High Precision Cosmology

Intrinsic Anisotropy in the CMB
Properties of the Flat Universe

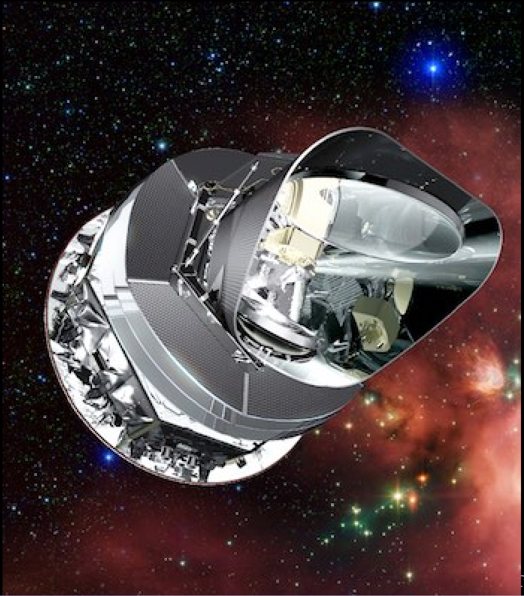
May 1, 2025

University of Rochester

High Precision Cosmology

- Intrinsic anisotropies in the CMB
- Baryonic acoustic oscillations: a standard ruler
- Properties of our flat Universe

Reading: Kutner Sec. 21.1, Ryden Sec. 24.3–24.4



Artist's drawing of the Planck spacecraft, which mapped the cosmic microwave background from the L_2 point between July 2009 and October 2013.

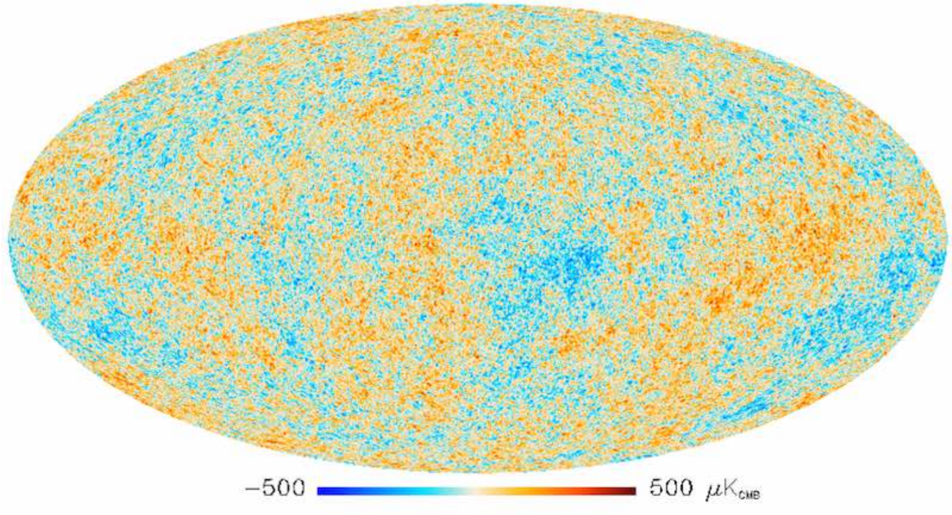
Small-scale anisotropy in the CMB

The satellite observatories WMAP and Planck have both mapped the cosmic background radiation over the whole sky at an angular resolution of a few arcminutes.

The resulting images have resolved the small-amplitude anisotropies in the background radiation.

- The anisotropies are thought to be density-temperature fluctuations due to adiabatic **acoustic oscillations**, endemic in the Universe before decoupling.
- The WMAP and Planck images represent the fluctuations at the instant **decoupling** forever stopped the oscillations.

Small-scale anisotropy in the CMB



Sky image from Planck based on 30 months of data (Planck Collaboration 2014). Due to its better angular resolution, very small anisotropies appear brighter to Planck, necessitating the larger temperature scale.

Small-scale anisotropy in the CMB

Origin of the small-scale anisotropies: **inhomogeneities**, or peaks and troughs in the density, which are to be expected in an expanding gas like that of the Big Bang, even if their contrast is small.

These inhomogeneities oscillate acoustically — i.e., they ring like a bell, driving sound waves into their surroundings.

- Gravity tends to collapse the density peaks, heating them up and increasing the temperature of the radiation (light) within.
- This radiation pressure pushes back against gravity, forming a bubble. As the bubble expands, the energy density of the radiation decreases and gravity starts pulling the material back in. This process repeats for as long as the radiation and matter are in thermal equilibrium.
- The opposite process happens in the density troughs.

Acoustic peaks in the CMB power spectrum

Acoustic oscillations in the primordial plasma proceeded until **decoupling**, at which point the radiation escaped from the matter.

Think of the Universe before decoupling like a **resonant cavity**, much like the radially pulsating stars discussed earlier in the semester. There would be a **fundamental mode** evident in the sound spectrum.

Thus, the CMB provides a last snapshot of the Universe in the act of this "ringing," which is preserved in the anisotropy.

Note: there are lots of resonances, but wavelengths larger than $\frac{2ct}{a}$ at decoupling will not appear in the CMB. That is,

$$\frac{\lambda_{\max}}{2} = \ell_d \approx \frac{ct_d}{a_d}$$

The quantity ℓ_d is called the **acoustic horizon**. It is the distance limit for cause and effect before decoupling.

The acoustic horizon

The acoustic horizon turns out to be independent of curvature. For $a \ll 1$, the relations between t and a for all universes we consider reduce to the same formula, which we will obtain for the **flat universe**. Recall our solution:

$$t(a) = \frac{2}{3H_0\sqrt{1-\Omega}} \sinh^{-1}\left(\sqrt{\frac{1-\Omega}{\Omega}a^3}\right)$$

Noting that

$$\sinh^{-1} x = x - \frac{1}{6}x^3 + \dots$$

Our solution to first order in $x = \sqrt{(1-\Omega)a^3/\Omega} \ll 1$ becomes

$$t(a) \approx \frac{2a^{3/2}}{3H_0\Omega^{1/2}}$$

The acoustic horizon

Since $a \ll 1$ at decoupling,

$$\ell_d = \frac{ct_d}{a_d} = \frac{2ca_d^{1/2}}{3H_0\Omega^{1/2}}$$

no matter the universe. The redshift of the decoupling surface is

$$z_d \approx 1090 \implies 1 + z_d = \frac{R_0}{R_d} = \frac{a_0}{a_d} = \frac{1}{a_d}$$

So the **acoustic horizon** — the scale length of the fundamental mode of oscillation — is

$$\ell_d = \frac{2c}{3H_0\sqrt{\Omega(1+z_d)}} \approx 150 \text{ Mpc}$$

for $H_0 = 74.03 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega = \Omega_{M_0} = 0.3$.

The acoustic horizon

Note that because Hubble's constant H_0 is experimentally determined, it is often expressed as

$$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$$

where h corresponds to the measured part of the quantity (caution: it is not Planck's constant!). Given the SH0ES measurement of H_0 (Riess et al. 2019), $h = 0.7403 \pm 0.0142$.

In terms of h, the acoustic horizon is

$$\ell_d \approx 110h^{-1} \text{ Mpc}$$

This expression is often used in the literature to back out the explicit dependence of ℓ_d on measurements of H_0, which (as you will recall) have historically suffered from major systematic uncertainties.

The acoustic horizon

A histogram of the small-scale anisotropies as a function of projected linear size will have a strong peak near 150 Mpc or $110h^{-1}$ Mpc, and other peaks for higher-order modes of oscillation.

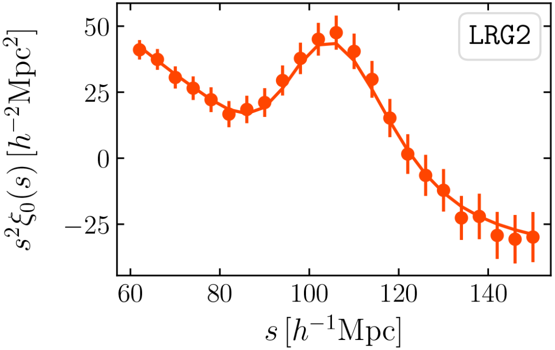
- The peaks comprise a **standard ruler**, projected onto the decoupling surface.
- At decoupling, the oscillation stops because the photons escape, but the matter concentrations ("bubbles") will still tend to have a characteristic size $\sim$150 Mpc; this will show up in the distributions of galaxies.
- Sure enough, these **baryon acoustic oscillations** have been seen in spectroscopic measurements of the 3-D distribution of galaxies since 2005 (Eisenstein et al. 2005), with increasing refinement ever since (Anderson et al. 2012, Vargas-Magana et al. 2016, Ross et al. 2017, Beutler et al. 2017, DESI Collaboration 2025).

A new standard ruler

The two-point correlation function, $\xi(s)$, is the **excess** probability that objects in a 3D survey lie a distance $s \pm \Delta s$ apart:

$$\xi(s) = \frac{n_R N_D(s)}{n_D N_R(s)} - 1$$

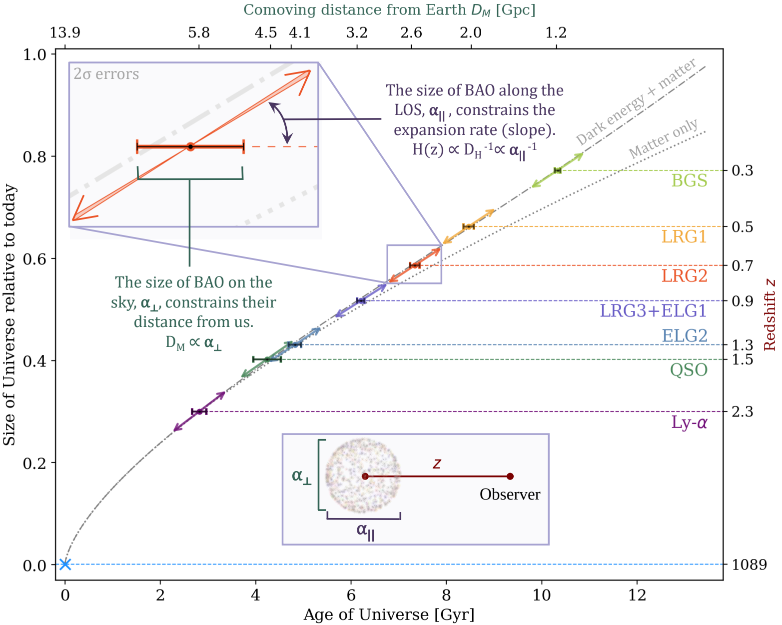
where n_D is the number of objects in the data, $N_D(s)$ is the number of **pairs** of objects in the data with separation $s \pm \Delta s$, n_R is the number of objects in mock data (distributed randomly), and $N_R(s)$ is the number of objects in the mock data with separation $s \pm \Delta s$.



Correlation function of luminous red galaxies (LRG) in DESI DR2 at $0.6 < z < 0.8$ (DESI Collaboration 2025).

A new standard ruler

As standard candles (like SNIa) can be used to construct a Hubble diagram, so too can standard rulers (like the BAO). Shown on the right is how measurements of the BAO at different redshifts constrain the expansion history of the universe. From the DESI Collaboration (2025).



A new standard ruler

The acoustic horizon is a standard ruler and is the same in any universe, but the **distance to the decoupling surface** and the apparent size of any given fluctuation depends on the **curvature** of the universe. Thus we can measure k.

Since the absolute interval for light $ds^2 = 0$, we can use the R-W metric to calculate the distance that light has traveled from the decoupling surface:

$$\Delta r = \int_{r_d}^{r_0} dr = c \int_{t_d}^{t_0} \frac{dt}{a(t)}$$

The results (where $\theta_d = \ell_d / \Delta r$ is the angular size of the BAO):

Universe	k	Δr [Gpc]	θ_d [°]
$\Omega_{M_0} = 0.3, \Omega_\Lambda = 0$	-1	24.62	0.35
$\Omega_{M_0} = 0.3, \Omega_\Lambda = 0.7$	0	13.05	0.66
$\Omega_{M_0} = 0.3, \Omega_\Lambda = 1.0$	$+1$	7.18	1.20

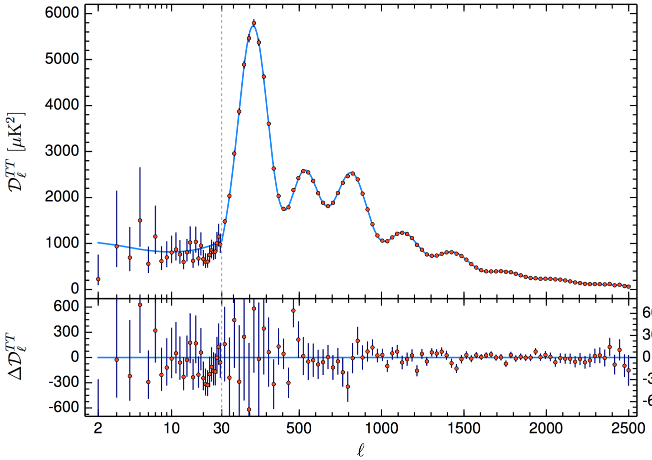
A new standard ruler

In the CMB power spectrum (histogram of fluctuations as a function of angular size), the fundamental acoustic mode appears at 0.66° (Planck Collaboration 2020). Thus the Universe appears to be accurately and precisely flat ($k = 0$)!

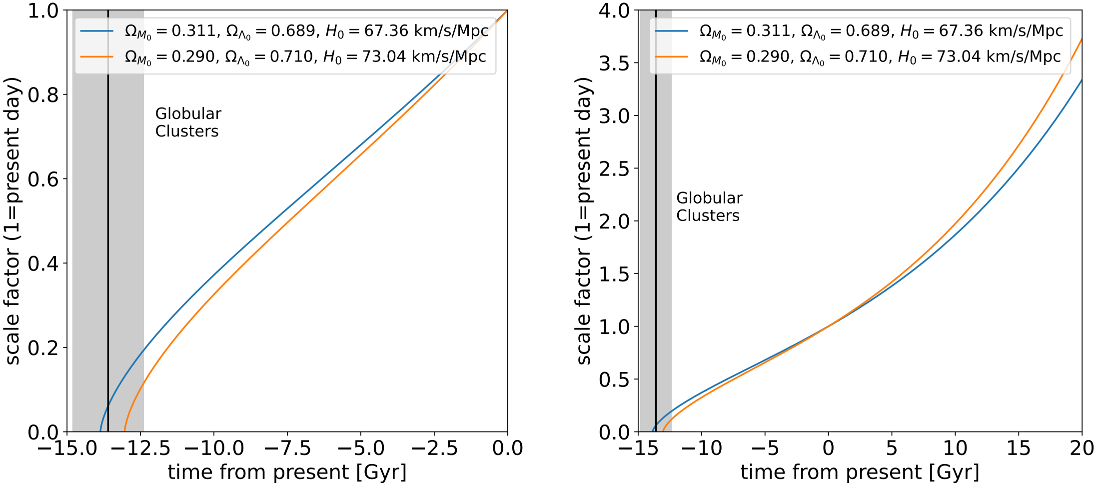
TT power spectrum of the CMB from the Planck mission.

The solid line shows the best-fit ΛCDM model, a flat universe dominated by dark energy ($\Omega_{\Lambda_0} \sim 0.7$) and a cold dark matter ($\Omega_{M_0} \sim 0.3$).

The additional peaks (at smaller angles) are harmonics of the oscillation; their amplitudes are sensitive to the Ωs.



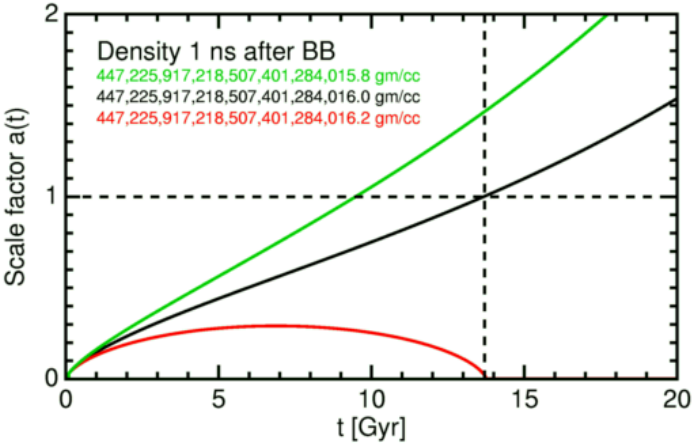
Properties of our flat Universe



Disquieting features of our flat Universe

The flatness-oldness problem

- Ω_Λ grows as the Universe expands, while Ω_M is constant.
- Yet the Universe is flat: $\Omega_{\Lambda_0} + \Omega_{M_0} = 1$ to better than 1% precision.
- How is this degree of fine-tuning possible, given that the Universe is 13.8 Gyr old?



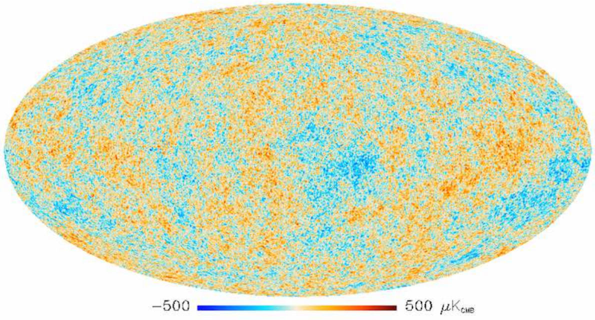
Ned Wright's cosmology tutorial

Disquieting features of our flat Universe

The horizon problem: CMB radiation arrives from all over the sky, having been radiated at the decoupling surface almost 13.8 Gyr ago.

Emission sources separated more widely than the acoustic-oscillation scale have been more than a light-travel time apart — thus out of causal contact — since the beginning.

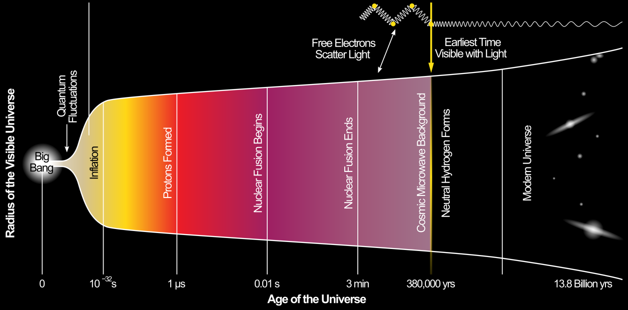
Yet the CMB manages to be smooth to better than one part in 10^5. How is this possible?



Inflation

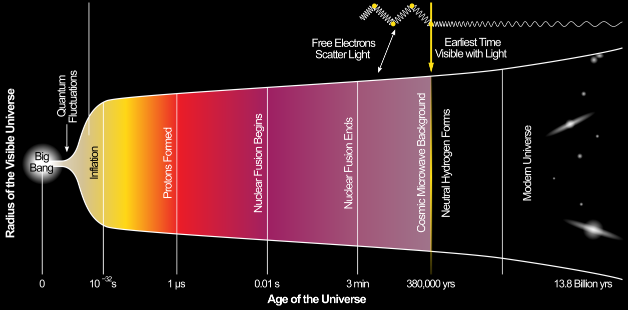
Both of these problems have at least a theoretical solution: inflation, originated mainly by Alan Guth (1981).

- The idea is that the vacuum can have different states with large differences in energy density among them.
- Very shortly after the Big Bang — i.e., well before decoupling — the vacuum underwent a phase transition to a state with large energy density, which acts like a very large cosmological constant, Λ.



Inflation

- The vacuum did not remain in this state for long, but while it did, the Universe expanded exponentially, becoming very large.
- This explains flatness: The Universe appears flat because what is presented to us in the CMB is like the surface of a large sphere which appears flat "locally," like the surface of the Earth does.
- It also allows the whole universe to have been in causal contact before the evolutionary epoch, which would resolve the horizon problem.



Disquieting features of our flat Universe

The cosmological constant itself is troublesome, because it has been put into GR *ad hoc* in Einstein's time and ours.

There was a questionable motivation for Λ originally, and we still have no consistent explanation of it in terms of experimentally identified particles and fields.

In fact, if you try to identify Λ with the energy density of the vacuum, an answer 10^{120} times bigger than the observed value of ρ_Λ is obtained. (No, this is not a typo.)

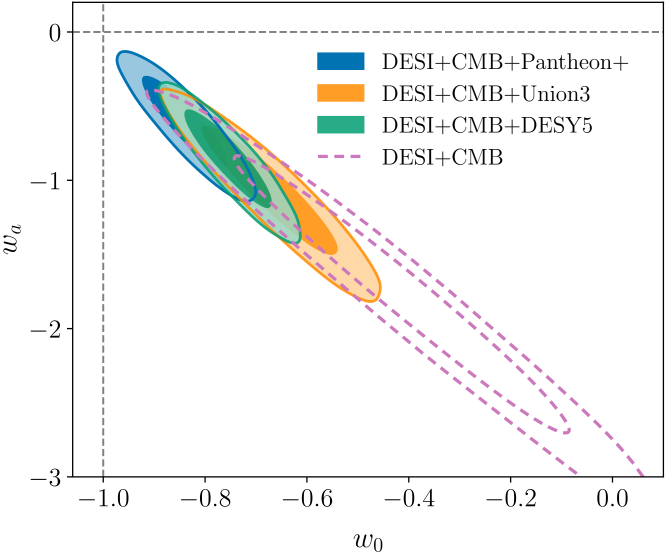
Beyond the lack of theoretical consensus is also the fact that dark energy has a large magnitude, comprising 70% of the Universe's total mass-energy.

M. Turner (U. Chicago): "It could be a 22nd-century problem we stumbled upon in the 20th century."

Is the cosmological constant constant?

Recent measurements from the DESI collaboration (DESI Collaboration 2025) seem to indicate that Λ is *not* a constant, but instead evolves with time.

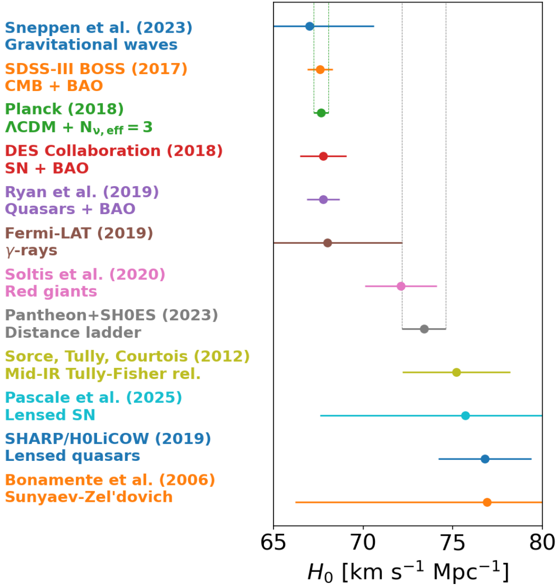
One model of Λ depends on $w(a) = w_0 + w_a(1 - a)$, where $w(a)$ evolves from a value $\sim (w_0 + w_a)$ at high redshift (small a) to w_0 today. If Λ is constant, then $w_a = 0$ and $w_0 = -1$.



Disquieting features of our flat Universe

The Hubble constant derived from CMB anisotropies is significantly smaller than the value from Cepheids and SNe Ia; they disagree at > 99% confidence, as we already discussed.

BAO results for H_0 tend to agree with CMB results, while other measures (gravitational lensing, S-Z clusters, etc.) agree with the distance ladder.



Disquieting features of our flat Universe

The **distance indicators** have issues. Our standard ruler is an acoustic oscillation in expanding media with gravitation and radiation in equilibrium. Our standard candle is the SN Ia.

These measures are physically more complex and less accessible for detailed study than the tools developed by astronomers over 50 years for use on smaller (Galactic) scales.

More detailed observations of BAOs will certainly help sort out the physics of the CMB anisotropies. Unfortunately, the blast physics in SNe Ia is poorly understood, and the effect of low metallicity in the early Universe on SN Ia yield is unknown.

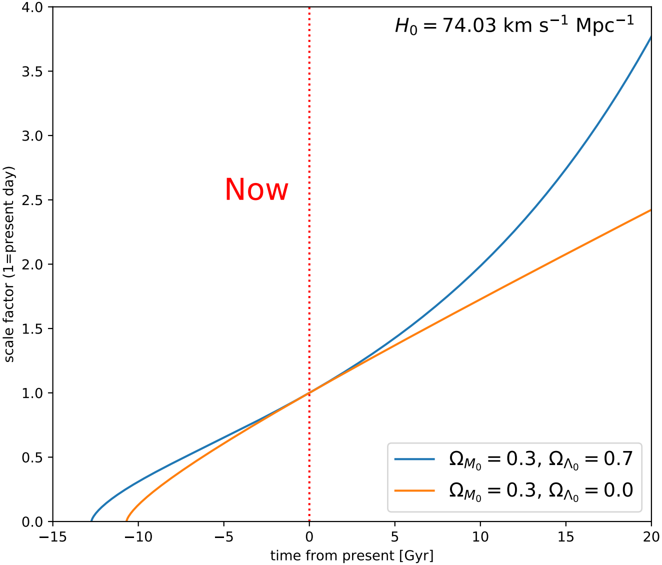
Pulsating stars are physically simple by comparison, and yet consider how many decades it took for them to be tamed for use as standard candles!

Disquieting features of our flat Universe

Anti-Copernicanism: Ω_M decreases with time while Ω_Λ increases with time.

We seem to live at a “special” time when the effects of matter and dark energy happen to be close in magnitude.

This kind of fine tuning always arouses suspicion, and often (though not always) points towards a conceptual problem or inconsistency.



Disquieting features of our flat Universe

Cosmology eventually becomes impossible, because further expansion will be exponential.

In a few Hubble times, distant galaxies and the CMB will be redshifted into undetectability.

So carve your results into stone, save many copies in safe places, and cultivate a reputation of honesty. Your successors will have to take your word for all this Big Bang stuff.