# Astronomy 142 — Practice Midterm Exam #1

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Name: _____

You may consult *only* one page of formulas and constants and a calculator while taking this test. You may *not* consult any books, digital resources, or each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly (answers must be circled or boxed). You will have one hour and fifteen minutes to complete the exam. Good luck!

- First, work on the problems you find the easiest. Come back later to the more difficult or less familiar material. Do not get stuck.
- The amount of space left for each problem is not necessarily an indication of the amount of writing it takes to solve it.
- Numerical answers are incomplete without units and should not be written with more significant figures than they deserve.
- Remember, you can earn partial credit for being on the right track. Be sure to show enough of your reasoning that we can figure out what you are thinking.

$R_\odot = 6.96 \times 10^{10}$ cm
$M_\odot = 1.989 \times 10^{33}$ g
$L_\odot = 3.827 \times 10^{33}$ erg/s
$T_e = 5772$ K
1 AU $= 149,597,870$ km
$k = 1.38 \times 10^{-16}$ erg/K
$G = 6.674 \times 10^{-8}$ dyn cm$^2$ g$^{-2}$
$h = 6.6261 \times 10^{-27}$ erg s
$m_n = 1.6749 \times 10^{-24}$ g
$q_e = 4.803 \times 10^{-10}$ esu

$M_{\rm bol} = 4.74$
$m_V = -26.71$
$M_V = 4.86$
$BC_V = -0.12$
1 pc $= 206,625$ AU
$\sigma = 5.6704 \times 10^{-5}$ erg s$^{-1}$ cm$^{-2}$ K$^{-4}$
$c = 3 \times 10^{10}$ cm/s
$m_p = 1.6726 \times 10^{-24}$ g
$m_e = 9.1094 \times 10^{-28}$ g

1. **Short answers.** Please write in complete sentences, and feel free to use equations and/or sketches to help explain your thoughts.

   (a) (5 points) The "twin" stars in Gemini, Castor and Pollux, differ in visual magnitude by 0.5. By what factor do they differ in visual flux?

   (b) (5 points) Give *two* pieces of evidence or arguments that the Sun is undergoing nuclear reactions in its interior.

(c) (5 points) Leaving you to watch from a large distance, I assume a stationary position between you and a nonspinning black hole's event horizon and arrange two meter sticks: one pointing directly (radially) away from the black hole, one perpendicular to this direction. Describe the difference in appearance of the meter sticks from your point of view.

(d) (5 points) Estimate the dates, during the next year, for which the sidereal time at midnight (standard time) is 0h, 6h, 12h, and 18h. What is your reasoning?

2. The HR diagram is the single most useful aid in the characterization of stars.

   (a) (10 points) Draw an HR diagram that contains only main sequence stars. Label the axes in physical units. Indicate the location of the O stars, the M stars, and the Sun.

   (b) (5 points) Where are the most luminous stars on this HR diagram? Where might you find the oldest stars? Are all the stars in this vicinity old? Why or why not? Where are the newest stars for sure?

3. **Stellar magnitudes**: Two stars are observed that appear to be companions. They are very close in the sky and both have an annual parallax of 0.1 arcsec. One of the stars is much brighter than the other.

   (a) (10 points) The brighter of the two stars has an apparent bolometric magnitude of 4.74 and its spectrum peaks at 0.5 $\mu$m. Estimate the surface temperature and radius of the star, and indicate whether or not it is likely to lie on the main sequence. Express your numerical answers in units of K and $R_\odot$.

   (b) (10 points) The dimmer companion has an apparent magnitude of 10.69 and its intensity peaks at 0.29 $\mu$m. Estimate the surface temperature and radius of the star (in units of K and $R_\odot$, respectively).

4. **Stellar Pulsations**: A spherical star with radius $R$ has uniform density $\rho$ and is supported by ideal gas pressure.

   (a) (10 points) Derive a formula for the pressure $P$ as a function of radius $r$ from the center of the star.

   (b) (5 points) Derive a formula for the temperature $T$ as a function of radius $r$ from the center of the star.

(c) (15 points) Derive a formula for the period $\Pi$ of the fundamental mode of radial pulsations for this star. *Hint: Recall that the adiabatic speed of sound is* $v_s = \sqrt{\gamma P/\rho}$. *You may also find the identity* $\int_0^1 du/\sqrt{1-u^2} = \pi/2$ *useful.*

5. (15 points) **Double-line eclipsing spectroscopic binary.** Suppose that you can measure the orbital period, $P$, and the velocity amplitudes, $v_1$ and $v_2$, for double-line eclipsing spectroscopic binary stars. Derive formulas for the distance $r$ between the stars, and the masses $m_1$ and $m_2$, in terms of these measurements, under the assumption that the orbits are circular.