Astronomy 142 — Practice Midterm Exam #2

Professor Kelly Douglass

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Name: ____

You may consult *only* one page of formulas and constants and a calculator while taking this test. You may *not* consult any books, digital resources, or each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly (answers must be circled or boxed). You will have one hour and fifteen minutes to complete the exam. Good luck!

- First, work on the problems you find the easiest. Come back later to the more difficult or less familiar material. Do not get stuck.
- The amount of space left for each problem is not necessarily an indication of the amount of writing it takes to solve it.
- Numerical answers are incomplete without units and should not be written with more significant figures than they deserve.
- You must show your work and/or explain your answers to receive full credit.
- Remember, you can earn partial credit for being on the right track. Be sure to show enough of your reasoning that we can figure out what you are thinking.

$R_\odot = 6.96 \times 10^{10}$ cm

$M_\odot = 1.989 \times 10^{33}$ g

$L_\odot = 3.827 \times 10^{33}$ erg/s

$T_e = 5772$ K

$R_\oplus = 6.371 \times 10^{8}$ cm

1 AU $= 149,597,870$ km

$k = 1.38 \times 10^{-16}$ erg/K

$G = 6.674 \times 10^{-8}$ dyn cm^2 g^{-2}

$h = 6.6261 \times 10^{-27}$ erg s

$m_n = 1.6749 \times 10^{-24}$ g $= 939.6$ MeV/c^2

$e = 4.803 \times 10^{-10}$ cm$^{3/2}$ g$^{1/2}$ s^{-1}

$M_{\text{bol}} = 4.74$

$m_V = -26.71$

$M_V = 4.86$

$BC_V = -0.12$

$M_\oplus = 5.9736 \times 10^{27}$ g

1 pc $= 206,625$ AU

$\sigma = 5.6704 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}

$c = 3 \times 10^{10}$ cm/s

$m_p = 1.6726 \times 10^{-24}$ g $= 938.3$ MeV/c^2

$m_e = 9.1094 \times 10^{-28}$ g $= 0.511$ MeV/c^2

1. **Short answers.** Please write in complete sentences, and feel free to use equations and/or sketches to help explain your thoughts.

   (a) (5 points) Who invented standard candles, and how?

   > **Solution:** Henrietta Leavitt, one of the "computers" employed by Edward Pickering at Harvard College Observatory. She discovered the relation between the apparent magnitude and pulsation period of Cepheids in the Magellanic clouds and realized that since all the Cepheids in each cloud are at the same distance from us, this is really a period-luminosity relation.

   (b) (5 points) Are the Milky Way's globular clusters made primarily of Population I or II stars? Approximately how old are the oldest globular clusters?

   > **Solution:** The globular clusters are Pop II stars, i.e., objects with low metallicity and relatively large velocity dispersion. The best estimates from main-sequence turnoff indicate they are $\sim$13 Gyr old.

(c) (5 points) Give two strong pieces of observational evidence for the existence of black holes within active galactic nuclei.

Solution: Any two out of these three answers are acceptable:

1. Galaxy-sized luminosity emitted from a volume the size of the Solar System cannot be produced by any stellar process.
2. Matter is ejected from AGN at speeds close to c, implying escape velocity from a black hole.
3. Rotation curves of nearby AGN indicate the presence of central pointlike masses in excess of $10^8 M_\odot$.

(d) (5 points) Give two strong pieces of observational evidence for the existence of dark matter in the Universe.

Solution: Any two out of these three answers are acceptable:

1. Rotation curves of *all* spiral galaxies are flat out to the largest detectable galactocentric radii, consistent with a spherical mass distribution. Also, the inferred M/L at large radii is far greater than can be accounted for by stars.
2. The virial mass of galaxy clusters greatly exceeds the luminous mass (measured by starlight and hot X-ray emitting gas).
3. The thermal velocities of hot gas in galaxy clusters generally exceed the escape velocity if the only mass is the luminous mass, but is bound to the cluster if there is additional dark matter (the amount indicated by the virial mass).

2. **Accretion from a disk**

(a) (5 points) A very young $1M_\odot$ protostar, still completing its formation, accretes matter from a surrounding disk at a rate limited by radiation pressure. What is the resulting luminosity (in $L_\odot$)?

Solution:

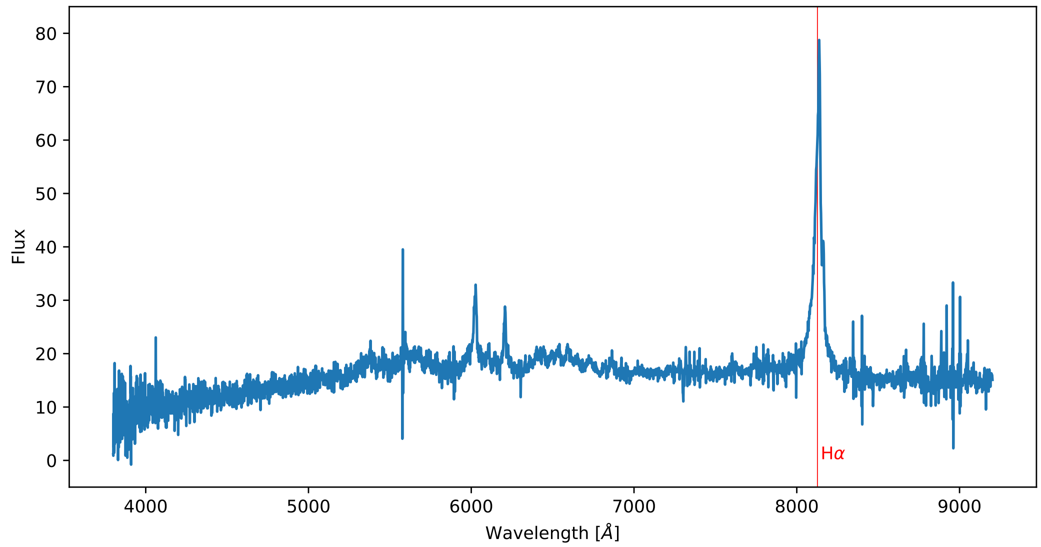
$$L = L_E = \frac{GMm_pc}{2r_e^2}$$
$$L = 1.3 \times 10^{38} \text{ erg/s} = 3.4 \times 10^4 L_\odot$$

(b) (5 points) What is the rate (in $M_\odot$/year) at which this protostar must accrete mass in order to produce its luminosity?

Solution: Even though the star has not yet fallen onto the main sequence, its mass and radius are related just the same as when on the MS. Thus, $M \propto R$, so $R \sim R_\odot$. Just like in Problem Set 8,

$$\begin{aligned} L &= \frac{dE_{\text{radiated}}}{dt} = -\frac{dU}{dt} \\ &= \frac{GM}{R}\frac{dm}{dt} - \frac{GM}{r}\frac{dm}{dt} \approx \frac{GM}{R}\frac{dm}{dt} \\ \frac{dm}{dt} &= \frac{LR}{GM} \\ \frac{dm}{dt} &= 6.8 \times 10^{22} \text{ g/s} = 1.1 \times 10^{-3} M_\odot/\text{year} \end{aligned}$$

3. An object with an apparent magnitude $m_r = 17.90$ has a measured spectrum that is shown below. The most prominent line in this object's spectrum is the Hα line, observed at 8137.67Å. (The Hα line is observed at 6562.8Å in the lab.)



(a) (5 points) Is this object an elliptical galaxy, a spiral galaxy, or an AGN? Explain your answer.

Solution: This object is an AGN. Its broad emission features tell us that there is ionized gas present, and that there is a significant amount of rotation in this gas. These are the signatures of an AGN spectrum, where the majority of the emission is coming from the accretion disk surrounding the central supermassive black hole.

(b) (5 points) What is the redshift of this object?

Solution:

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{8137.67\text{ Å} - 6562.79\text{ Å}}{6562.79\text{ Å}}$$

$$\boxed{z = 0.240}$$

(c) (5 points) How far away is the object?

Solution:

$$d = \frac{cz}{H_0} = \frac{(0.237)(3 \times 10^{10}\ \text{cm/s})}{73.04\ \text{km s}^{-1}\ \text{Mpc}^{-1}}$$

$$\boxed{d = 719.9\ \text{Mpc/h} = 985.6\ \text{Mpc}}$$

(d) (5 points) What is the object's absolute magnitude M_r?

Solution:

$$\begin{aligned} M_r &= m_r + 5 - 5\log\left(\frac{d}{\text{pc}}\right) \\ &= 17.90 + 5 - 5\log\left(9.8227 \times 10^{8}\right) \\ M_r &= -22.07 \end{aligned}$$

4. A giant elliptical galaxy lies 20 Mpc away. Observations are made of the total flux from starlight, and stellar velocity dispersion along the line of sight, within circles on the sky that correspond to radii of 100 and 1000 pc. Within these two areas, the results are as follows:

	100 pc radius	1000 pc radius
Total flux [erg s^{-1} cm^{-2}]	2.1×10^{-11}	1.5×10^{-9}
Radial velocity dispersion [km/s]	290	250

Use the above data to answer the following questions about the galaxy.

(a) (10 points) Calculate the mass, luminosity, and mass-to-light ratio, all in solar units, for the part of the galaxy contained within the large (1 kpc radius) area.

Solution:

$$M = \frac{6R\overline{v_r^2}}{G} = \frac{6(1000\text{ pc})(3.09 \times 10^{18}\text{ cm/pc})(250 \times 10^5\text{ cm/s})^2}{6.674 \times 10^{-8}\text{ dyn cm}^2\text{ g}^{-2}}$$

$$\boxed{M = 1.73 \times 10^{44}\text{ g} = 8.7 \times 10^{10} M_\odot}$$

$$L = 4\pi r^2 f = 4\pi(20 \times 10^6\text{ pc})^2(3.09 \times 10^{18}\text{ cm/pc})^2(1.5 \times 10^{-9}\text{ erg/s/cm}^2)$$

$$\boxed{L = 7.2 \times 10^{43}\text{ erg/s} = 1.9 \times 10^{10} L_\odot}$$

$$\boxed{\frac{M}{L} = 4.6 M_\odot L_\odot^{-1}}$$

This is close to the Solar-neighborhood value in our galaxy.

(b) (5 points) Repeat for the small (100 pc radius) area.

Solution:

$$M = \frac{6R\overline{v_r^2}}{G} = \frac{6(100\text{ pc})(3.09 \times 10^{18}\text{ cm/pc})(290 \times 10^5\text{ cm/s})^2}{6.674 \times 10^{-8}\text{ dyn cm}^2\text{ g}^{-2}}$$

$$\boxed{M = 2.3 \times 10^{44}\text{ g} = 1.2 \times 10^{10} M_\odot}$$

$$L = 4\pi r^2 f = 4\pi(20 \times 10^6\text{ pc})^2(3.09 \times 10^{18}\text{ cm/pc})^2(2.1 \times 10^{-11}\text{ erg/s/cm}^2)$$

$$\boxed{L = 1.0 \times 10^{42}\text{ erg/s} = 2.6 \times 10^8 L_\odot}$$

$$\boxed{\frac{M}{L} = 45 M_\odot L_\odot^{-1}}$$

(c) (10 points) On the basis of your answers to parts a–b, argue for or against the presence of a supermassive black hole at the center of this galaxy. If a black hole is likely to be present, estimate its mass.

> **Solution:** We may assume that the mass within the large area is dominated by luminous matter, since the mass-to-light ratio is similar to that of the Solar neighborhood. The much larger mass-to-light ratio in the smaller area indicates dominance by a dark compact mass; this could be a supermassive black hole, though one that is not in quasar mode, else it would be thousands of times more luminous.
>
> Most of the mass in the 100-pc area should thus be that of the black hole, so the lower of the two masses would be a decent estimate. But one could perhaps get closer by assuming that the same stellar M/L applies in the small area as the large, and the luminosity there is all from stars. This gives a mass of $(4.6)(2.6 \times 10^8 M_\odot) = 1.2 \times 10^9 M_\odot$ for the stars, so about $1.1 \times 10^{10} M_\odot$ for the black hole.

(d) (5 points) Estimate the stellar relaxation time for the 100 pc area and compare this to the likely age of the galaxy. Comment on the validity of the virial theorem in this case.

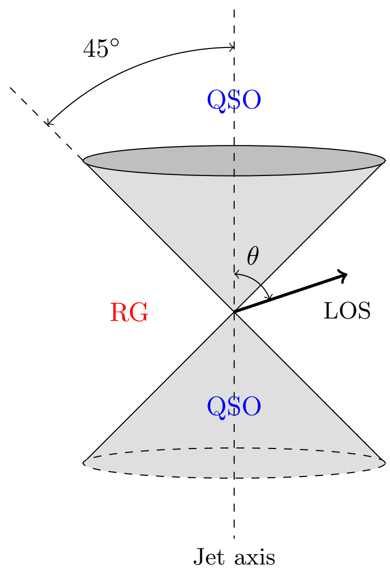
> **Solution:** With the mass in stars from part c and a crude approximation that they are all $1 M_\odot$ stars, the number of stars in the central 100 pc is $N = 1.2 \times 10^9$, which leads to
>
> $$t_c = \frac{2R}{\sqrt{3} v_r} \frac{N}{24 \ln\left(\frac{N}{2}\right)}$$
>
> $$\boxed{t_c = 3.0 \times 10^{19} \text{ s} = 9.6 \times 10^{11} \text{ yr}}$$
>
> This is much longer than the likely age of this galaxy, since it is longer than the age of the Universe. But giant elliptical galaxies, like galaxy clusters themselves, are dominated in their structure by dark matter: the galaxy's dark halo came to equilibrium very early in the Universe's history, and the motions of the stars within the galaxy thus trace an equilibrium distribution.

5. **The unified AGN model**

Suppose that the jets of an AGN that are within 45° of our line of sight appear as quasars, while those outside of this cone appear as radio galaxies.

(a) (5 points) Sketch a diagram containing an AGN and its jets, and label the regions of the sky where we would detect it as either a quasar or a radio galaxy.

Solution:



(b) (5 points) What is the solid angle of the sky that corresponds to our perception of the AGN as a radio galaxy? As a quasar?

Solution:

$$\Omega_{\rm RG} = \int d\Omega = \int_0^{2\pi} d\phi \int_{\pi/4}^{3\pi/4} \sin\theta d\theta$$
$$= 2\pi \left[-\cos\theta\right]_{\pi/4}^{3\pi/4}$$
$$\boxed{\Omega_{\rm RG} = 2\pi\sqrt{2}}$$

We will perceive the AGN as a quasar for all parts of the sky within the cones, which corresponds to the rest of the sky where we do not see the AGN as a radio galaxy.

$$\Omega_{\rm QSO} = 4\pi - \Omega_{\rm RG}$$
$$\boxed{\Omega_{\rm QSO} = 2\pi(2-\sqrt{2})}$$

(c) (5 points) Are we more likely to observe an AGN as a radio galaxy or a quasar? By how much? Be sure to state any assumptions that you make. (*Hint:* Use your results from part b.)

Solution: Our chance of seeing the AGN as a quasar is the same as our chance of being within the 45° cone around the jets. This is the same as the fraction of the AGN's sky solid angle that the cones occupy, so we should expect to observe the AGN as a radio galaxy instead of a quasar the same as the ratio of the solid angle of the radio galaxy perception to the solid angle of the quasar perception.

$$\frac{N(\mathrm{RG})}{N(\mathrm{QSO})} = \frac{\Omega(\mathrm{RG})}{\Omega(\mathrm{QSO})}$$

$$= \frac{2\pi\sqrt{2}}{2\pi(2-\sqrt{2})} = \frac{1}{\sqrt{2}-1}$$

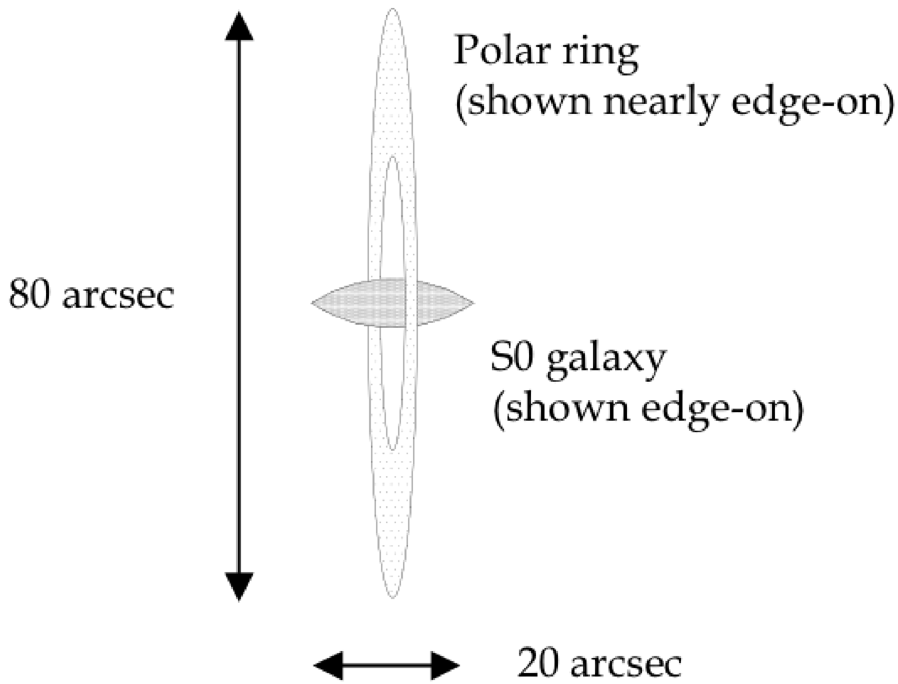
$$\boxed{\frac{N(\mathrm{RG})}{N(\mathrm{QSO})} = 2.41}$$

(d) (5 points) Within the redshift range 0.5–1, we have observed 2.44 times as many radio galaxies as quasars. By comparing this to your answer from part c, what conclusions can you make about the unified AGN model? What can you say about the inclusion of blazars into this model?

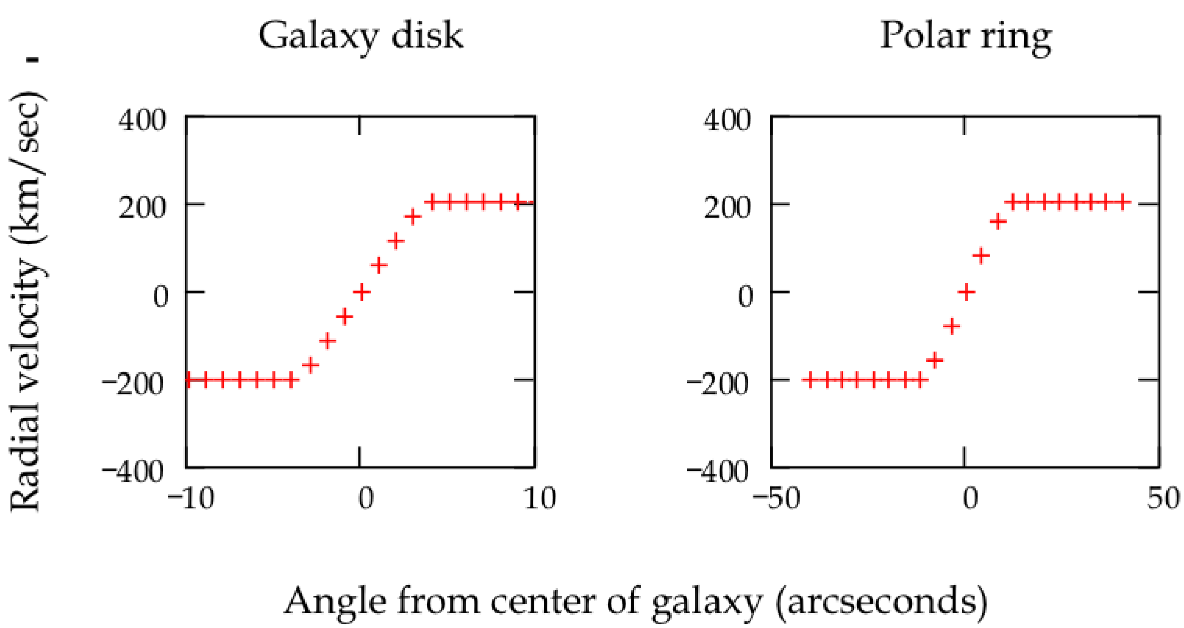
Solution: Our answer is only slightly less than what has been observed, so it is likely that quasars are AGN whose jets are within 45° of our line of sight.

If blazars are when we view an AGN with its jets almost perfectly along our line of sight, then this would subtract slightly from the number count of the quasars. This would reduce the number of quasars observed, which would actually bring our answer closer to what is observed.

6. A polar-ring S0 galaxy is observed to have both rotation axes nearly perpendicular to the line of sight. The disk is 20 arcsec long in the direction perpendicular to its rotation axis; the ring is about 80 arcsec long in the direction perpendicular to its own rotation axis. The system lies $d = 100$ Mpc away from us. A sketch of the appearance of this system at visible wavelengths is shown below.



Radial velocities have been measured for the entire visible extents of the galaxy disk and the polar ring, and appear below.



(a) (10 points) How much mass is seen?

Solution: The angles involved are all small, ($\tan\theta \cong \sin\theta \cong \theta$), so at the distance of this system, 40 arcsec is the subtense of a length

$$\begin{aligned} d(40'') &= 100 \text{ Mpc} \left(40'' \frac{1^\circ}{3600''} \frac{\pi \text{ radians}}{180^\circ} \right) \\ &= 19.4 \text{ kpc} = 6.0 \times 10^{22} \text{ cm} \end{aligned}$$

The most distant test particles — in the ring, 40 arcsec from the center of the system — have orbits which encompass the most mass, given by

$$\begin{aligned} F = \frac{GMm}{r^2} &= m\frac{v^2}{r} \\ M &= \frac{v^2 r}{G} = \frac{v_r^2 r}{G} \text{ (since the orbit is nearly edge on)} \\ &= \frac{(200 \text{ km/s})^2 (10^5 \text{ cm/km})^2 (6.0 \times 10^{22} \text{ cm})}{6.674 \times 10^{-8} \text{ dyn cm}^2 \text{ g}^{-2}} \\ M &= 3.6 \times 10^{44} \text{ g} = 1.8 \times 10^{11} M_\odot \end{aligned}$$

(b) (5 points) What is the shape of the mass distribution? Is there likely to be very much dark matter in the system?

Solution: The rotation curves of both the galaxy disk and the polar ring are flat in their outer parts, and both flatten out at a radial velocity of about 200 km/s. Since the radial velocity amplitudes are the same in the perpendicular directions, we infer that the mass is distributed spherically symmetric in the system. From the flatness we also infer that, outside the central regions, the mass density is of the form

$$\rho(r) = \rho_0 \left(\frac{r_0}{r}\right)^2 \propto \frac{1}{r^2}$$

Both the disk and the ring rotation curves have nonzero slope linear sections in their centers. In the case of the disk, this is probably the "solid body" part of the rotation curve, owing to an approximately constant mass density within the core. In the case of the ring, this is probably the signature of the inner edge of the ring, approximately 5 kpc away from the center.

The mass is distributed in a spherically-symmetric fashion. The light is obviously not, so most of the mass in this system is dark.