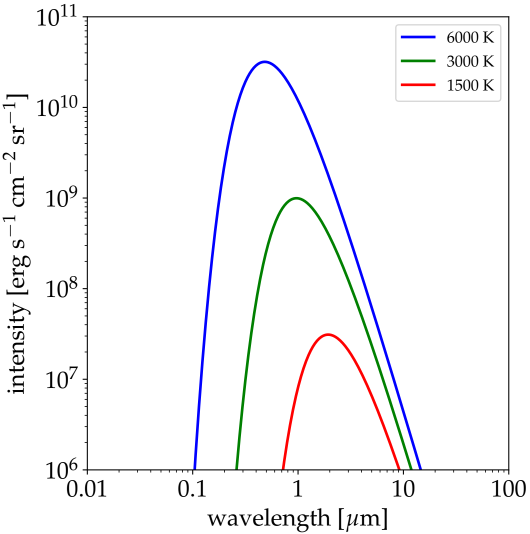
By the same token, matter below 3000 K is very transparent to radiation at the same temperature.

Therefore, radiation and matter decouple — live separate thermodynamic lives, no longer in equilibrium — after the Universe cools past this point.

Alpher, Herman, & decoupling

Alpher and Herman realized this meant that the Universe has a photosphere, just like a star does: the **decoupling surface**.

- Since the Universe is transparent after decoupling, we should be able to see all the way to the 3000 K surface!
- Since the Universe is opaque before decoupling, the surface emits like any other 3000 K blackbody.
- The light started off in the visible range but, due to our great distance to the source, it would be redshifted to much longer wavelengths.

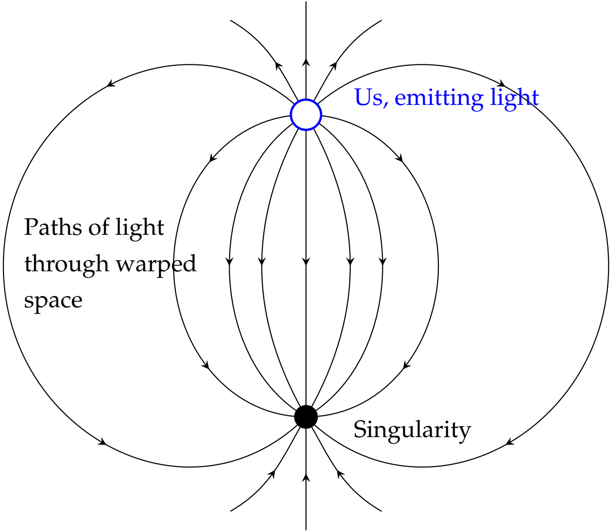


Alpher, Herman, & decoupling

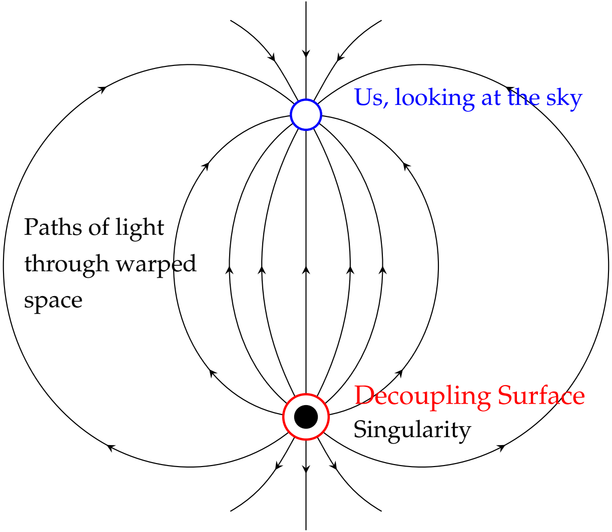
Alpher and Herman also realized that since the decoupling surface is close to the original singularity, the light should appear **isotropic** to us. In analogy with a black-hole singularity:

- Light emitted from within a black hole horizon cannot escape and therefore must fall into the singularity no matter what direction it is emitted. All light paths end at the singularity.
- By the same token, since light can travel in either direction along these paths, light emitted from the surroundings of the singularity would seem to an observer inside the horizon to arrive from all directions rather than one particular direction.
- It would look as if the surroundings of the singularity filled the entire sky.

Within a black hole horizon



Analogy with the Universal singularity



Alpher, Herman, & decoupling

Finally, Alpher and Herman realized that the spectrum of redshifted blackbody radiation would appear to be that of a colder blackbody; they estimated $\sim$5 K.

They also noted that a steady-state Universe would be incapable of producing such radiation, allowing for the hypothesis to be *falsified*.

Recall that

$$\frac{\lambda_0}{\lambda} = 1 + z$$

At the decoupling surface, the energy per unit time, area, solid angle, and bandwidth S_λ is given by the Planck function at the decoupling temperature T:

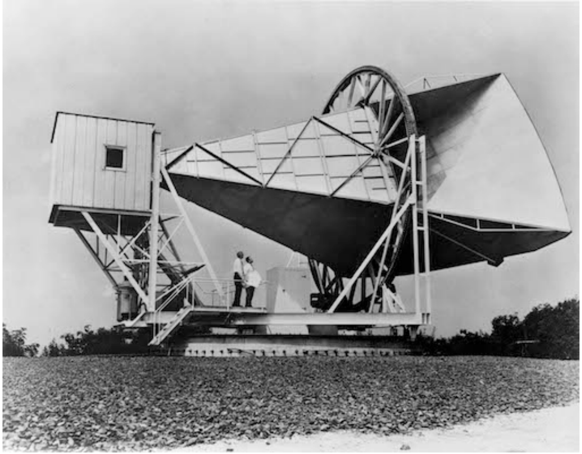
$$S_\lambda = B_\lambda(T)$$

The spectrum of the decoupling surface, as presently observed, should also be a blackbody at temperature $T_0 = T\frac{\ell}{\ell_0}$, where ℓ is some characteristic length.

The Cosmic Microwave Background

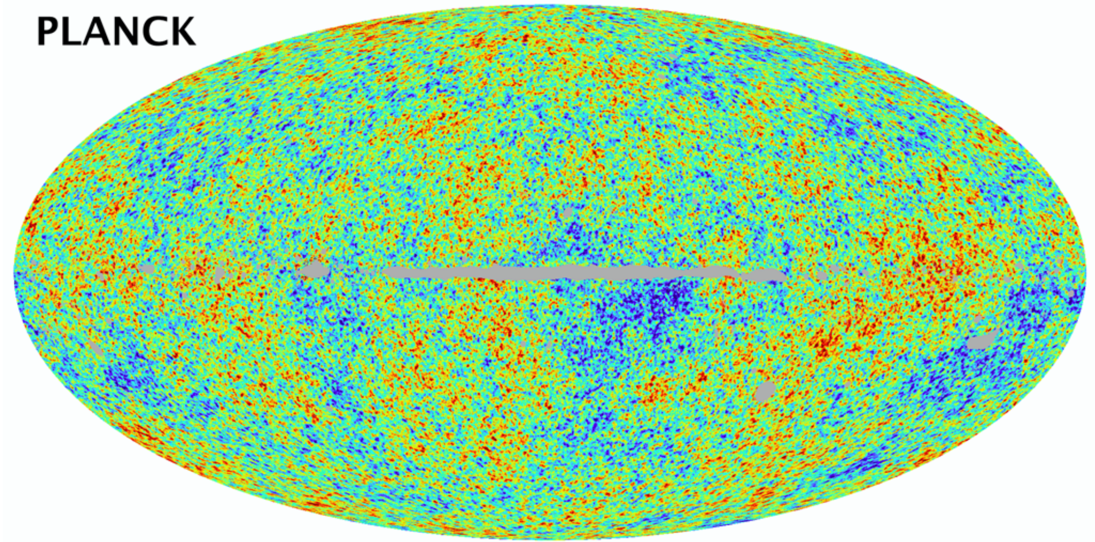
While building a microwave antenna and receiver for satellite communication, Arno Penzias and Robert Wilson of Bell Labs accidentally detected the relic radiation from the Big Bang (Penzias & Wilson 1965), now called the cosmic microwave background (CMB).

Thus the blast from the Big Bang is seen directly. This is the strongest support for Big Bang models. For their immensely influential discovery, Penzias and Wilson shared the 1978 Nobel Prize in Physics.



The CMB intrinsic anisotropy

The CMB is extremely isotropic; its temperature fluctuates on the order of 1 part in 10^{-5}.

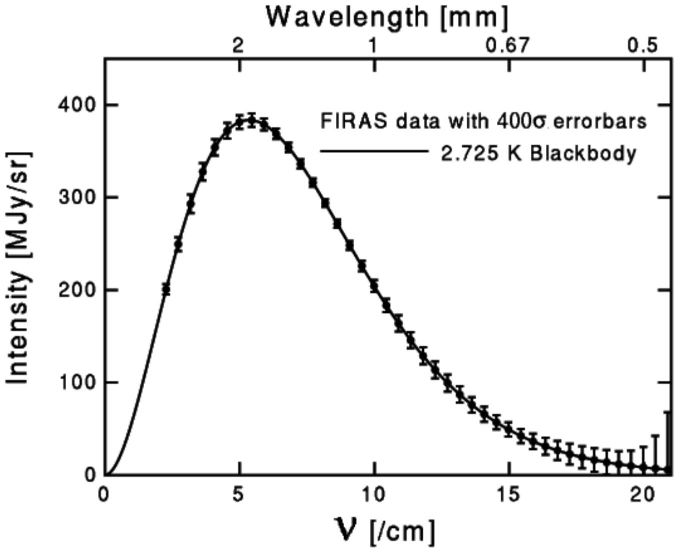


The Cosmic Microwave Background

Penzias and Wilson observed an astrophysical background of 3.5 ± 1.0 K (Penzias & Wilson 1965).

It turns out the CMB is a perfect blackbody with an effective temperature of $T_e \approx 2.73$ K (Mather et al. 1994, Wright et al. 1994, Fixsen et al. 1996).

For the measurement with COBE, John Mather and George Smoot shared the 2006 Nobel Prize in Physics.



How distant is the decoupling surface?

The redshift of decoupling can be obtained almost trivially:

$$1 + z = \frac{\lambda_0}{\lambda} = \frac{T}{T_0} \approx \frac{3000\ \mathrm{K}}{2.73\ \mathrm{K}} \approx 1100$$
$$\therefore z \approx 1100$$

Measurements with Planck, combined with weak lensing and other non-CMB measurements, give $z = 1089.80 \pm 0.21$ (Planck Collaboration 2020).

The most distant galaxy yet detected has $z = 13.27$. So the last scattering surface is substantially further away than any galaxy!

Questions about the Universe

We now know that the Universe is:

- **Expanding**, **isotropic** (uniform in all directions), and **homogeneous** (smooth, similar on large scales)
- Ruled by gravity and the kinetic energy of galaxies and clusters

But this raises several questions:

- How can we use the observations of galaxy motions and distributions, along with our theories of gravity, to determine the density of mass and energy on large scales?
- What is the age of the Universe?
- What is the fate of the Universe? How will the expansion change with time?

A Newtonian universe?

In an infinite, **homogeneous**, and **isotropic** Universe, the gravitational acceleration g should be zero everywhere. Why?

Averaged over large enough scales, a test mass should be equally attracted to all directions in a homogeneous and isotropic universe.

If space is Euclidean and gravity is Newtonian, the only way to get this property is if **space is completely empty**. Reason:

- A large sphere can be drawn through any point.
- The acceleration g at this point due to matter outside the sphere is zero (that matter is uniform, infinite,...).
- The acceleration g due to matter within the sphere is

$$g = \frac{GM(r)}{r^2}$$

  and so $g = 0$ iff the sphere is empty — $M(r) = 0$ — and all other such spheres are empty, by homogeneity.

General Relativistic universes

The Universe is not empty, so it cannot be Newtonian. Fortunately, General Relativity offers a good description, and large-scale structure was the first problem to which Einstein applied GR.

- Note: GR and the solutions to the **Einstein field equations**

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

  are covered in PHYS 231 and upper-level physics courses.

- The field equations are nonlinear second-order differential equations involving symmetric second-rank tensors. They can be solved uniquely, like any other differential equation, given enough boundary conditions.
- The solution to Einstein's equations is a second-rank tensor called the **metric**, which describes the measures and curvatures of spacetime under the given boundary conditions.

General Relativistic universes

For any given metric, we can define an **absolute interval** between two events in spacetime.

Flat spacetime

In flat spacetime with no mass present — the limit where GR converges to special relativity — the infinitesimal absolute interval is

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

where dx, dy, dz, and dt are infinitesimal intervals between two events, measured in one reference frame.

The combination ds^2 is independent of reference frame, as you will show in the homework.

The flat-space interval is sometimes called the **Minkowski interval**.

General Relativistic universes

Curved spacetime outside a spherical mass M

The **Schwarzschild metric** describes the solution to Einstein's equations outside a spherical mass M with zero charge and angular momentum (and $\Lambda = 0$). Its absolute interval, in spherical coordinates, is

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

where $r > \frac{2GM}{c^2}$.

The expressions for the spacetime structure outside a black hole, discussed when we covered solar-mass black holes, were derived from this absolute interval.

General Relativistic universes

Isotropic and homogeneous universe

The solution to Einstein's equations in an expanding, isotropic, and homogeneous Universe are called the **Robertson-Walker metric**, which corresponds to an absolute interval

$$ds^2 = c^2dt^2 - R(t)^2\left[\frac{dr_*^2}{1-kr_*^2} + r_*^2 d\theta^2 + r_*^2 \sin^2\theta d\phi^2\right]$$

where

$R(t)$	is the **scale factor**: radius of curvature of the Universe
$k = 0, \pm 1$	depending on the curvature
r_*, θ, ϕ	are spherical "comoving" coordinates (dimensionless)