Astronomy 142 — Practice Midterm #2

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Name: ____

You may consult *only* one page of formulas and constants and a calculator while taking this test. You may *not* consult any books, digital resources, or each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly (answers must be circled or boxed). You will have one hour and fifteen minutes to complete the exam. Good luck!

- First, work on the problems you find the easiest. Come back later to the more difficult or less familiar material. Do not get stuck.
- The amount of space left for each problem is not necessarily an indication of the amount of writing it takes to solve it.
- You must show your work and/or explain your answers to receive full credit.
- Numerical answers are incomplete without units and should not be written with more significant figures than they deserve.
- Remember, you can earn partial credit for being on the right track. Be sure to show enough of your reasoning that we can figure out what you are thinking.

$$R_\odot = 6.96 \times 10^{10} \text{ cm}$$
$$M_\odot = 1.989 \times 10^{33} \text{ g}$$
$$L_\odot = 3.827 \times 10^{33} \text{ erg/s}$$
$$T_e = 5772 \text{ K}$$
$$R_\oplus = 6.371 \times 10^{8} \text{ cm}$$
$$1 \text{ AU} = 149,597,870 \text{ km}$$
$$k = 1.38 \times 10^{-16} \text{ erg/K}$$
$$G = 6.674 \times 10^{-8} \text{ dyn cm}^2 \text{ g}^{-2}$$
$$h = 6.6261 \times 10^{-27} \text{ erg s}$$
$$m_n = 1.6749 \times 10^{-24} \text{ g} = 939.6 \text{ MeV}/c^2$$
$$e = 4.803 \times 10^{-10} \text{ cm}^{3/2} \text{ g}^{1/2} \text{ s}^{-1}$$
$$M_{\text{bol}} = 4.74$$
$$m_V = -26.71$$
$$M_V = 4.86$$
$$BC_V = -0.12$$
$$M_\oplus = 5.9736 \times 10^{27} \text{ g}$$
$$1 \text{ pc} = 206,265 \text{ AU}$$
$$\sigma = 5.6704 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$$
$$c = 3 \times 10^{10} \text{ cm/s}$$
$$m_p = 1.6726 \times 10^{-24} \text{ g} = 938.3 \text{ MeV}/c^2$$
$$m_e = 9.1094 \times 10^{-28} \text{ g} = 0.511 \text{ MeV}/c^2$$

1. **Short answers.** Please write in complete sentences, and feel free to use equations and/or sketches to help explain your thoughts.

   (a) (5 points) Emission of a range of local standard of rest (LSR) velocities from interstellar atomic hydrogen (21 cm line) is observed at a Galactic longitude of 30°. How far away (in kpc) from the Galactic Center is the emission with the largest velocity with respect to the LSR?

   Solution: The largest velocity arises at the tangent point:

   $$r = r_{\odot} \sin \ell = 4.2 \text{ kpc}$$

   (b) (5 points) The galaxies in a cluster have an average radial velocity of 15000 km/s. How far away is the cluster?

   Solution: From Hubble's Law,

   $$d = v_r / H_0 \approx 200 \text{ Mpc}$$

(c) (5 points) Describe two observational differences between Type Ia and Type II supernovae.

Solution:

1. SNe II leave a neutron star remnant; SNe Ia leave no remnant.
2. H emission is prominent in SNe II spectra but not SNe Ia spectra.
3. The shapes of the light curves are different; SNe Ia achieve a much higher peak luminosity but fades faster.

(d) (5 points) Over a certain range of radius, the rotational velocities in a certain galaxy increase linearly with radius. What does this say about the mass density of the galaxy in this radial range?

Solution: This is a solid-body rotation curve, so the density is constant within this radial range.

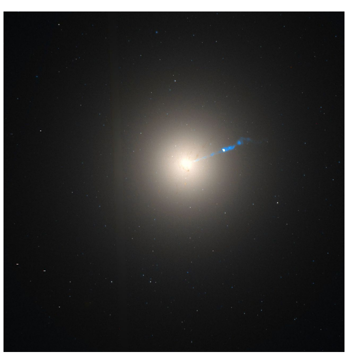
2. (10 points) Give the Hubble type for each of the following galaxies. If the classification is ambiguous, justify your response.

(a) 

Solution: Sa (NGC 1302)

(b) 

Solution: SBc (NGC 613)

(c) 

Solution: E0 (NGC 4486)

(d) 

Solution: Sb (NGC 4450)

(e) 

Solution: Sc (NGC 4303)

3. Two regularly-variable stars are observed in the Andromeda Galaxy, M31. Their light curves have the same shape and both have a period $\Pi = 30$ days. Their average V magnitudes are 18.6 and 20.1.

   (a) (5 points) Determine the type of each of these variable stars.

   Solution: They are both in M31 so to a good approximation they both lie at the same distance. They have the same period but different magnitudes so they cannot be the same type of pulsating star.

   Thirty days is in the range of a Cepheid and is too long to be an RR Lyr or a δ Scu star. Since they differ by 1.5 magnitudes, it seems safe to identify them as follows:

   1. The brighter one ($\overline{m_V} = 18.6$) is a classical Pop I Cepheid.
   2. The dimmer one ($\overline{m_V} = 20.1$) is a W Virginis star (Pop II Cepheid).

   (b) (5 points) Use these magnitudes to calculate the distance to M31. Give your answer in kiloparsecs.

   Solution: Leavitt's Law can be used to obtain the absolute V magnitude for the brighter one (not the dimmer one):

$$\overline{M_V} = -2.77 \log \Pi - 1.69 = -5.78$$

   so the distance for the 18.6 magnitude Cepheid is

$$\mu = \overline{m_V} - \overline{M_V} = 18.6 + 5.78 = 24.4$$
$$d = 10 \text{ pc} \cdot 10^{\mu/5} = 752 \text{ kpc}$$

(c) (5 points) S Andromedae was a SN Ia which occurred in M31 in February 1885. What was its apparent V magnitude at maximum brightness?

Solution: The absolute V magnitude of a SN Ia at peak brightness is $M_V = -19.14$, so

$$m_V = \mu + M_V = 24.4 - 19.14 = 5.2$$

This is visible to the naked eye in a dark sky, and if the sky were dark enough to see M31 with the naked eye — not too rare even these days — it would have easily been recognized as something new.

4. A distant galaxy is observed to have spectral lines with **full** widths of 600 km/s arising from a region 100 kpc in diameter.

   (a) (5 points) Assume the galaxy is rotating and has no random internal motions, and that the plane of rotation is viewed edge-on. Estimate the mass within the central 100 kpc diameter region.

   Solution: Assume the 600 km/s spread of velocities is due to rotation. Thus the largest rotational velocities are $v_r = \pm 300$ km/s and the radius of the observed region is $r = 50$ kpc. Assuming the system is viewed edge-on, we estimate

$$M = \frac{rv_r^2}{G} = \frac{50 \times 10^3 (3.09 \times 10^{18} 300^2 (10^5)^2}{6.67 \times 10^{-8}} \text{ g}$$
$$= 1.0 \times 10^{12} M_\odot$$

   (b) (5 points) Now assume instead that the galaxy is not rotating and has only random internal motions. What is the mass within the central 100 kpc diameter region?

   Solution: Now the observed radial velocities are due to random motions of up to ± 300 km/s:

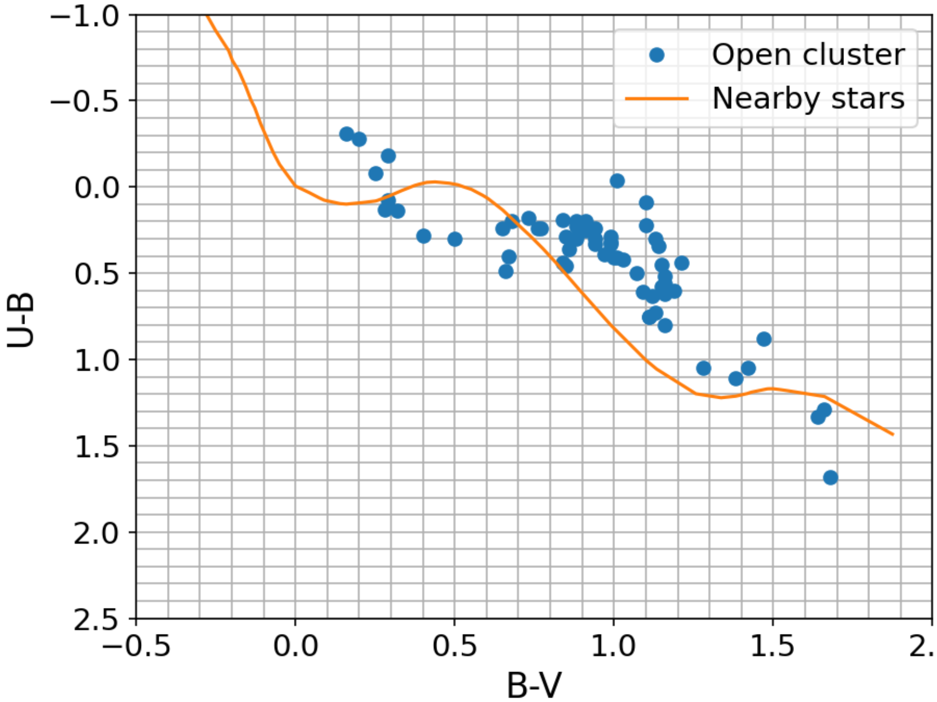
$$M = \frac{6r\overline{v_r^2}}{G} = \frac{6 \times 50 \times 10^3 (3.09 \times 10^{18} 300^2 (10^5)^2}{6.67 \times 10^{-8}} \text{ g}$$
$$= 1.3 \times 10^{46} \text{ g} = 6.3 \times 10^{12} M_\odot$$

(c) (5 points) What would be the shape and approximate Hubble type of the galaxy if the assumptions in part a were true? What if the assumptions in part b were true?

Solution: The purely rotating galaxy is pancake-flat, with smaller scale height given the small random motions. So it is probably a late-type spiral (Sb or c, SBb or c).

If in the latter case the purely-random motions are random in direction, then the galaxy is spherical (E0 elliptical).

5. You have measured the U, B, and V magnitudes of many stars in an open cluster. You plot $U - B$ against $B - V$ and compare this graph to that for nearby main-sequence stars with no extinction. Here is what you get:



(a) (10 points) Estimate the color excess and extinction at visible wavelengths between us and the open cluster.

Solution: Judging from the bend that appears around (0,0) in the nearby stars and around (0.3, 0.2) in the cluster stars, it looks like the points would be centered around the curve if we shifted the points by about -0.3 in B–V and about -0.2 in U–B. The B–V color excess of the cluster is therefore $E(B - V) = 0.3$, so

$$A_V = 3.06E(B - V) = 0.9 \text{ mag}$$

6. **Dust heated by starlight**: Dust grains made of graphite will sublime (that is, turn from solid to gas) at a temperature $T \approx 1500$ K. The albedo (the fraction of incident light reflected by an object's surface) of graphite is $A \approx 0.04$.

   (a) (10 points) How close to an O5 V star ($T_e = 42\ 000$ K, $R = 12R_\odot$) can graphite grains survive? *Hint: assume the grains are not rotating, so that only one part of a grain's surface ever faces the star.*

   Solution: Equate the power incident on the grain and the power it outputs as blackbody radiation. Assuming it has a surface area a facing the star and observes a flux $f = L/4\pi r^2$, where r is the distance to the star,

$$\begin{aligned} P_{\text{in}} &= P_{\text{out}} \\ f \cdot a(1-A) &= a\sigma T_{\text{g}}^4 \\ \frac{L}{4\pi r^2}(1-A) &= \sigma T_{\text{g}}^4 \\ \frac{4\pi R^2 \sigma T_e^4}{4\pi r^2}(1-A) &= \sigma T_{\text{g}}^4 \\ r &= R(1-A)^{1/2}\left(\frac{T_e}{T_{\text{g}}}\right)^2 \\ &= 12R_\odot(0.96)^{1/2}\left(\frac{42000}{1500}\right)^2 \\ &= 9216R_\odot = 6.412\times 10^{14}\text{ cm} \approx 43\text{ AU} \end{aligned}$$

   (b) (5 points) How close to an M2 III star ($T_e = 3540$ K, $R = 0.5R_\odot$) can graphite grains survive?

   Solution: Using the same solution as above,

$$\begin{aligned} r &= R(1-A)^{1/2}\left(\frac{T_e}{T_{\text{g}}}\right)^2 \\ &= 0.5R_\odot(0.96)^{1/2}\left(\frac{3540}{1500}\right)^2 \\ &= 2.73R_\odot = 1.899\times 10^{11}\text{ cm} \approx 0.013\text{ AU} \end{aligned}$$

7. The object at the center of the Milky Way, Sgr A*, is a black hole with mass $M = 4.154 \times 10^6 M_\odot$. However, it currently has a fairly small luminosity of $L = 10^3 L_\odot$, many orders of magnitude less than the black holes in quasars like 3C 273, which has $L = 10^{12} L_\odot$.

   (a) (10 points) If Sgr A* turns accreted mass into radiation with efficiency $\eta = 0.1$, what is its current mass accretion rate? What accretion rate is required to make Sgr A* have the same luminosity as 3C 273? Express your answers in $M_\odot\ \mathrm{yr}^{-1}$.

Solution:

$$\left(\frac{dM}{dt}\right)_{\mathrm{3C\ 273}} = \frac{10^{12} L_\odot}{\eta c^2} = 0.7 M_\odot\ \mathrm{yr}^{-1}$$
$$\left(\frac{dM}{dt}\right)_{\mathrm{Sgr\ A*}} = \frac{10^{3} L_\odot}{\eta c^2} = 7 \times 10^{-10} M_\odot\ \mathrm{yr}^{-1}$$

   (b) (5 points) What is the maximum possible accretion-driven luminosity of the Milky Way's central black hole? Can it produce $10^{12} L_\odot$?

Solution: The Eddington luminosity — the maximum that can be produced by accretion onto the black hole in Sgr A* — is "only"

$$\begin{aligned} L = L_E &= \frac{3GMm_p m_e^2 c^5}{2e^4} \\ &= 32821 \frac{M}{M_\odot} L_\odot \\ &= 1.3 \times 10^{11} L_\odot \end{aligned}$$

So it cannot produce such a high luminosity.

(c) (5 points) What is the maximum rate in $M_\odot\ \mathrm{yr}^{-1}$ at which the black hole Sgr A* is capable of accreting material? Can it accrete at the same rate as 3C 273?

> **Solution:** The black hole in Sgr A* can accrete matter at a rate no larger than
>
> $$\left(\frac{dM}{dt}\right)_E = \frac{L_E}{\eta c^2} = 0.09 M_\odot\ \mathrm{yr}^{-1}$$
>
> It cannot achieve the rate of accretion of 3C 273.