Ever Since Decoupling

Constraining the constants in the Friedmann Equation
Intrinsic Anisotropy in the CMB
Properties of the Flat Universe

April 30, 2024

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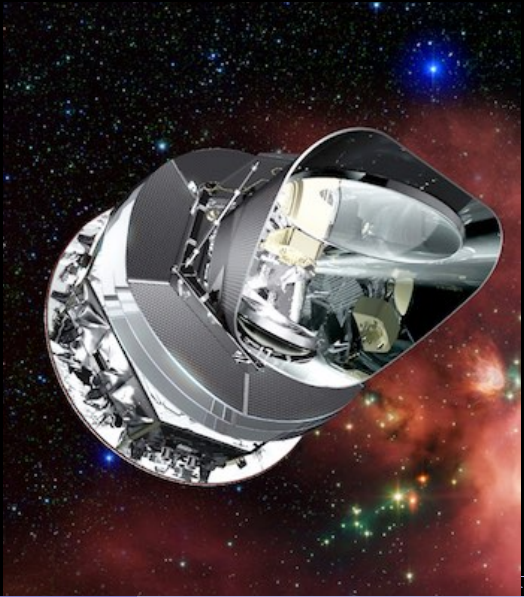
Ever since decoupling

- The cosmological constant
- Constraining the constants in the Friedmann equation: Acceleration of high-redshift galaxies
- Intrinsic anisotropies in the CMB
- Baryon acoustic oscillations: a standard ruler
- Properties of the flat Universe

Reading: Kutner Sec. 21.1, Ryden Sec. 24.3–24.4



Artist's drawing of the Planck spacecraft, which mapped the cosmic microwave background from the L_2 point between July 2009 and October 2013.

Notes on the cosmological constant

Despite the appearance that it functions as just another energy density, the cosmological constant Λ is very different from the density terms.

- The total mass of a physical universe is constant — energy is conserved — so the mass density decreases monotonically with time in an expanding universe.
- But Λ, being constant, *does not decrease with time*. As the volume of an expanding universe increases, the total energy represented by Λ, which we call **dark energy**, increases.
- The dark energy in a universe is not necessarily conserved. Its increase is responsible for the exponential, accelerating expansion in flat universes with nonzero Ω_Λ.

Is our Universe among these choices?

$$\dot{a}^2 = H_0^2 \left[1 + \Omega_{M_0} \left(\frac{1}{a} - 1 \right) + \Omega_{\Lambda_0} \left(a^2 - 1 \right) \right]$$

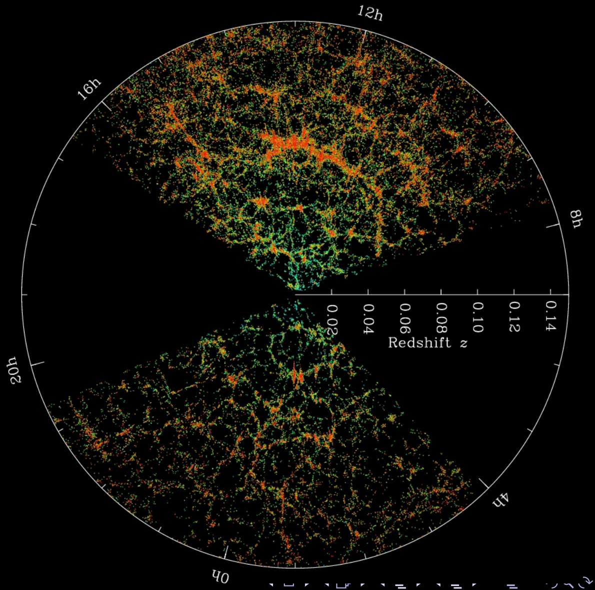
A wide range of ages and fates is consistent with homogeneity, isotropy, and expansion at 73.04 km/s/Mpc. So we need more experimental constraints:

- Independent measurements of the **age of the Universe**. For example, the age of the Milky Way's globular clusters, derived from main-sequence turnoff, is ~ 13 Gyr (Krauss & Chaboyer 2003, Marin et al. 2009). This sets a lower bound on the age of the Universe, which must be at least a bit older than its contents.
- Measurements of the **mass density** Ω_M.
- Measurements of the **acceleration** or **deceleration**, $\dot{a}$, of the expansion of the Universe.
- Measurements of the **curvature**, k, of the Universe.

Measurements of the mass density

One way to determine the mass density, ρ_{M_0} or Ω_{M_0}, is to measure fluctuations in the 3-D distributions of galaxies or galaxy clusters determined from sky surveys and radial velocity measurements.

- Principle: relate statistical variation in galaxy distribution to average values.
- Statistical variation is measured by galaxy or cluster **correlation functions**, or their corresponding **power spectra**.
- Compare to statistics of gravitational structure formation models to get the total average density, including dark matter.

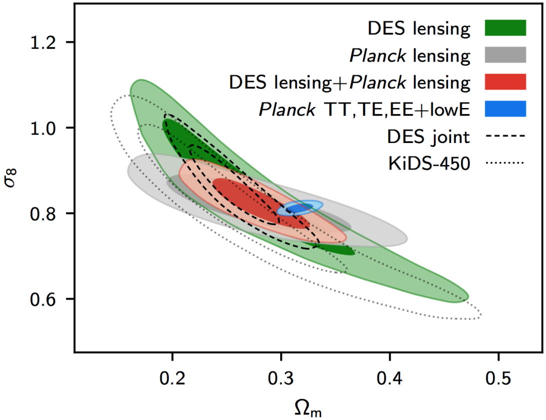


Measurements of the mass density

Observational bounds on Ω_M made from the number density of galaxy clusters (Planck Collaboration 2018) and hence sensitive to luminous and dark matter:

$$\Omega_M = 0.3111 \pm 0.0056$$

The clusters were observed in measurements by the Planck satellite using the Sunyaev-Zel'dovich effect (Sunyaev & Zel'dovich 1970).



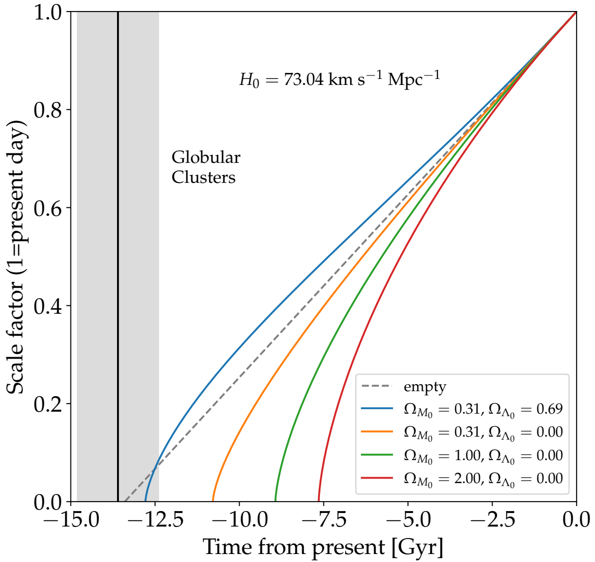
The contours describe different methods of measuring the parameters; the blue ellipses show fit results from 2018 (Planck Collaboration 2018). Note that σ_8 is the measured RMS in galaxy numbers within spheres of radius 8 h^{-1} Mpc, where $H_0 = 100h$ km s^{-1} Mpc^{-1}.

Measurements of the mass density

Globular cluster ages and measurements of Ω_{M_0} firmly rule out the possibility that we are in a matter-dominated universe, but universes with a nonzero cosmological constant appear to solve the age problem fairly easily.

The expansion of the universe decelerates for large Ω_{M_0} due to the effect of gravity.

But, if $\Omega_{\Lambda_0} \neq 0$, the accumulation of dark energy eventually accelerates the expansion. So, look for observational evidence of acceleration.

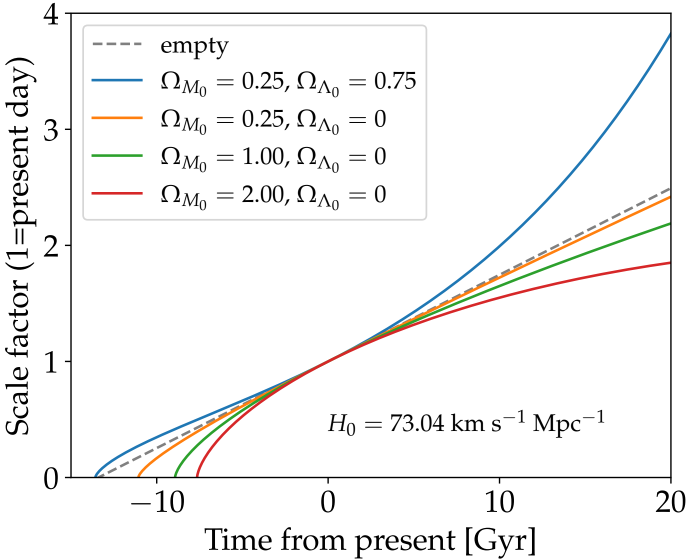


Universal acceleration

In our usual plots of a and t, **deceleration** appears as concave-downward curvature, as in the universes with $\Lambda = 0$.

And **acceleration** appears as concave-upward curvature, as in our results for flat universes several Gyr after the Big Bang.

So we should look for observational evidence of $\Lambda \neq 0$.



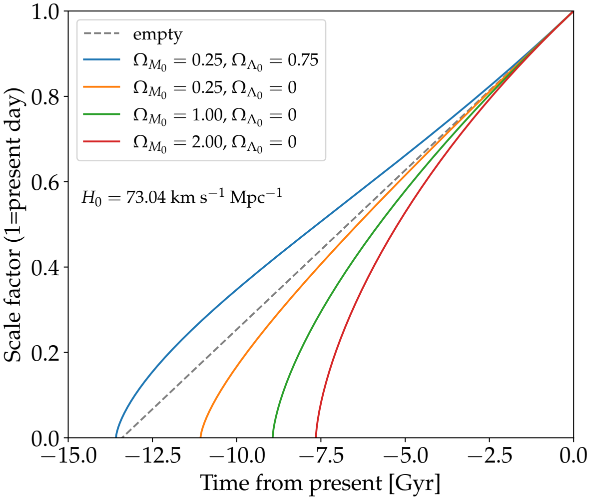
Universal acceleration

Recall that the normalized scale factor a is proportional to the typical distance between galaxies in the Universal expansion.

Thus acceleration and deceleration appear as departures from the Hubble diagram, which is linear in this rendition.

- Above the empty universe: objects will appear fainter than expected from their distances.
- Below the empty universe: objects will appear brighter than expected.

Note how the "empty" a-t plot has slope H_0 and t-intercept $1/H_0$.

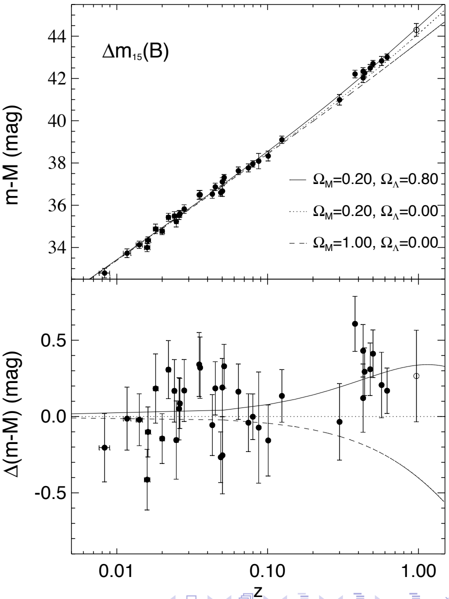


Universal acceleration

Using SNe Ia found in $z \sim 0.1 - 1$, evidence was found for **acceleration** (Knop et al. 2003). Note the presence of systematic uncertainty due to lower metallicity in the distant past.

Big caveat: The distances to these galaxies are still, of course, measured by SNe Ia luminosity.

- Their light was emitted far enough in the past that metal abundances were substantially smaller than today.
- So we probably do not know the SN Ia luminosity of these objects accurately enough to confidently use them as standard candles at high redshift.

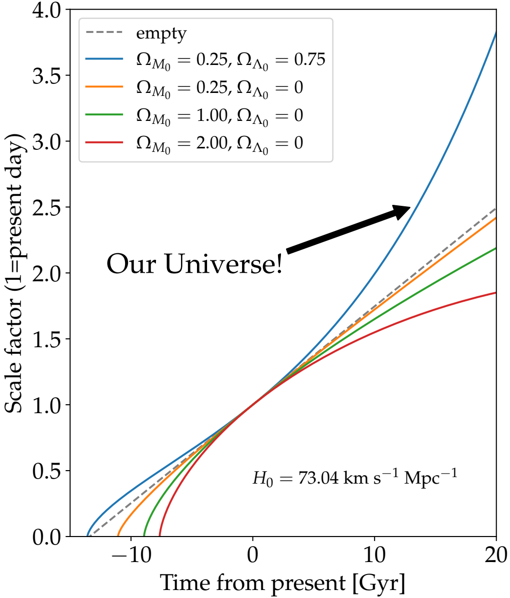


SNe Ia observations in 1998 from the Supernova Search Team (Riess et al. 1998) .

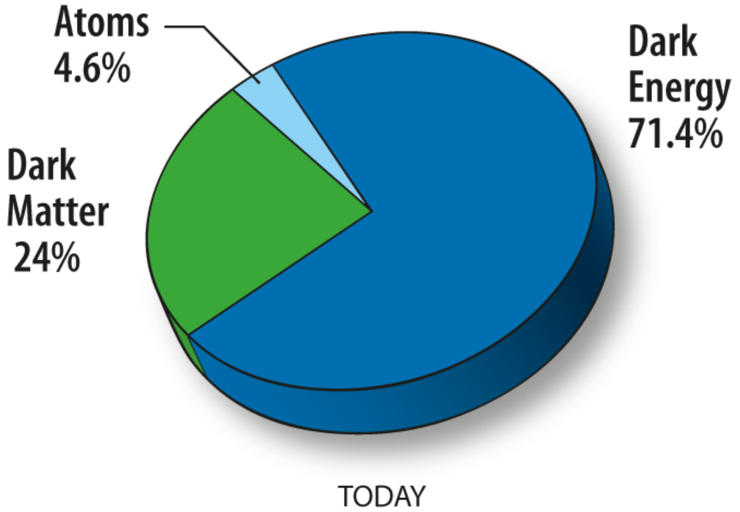
Universal acceleration

Despite the considerable and ongoing uncertainties about high-redshift SNe Ia, the observation of Universal acceleration was the first respectable indication of a non-zero cosmological constant.

The 2011 Nobel Prize in Physics went to Saul Perlmutter, Adam Riess, and Brian Schmidt, who led the SNe Ia surveys.



Summary: The Universe today



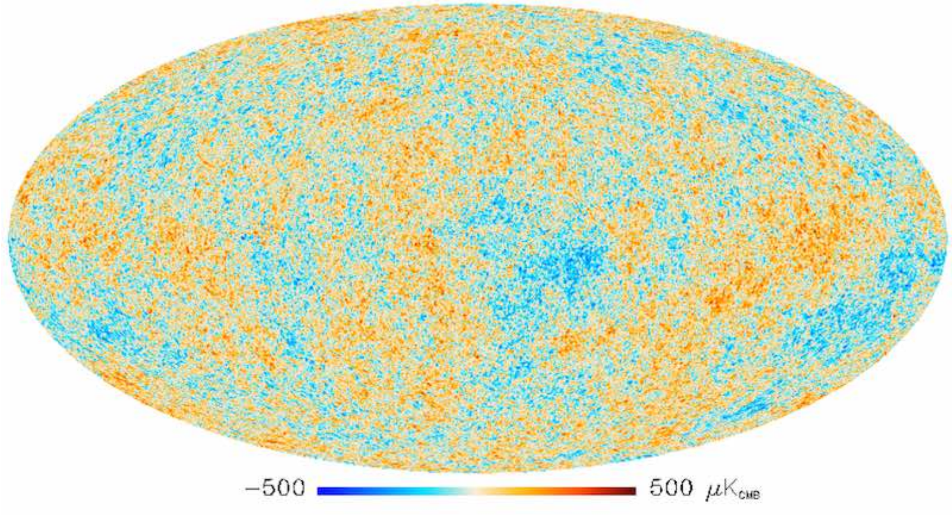
Small-scale anisotropy in the CMB

The satellite observatories WMAP and Planck have both mapped the cosmic background radiation over the whole sky at an angular resolution of a few arcminutes.

The resulting images have resolved the small-amplitude anisotropies in the background radiation.

- The anisotropies are thought to be density-temperature fluctuations due to adiabatic **acoustic oscillations**, endemic in the Universe before decoupling.
- The WMAP and Planck images represent the fluctuations at the instant **decoupling** forever stopped the oscillations.

Small-scale anisotropy in the CMB



Sky image from Planck based on 30 months of data (Planck Collaboration 2014). Due to its better angular resolution, very small anisotropies appear brighter to Planck, necessitating the larger temperature scale.

Small-scale anisotropy in the CMB

Origin of the small-scale anisotropies: **inhomogeneities**, or peaks and troughs in the density, which are to be expected in an expanding gas like that of the Big Bang, even if their contrast is small.

These inhomogeneities oscillate acoustically — i.e., they ring like a bell, driving sound waves into their surroundings.

- Gravity tends to collapse the density peaks, heating them up and increasing the temperature of the radiation (light) within.
- This radiation pressure pushes back against gravity, forming a bubble. As the bubble expands, the energy density of the radiation decreases and gravity starts pulling the material back in. This process repeats for as long as the radiation and matter are in thermal equilibrium.
- The opposite process happens in the density troughs.

Acoustic peaks in the CMB power spectrum

Acoustic oscillations in the primordial plasma proceeded until **decoupling**, at which point the radiation escaped from the matter.

Think of the Universe before decoupling like a **resonant cavity**, much like the radially pulsating stars discussed earlier in the semester. There would be a **fundamental mode** evident in the sound spectrum.

Thus, the CMB provides a last snapshot of the Universe in the act of this “ringing,” which is preserved in the anisotropy.

Note: there are lots of resonances, but wavelengths larger than $\frac{2ct}{a}$ at decoupling will not appear in the CMB. That is,

$$\frac{\lambda_{\max}}{2} = \ell_d \approx \frac{ct_d}{a_d}$$

The quantity ℓ_d is called the **acoustic horizon**. It is the distance limit for cause and effect before decoupling.

The acoustic horizon

The acoustic horizon turns out to be independent of curvature. For $a \ll 1$, the relations between t and a for all universes we consider reduce to the same formula, which we will obtain for the **flat universe**. Recall our solution:

$$t(a) = \frac{2}{3H_0\sqrt{1-\Omega}} \sinh^{-1}\left(\sqrt{\frac{1-\Omega}{\Omega}a^3}\right)$$

Noting that

$$\sinh^{-1} x = x - \frac{1}{6}x^3 + \ldots$$

Our solution to first order in $x = \sqrt{(1-\Omega)a^3/\Omega} \ll 1$ becomes

$$t(a) \approx \frac{2a^{3/2}}{3H_0\Omega^{1/2}}$$

The acoustic horizon

Since $a \ll 1$ at decoupling,

$$\ell_d = \frac{ct_d}{a_d} = \frac{2ca_d^{1/2}}{3H_0\Omega^{1/2}}$$

no matter the universe. The redshift of the decoupling surface is

$$z_d \approx 1090 \implies 1 + z_d = \frac{R_0}{R_d} = \frac{a_0}{a_d} = \frac{1}{a_d}$$

So the **acoustic horizon** — the scale length of the fundamental mode of oscillation — is

$$\ell_d = \frac{2c}{3H_0\sqrt{\Omega(1+z_d)}} \approx 150 \text{ Mpc}$$

for $H_0 = 74.03 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega = \Omega_{M_0} = 0.3$.

The acoustic horizon

Note that because Hubble's constant H_0 is experimentally determined, it is often expressed as

$$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$$

where h corresponds to the measured part of the quantity (caution: it is not Planck's constant!). Given the SH0ES measurement of H_0 (Riess et al. 2019), $h = 0.7403 \pm 0.0142$.

In terms of h, the acoustic horizon is

$$\ell_d \approx 110h^{-1} \text{ Mpc}$$

This expression is often used in the literature to back out the explicit dependence of ℓ_d on measurements of H_0, which (as you will recall) have historically suffered from major systematic uncertainties.

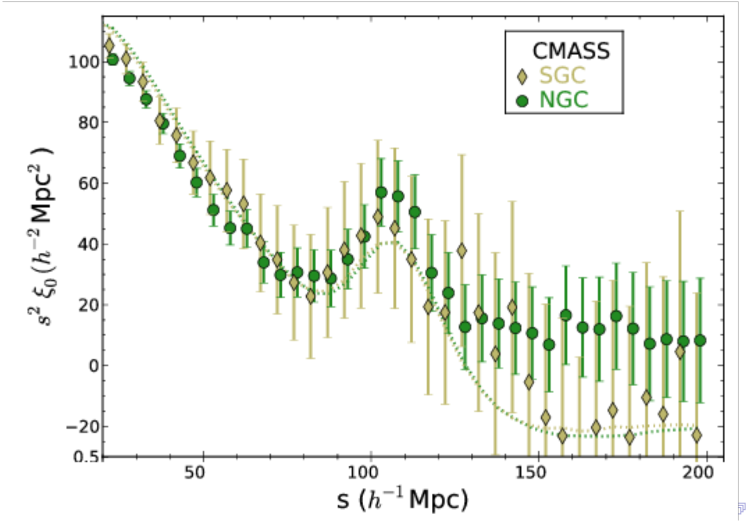
The acoustic horizon

A histogram of the small-scale anisotropies as a function of projected linear size will have a strong peak near 150 Mpc or $110h^{-1}$ Mpc, and other peaks for higher-order modes of oscillation.

- The peaks comprise a **standard ruler**, projected onto the decoupling surface.
- At decoupling, the oscillation stops because the photons escape, but the matter concentrations ("bubbles") will still tend to have a characteristic size $\sim$150 Mpc; this will show up in the distributions of galaxies.
- Sure enough, these **baryon acoustic oscillations** have been seen in spectroscopic measurements of the 3-D distribution of galaxies since 2005 (Eisenstein et al. 2005), with increasing refinement ever since (Anderson et al. 2012, Vargas-Magana et al. 2016, Ross et al. 2017, Beutler et al. 2017, DESI Collaboration 2024).

A new standard ruler

Correlation function of galaxies in SDSS DR12 BOSS (Ross et al. 2017).



A new standard ruler

The acoustic horizon is a standard ruler and is the same in any universe, but the **distance to the decoupling surface** and the apparent size of any given fluctuation depends on the **curvature** of the universe. Thus we can measure k.

Since the absolute interval for light $ds^2 = 0$, we can use the R-W metric to calculate the distance that light has traveled from the decoupling surface:

$$\Delta r = \int_{r_d}^{r_0} dr = c \int_{t_d}^{t_0} \frac{dt}{a(t)}$$

The results:

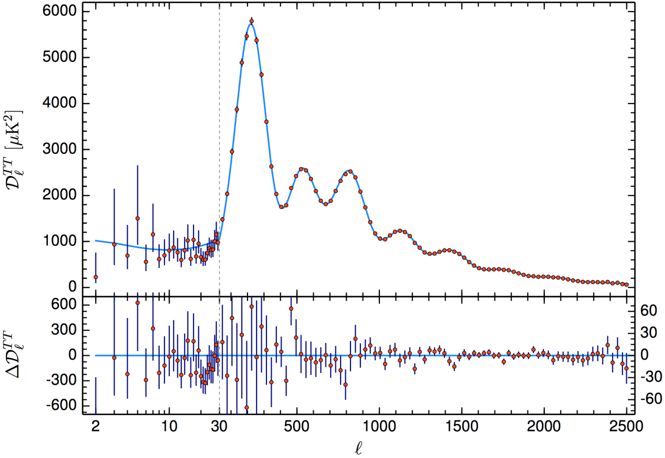
Universe	k	Δr [Gpc]	θ_d [°]
$\Omega_{M_0} = 0.3, \Omega_\Lambda = 0$	-1	24.62	0.35
$\Omega_{M_0} = 0.3, \Omega_\Lambda = 0.7$	0	13.05	0.66
$\Omega_{M_0} = 0.3, \Omega_\Lambda = 1.0$	$+1$	7.18	1.20

A new standard ruler

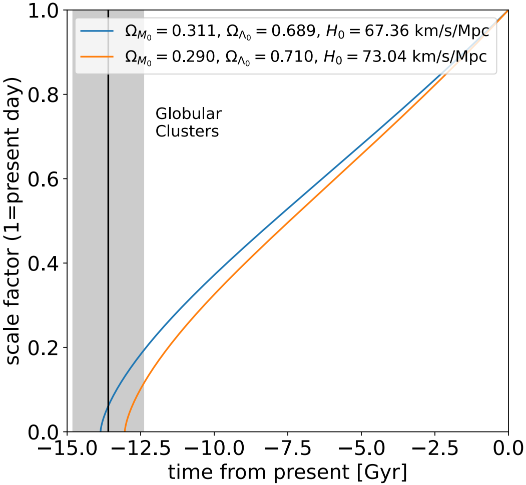
In the CMB power spectrum, the fundamental acoustic mode appears at 0.8°, but accounting for higher-order effects/details leads to 0.6° for the acoustic horizon (Page et al. 2003). Thus the Universe appears to be accurately and precisely flat ($k = 0$)!

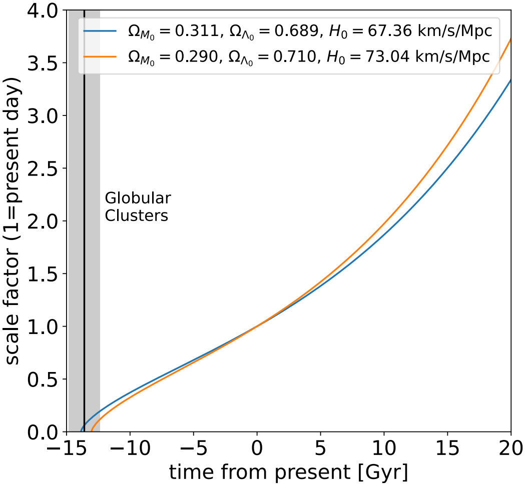
TT power spectrum of the CMB from the Planck mission.

The solid line shows the best-fit ΛCDM model, a flat universe dominated by dark energy ($\Omega_{\Lambda_0} \sim 0.7$) and a cold dark matter ($\Omega_{M_0} \sim 0.3$).



Properties of our flat Universe

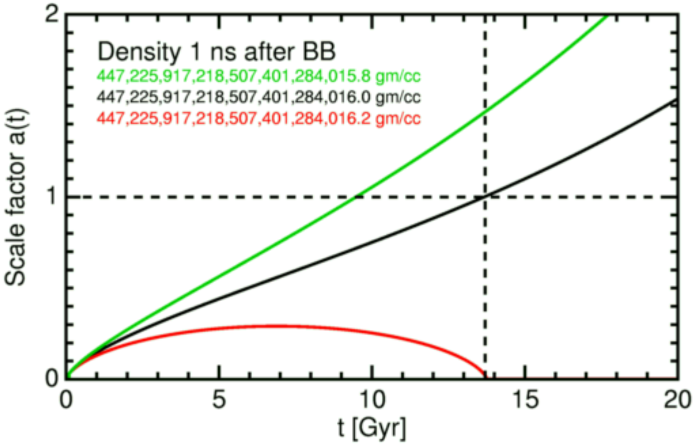




Disquieting features of our flat Universe

The flatness-oldness problem

- Ω_Λ grows as the Universe expands, while Ω_M is constant.
- Yet the Universe is flat: $\Omega_\Lambda + \Omega_M = 1$ to better than 1% precision.
- How is this degree of fine-tuning possible, given that the Universe is 13.8 Gyr old?



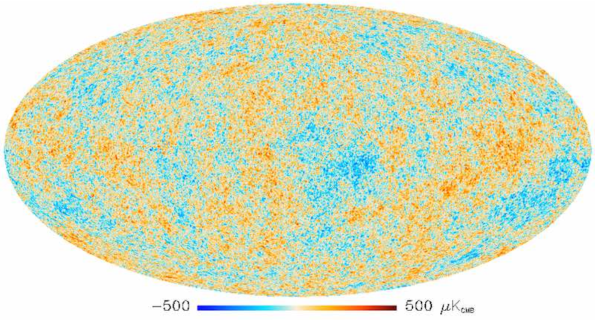
Ned Wright's cosmology tutorial

Disquieting features of our flat Universe

The horizon problem: CMB radiation arrives from all over the sky, having been radiated at the decoupling surface almost 13.8 Gyr ago.

Emission sources separated more widely than the acoustic-oscillation scale have been more than a light-travel time apart — thus out of causal contact — since the beginning.

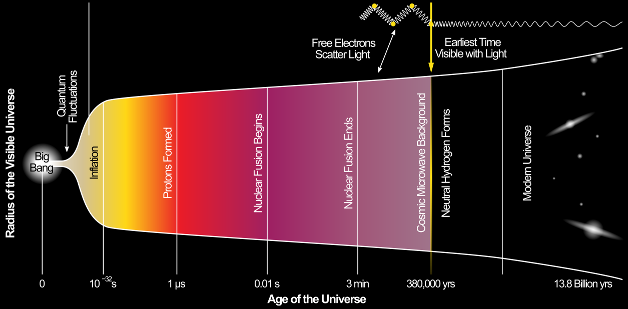
Yet the CMB manages to be smooth to better than one part in 10^5. How is this possible?



Inflation

Both of these problems have at least a theoretical solution: inflation, originated mainly by Alan Guth (1981).

- The idea is that the vacuum can have different states with large differences in energy density among them.
- Very shortly after the Big Bang — i.e., well before decoupling — the vacuum underwent a phase transition to a state with large energy density, which acts like a very large cosmological constant, Λ.



Inflation

- The vacuum did not remain in this state for long, but while it did, the Universe expanded exponentially, becoming very large.
- This explains flatness: The Universe appears flat because what is presented to us in the CMB is like the surface of a large sphere which appears flat "locally," like the surface of the Earth does.
- It also allows the whole universe to have been in causal contact before the evolutionary epoch, which would resolve the horizon problem.