Astronomy 142 — Practice Final Exam

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Name: ____

You may consult two pages of formulas and constants and a calculator while taking this test. You may *not* consult any books, digital resources, or each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly (answers must be circled or boxed). You will have three hours to complete the exam. Good luck!

- First, work on the problems you find the easiest. Come back later to the more difficult or less familiar material. Do not get stuck.
- The amount of space left for each problem is not necessarily an indication of the amount of writing it takes to solve it.
- Numerical answers are incomplete without units and should not be written with more significant figures than they deserve.
- You must show your work and/or explain your answers to receive full credit.
- Remember, you can earn partial credit for being on the right track. Be sure to show enough of your reasoning that we can figure out what you are thinking.

$R_\odot = 6.96 \times 10^{10}$ cm	$M_{\rm bol} = 4.74$
$M_\odot = 1.989 \times 10^{33}$ g	$m_V = -26.71$
$L_\odot = 3.827 \times 10^{33}$ erg/s	$M_V = 4.86$
$T_e = 5772$ K	$BC_V = -0.12$
$R_\oplus = 6.371 \times 10^{8}$ cm	$M_\oplus = 5.9736 \times 10^{27}$ g
1 AU $= 149,597,870$ km	1 pc $= 206,265$ AU
$k = 1.38 \times 10^{-16}$ erg/K	$\sigma = 5.6704 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}
$G = 6.674 \times 10^{-8}$ dyn cm^2 g^{-2}	$c = 3 \times 10^{10}$ cm/s
$H_0 = 73.04$ km/s/Mpc	$h = 6.6261 \times 10^{-27}$ erg s
$m_p = 1.6726 \times 10^{-24}$ g $= 938.3$ MeV/c^2	$m_n = 1.6749 \times 10^{-24}$ g $= 939.6$ MeV/c^2
$m_e = 9.1094 \times 10^{-28}$ g $= 0.511$ MeV/c^2	$e = 4.803 \times 10^{-10}$ cm$^{3/2}$ g$^{1/2}$ s^{-1}

1. Answer each of the following questions with a few complete sentences. Feel free to use diagrams and/or equations if you feel that they will help you answer the questions.
   (a) (5 points) Why is the time told on a sundial different by several minutes from time told on a clock?

   (b) (5 points) How is it possible for knots within quasar jets to appear to move faster than light?

(c) (5 points) Why is the cosmic background radiation essentially isotropic?

(d) (5 points) Give two strong pieces of observational evidence for the presence of dark energy in the Universe.

(e) (5 points) An isolated black hole has a mass of 10^8 g. At what temperature does it radiate?

(f) (5 points) Under what conditions can the Robertson-Walker absolute interval be used to calculate the properties of spacetime in a model universe?

2. (a) (15 points) Suppose that the Universe were flat — as observed — but besides a very small amount of normal matter, it contains only dark energy. Starting from the Friedmann equation, derive an expression for the relation between time, t, and the normalized scale factor, a.

(b) (15 points) Repeat for a flat, matter-dominated universe: one in which there is negligible dark energy but lots of matter.

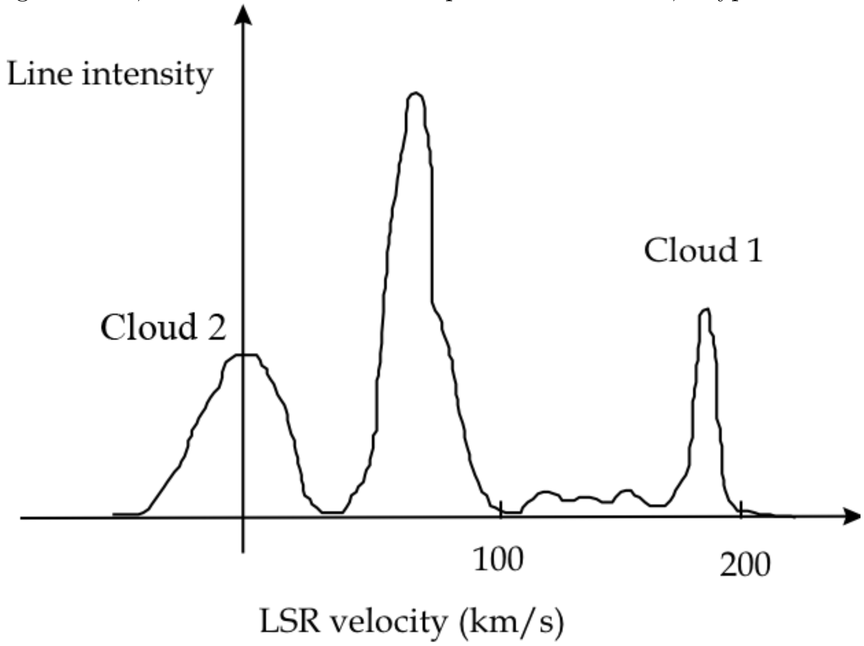
(c) (5 points) Which of these universes expands faster? What is the fate of each of these universes?

3. Consider a cylindrical, rotating cluster of stars that is gravitationally bound. Both its radius R and thickness $H \ll R$ are much larger than the typical distance between stars, and the mass density ρ of the cluster is uniform on scales much larger than the typical distance between stars.

   (a) (5 points) What is the rotation speed V of the stars in the cluster as a function of the cylindrical radius r? Your answer should include only given quantities and known constants.

(b) (10 points) What is the typical random stellar speed, v?

4. At galactic longitude 20°, a series of HI 21 cm line spectra are obtained; a typical result is shown below.



HI line intensity as a function of radial velocity with respect to the local standard of rest.

(a) (10 points) The highest Doppler velocities correspond to a peak at 180 km/s and belong to a diffuse cloud we will refer to as Cloud 1 (see above figure). Calculate the distance r from Cloud 1 to the Galactic center, and the distance d from the Solar System to Cloud 1. Express your answers in kiloparsecs.

(b) (10 points) Calculate the Galactic rotation velocity of Cloud 1 and the Galactic mass enclosed by its orbit. *(Hint:* $\Omega_\odot = 30$ *km/s/kpc)*

(c) (5 points) What is the galactocentric radius of Cloud 2?

5. A spherical clump of molecular hydrogen ($m_{H_2} = 3.3 \times 10^{-24}$ g) with mass $10M_\odot$ is observed to have a temperature $T = 10$ K and to be $r = 0.1$ pc in radius.

   (a) (5 points) Show that the clump is hydrostatically unstable and will collapse. Clearly state any assumptions that you make.

   (b) (5 points) How long will it take the core to collapse to stellar dimensions? Give your answer in years. Again, state any assumptions that you make.

(c) (10 points) Suppose that we have found a clump with very little rotational motion so that the material all collapses into a spherical protostar which eventually becomes a $10M_\odot$ star. Estimate the radius of this star, and from this estimate and the previous results, estimate the average luminosity of the object during the collapse that forms the star.

6. A star with mass M and outer radius R has a density given as a function of radius r given by

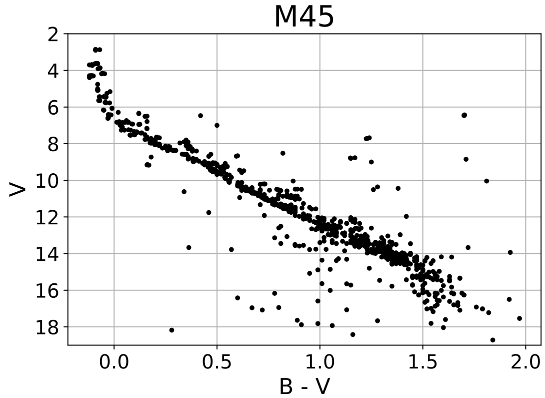
$$\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right)$$

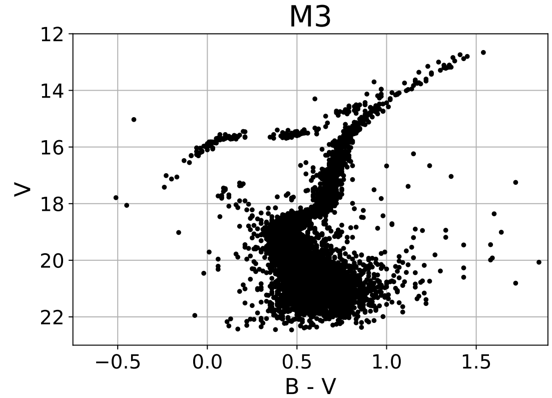
(a) (10 points) Find an expression for the central density ρ_0 in terms of M and R.

(b) (20 points) Find an expression for the central pressure P_0 in terms of M and R.

(c) (5 points) Find an expression for the temperature at the center of the star.

7. Below are H-R diagrams fro two star clusters, M45 (left) and M3 (right); the apparent V magnitude is plotted against $B - V = m_B - m_V$.





(a) (5 points) Rank the clusters in order of age. Roughly how old is the older one? (Use a sentence or two to explain each answer.)

(b) (5 points) The Sun has an absolute V magnitude of $+4.83$ and a $B - V$ color index of 0.65. Use this information along with the H-R diagrams to estimate the distance to M45. (Neglect extinction, and express your answer in parsecs.)

(c) (5 points) The nearby star RR Lyrae has an average, absolute V magnitude of $+0.75$ and a distance of 250 pc. Estimate the distance to M3.