Astronomy 142 — Practice Final Exam

Professor Kelly Douglass

Spring 2025

Name: _

You may consult two pages of formulas and constants and a calculator while taking this test. You may *not* consult any books, digital resources, or each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly (answers must be circled or boxed). You will have three hours to complete the exam. Good luck!

- First, work on the problems you find the easiest. Come back later to the more difficult or less familiar material. Do not get stuck.
- The amount of space left for each problem is not necessarily an indication of the amount of writing it takes to solve it.
- Numerical answers are incomplete without units and should not be written with more significant figures than they deserve.
- You must show your work and/or explain your answers to receive full credit.
- Remember, you can earn partial credit for being on the right track. Be sure to show enough of your reasoning that we can figure out what you are thinking.

$R_{\odot} = 6.96 \times 10^{10} \text{ cm}$	$M_{ m bol} = 4.74$
$M_{\odot} = 1.989 \times 10^{33} \text{ g}$	$m_V = -26.71$
$L_{\odot} = 3.827 \times 10^{33} \text{ erg/s}$	$M_V = 4.86$
$T_e = 5772 \text{ K}$	$BC_V = -0.12$
$R_{\oplus} = 6.371 \times 10^8 \text{ cm}$	$M_{\oplus} = 5.9736 \times 10^{27} \ {\rm g}$
1 AU = 149,597,870 km	1 pc = 206, 265 AU
$k = 1.38 \times 10^{-16} \text{ erg/K}$	$\sigma = 5.6704 \times 10^{-5} \ {\rm erg \ s^{-1} \ cm^{-2} \ K^{-4}}$
$G = 6.674 \times 10^{-8} \text{ dyn cm}^2 \text{ g}^{-2}$	$c = 3 \times 10^{10} \text{ cm/s}$
$H_0 = 73.04 \text{ km/s/Mpc}$	$h = 6.6261 \times 10^{-27} \text{ erg s}$
$m_p = 1.6726 \times 10^{-24} \text{ g} = 938.3 \text{ MeV}/c^2$	$m_n = 1.6749 \times 10^{-24} \text{ g} = 939.6 \text{ MeV}/c^2$
$m_e = 9.1094 \times 10^{-28} \text{ g} = 0.511 \text{ MeV}/c^2$	$e = 4.803 \times 10^{-10} \text{ cm}^{3/2} \text{ g}^{1/2} \text{ s}^{-1}$

- 1. Answer each of the following with a few complete sentences. Feel free to use diagrams and/or equations if you feel that they will help you answer the questions.
 - (a) (5 points) An object is observed at redshift z. What is the dimensionless scale factor which applies to this object?

(b) (5 points) How old is the Universe, according to current best estimates?

(c) (5 points) Under what conditions can the virial theorem be used to measure the mass of a cluster of stars or galaxies?

(d) (5 points) Who invented the use of standard candles, and how was it done?

(e) (5 points) Calculate the average density in g/cm^3 of a classical Cepheid variable star with a 10-day period.

(f) (5 points) With what speed do galaxies 100 Mpc away recede from us? What is their redshift?

2. The distance ladder

(a) (10 points) In the outer reaches of a certain galaxy, a variable star is detected with a period of 30 days and a total flux of 3.3×10^{-15} erg s⁻¹ cm⁻². Under the assumption that the star is a Cepheid I, calculate the distance to the galaxy.

(b) (10 points) In the same galaxy, a supernova remnant is monitored for two years. Visible spectral lines show that during this time the gas has a maximum velocity along the line of sight of 5000 km/s. Radio images show the remnant to be circular in appearance. During the two-year period, the diameter of the remnant increases by 0.84×10^{-3} arcseconds. Using these data, calculate the distance to the galaxy.

(c) (5 points) Which of these measurements is likely to be more accurate? If the two estimates are different, describe the source of the discrepancy.

- 3. Consider a universe in which $\Lambda = 0$ and the normalized matter density is $\Omega = \Omega_{M_0} > 1$.
 - (a) (20 points) Derive an expression for the comoving radial distance between z = 0 (that is, today) and $z_d = 1090$, the redshift of the decoupling surface.

(b) (5 points) Independent of the curvature of the universe, the acoustic horizon size is given as

$$\ell_{d} = \frac{2c}{3H_{0}}\sqrt{\frac{a_{d}}{\Omega_{M_{0}}}} = \frac{2c}{3H_{0}\sqrt{\Omega_{M_{0}}(1+z_{d})}}$$

Calculate the angular size, in degrees, of the fundamental mode of acoustic oscillations visible on the decoupling surface as the cosmic-background anisotropies, using the results of part a and a normalized matter density of $\Omega_{M_0} = 2$. How does your answer compare to the measured angular size of the fundamental mode, $\theta_d = 0.66^{\circ}$?

- 4. A $1M_{\odot}$ star with radius $1R_{\odot}$ has just reached the end of its main-sequence life, with a central temperature of 1.4×10^7 K and a core containing 10% of its total mass. Hydrogen fusion has recently shut off in the core, which we assume for simplicity to be composed completely of $\frac{4}{2}$ He.
 - (a) (5 points) What is the central pressure (in dyne/cm²) due to the weight of the outer layers of the star?

(b) (5 points) At the very center of the star, nonrelativistic electron degeneracy pressure balances gravity. What is the density of electrons there?

(c) (5 points) Suppose hydrogen fusion starts up in the shell just outside the helium core. The shell is the same temperature as the core itself. As this fusion proceeds, helium "ash" is added to the core. The temperature scales with the core mass M_C according to $T_C \propto M_C^{4/3}$. What is the mass of the helium core when the temperature reaches 10^8 K, high enough to ignite ⁴/₂He fusion?

5. There are three different basic types of Big Bang universes.

(a) (15 points) What are their pasts and futures, if they are matter-dominated?

(b) (5 points) What determines the fate of a universe?

(c) (5 points) What are the current ideas about the fate of our Universe?

6. Rotation curves

(a) (10 points) Draw a typical rotation curve for a spiral galaxy; label the axes and identify critical values of the curve on the axes. Indicate the different types of rotation observed.

(b) (5 points) Describe a rotation curve. What information can be gleaned from it?

7. Galaxy surface brightness

(a) (5 points) A spiral galaxy appears circular when seen face-on. The flux observed per steradian of the galaxy is given by the relation

$$\Sigma(r) = \Sigma_0 e^{-r/r_0}$$

where r is the angular distance in radians from the center of the galaxy. What is the total flux that we observe from the galaxy?

(b) (10 points) You observe an elliptical E0 galaxy. The flux you measure per steradian is given by the relation

$$\Sigma(r) = \Sigma_0 \exp\left[-(r/r_0)^{1/4}\right]$$

where r is the angular distance in radians from the center of the galaxy. What is the total observed flux? *Hint: You may wish to use the fact that*

$$\int_0^\infty x^n e^{-x} dx = n!$$

8. The mass density of a star with mass M and radius R decreases linearly from the center to the surface of the star and vanishes at the surface:

$$\rho(r) = \rho_C \left(1 - \frac{r}{R} \right)$$

(a) (10 points) Derive an expression for the density ρ_C at the center of the star, in terms of M and R.

(b) (20 points) Derive an expression for the pressure P_C at the center of the star, in terms of M and R.