Instrumental Sensitivity

January 18, 2024

University of Rochester

Instrumental sensitivity

Useful references

- ASTR 203 lectures 19–25 `http://www.pas.rochester.edu/~dmw/ast203/`

The Deer Lick galaxy group (Mees Observatory image)

Making astronomical images, simplified

Much more complicated than most picture-taking.

- Identify a date range and an object which is high in the sky at night for at least 3–4 hours on those dates.
- Compile, or plan to take, the calibration data.
  - Dark-current and bias frames, 32–256 frame sequences each, with the CCD operating at the temperature you intend to use ($T = -35^\circ$ C to -20° C).
  - Flat field frames, 32–64 frame sequences in each filter, using the telescope cover's built-in flat field lamp.
  - A few 8–10 mag A stars near your target.
- Plan to autoguide on all deep-sky targets; identify a 6–14 mag star about 12 arcmin from your target.

At the telescope

For scientifically-useful images, take

- Frames in R, G, B, and/or narrowband spectral-line filters, all binned 2×2 pixels. Relative numbers depend on desired sensitivity.
- Shorter-exposure sequences on the calibration standards, every so often.
- As many frame sequences as you have time for.
- 5 minute exposures in moonlight; 8–10 minute exposures in dark skies. All will be averaged together in the end.

For pretty pictures: as above, but

- 3–4 times as many frames in L as any of the other filters, in 1×1 binning.
- Again, as many frames as you have time for in > 4 hours.

Note that most deep-sky APOD images involve $\gg 4$ hours.

Process the images

For each object and filter,

1. Calibrate each image: subtract dark and bias charge, divide by flat field.
2. Remove images in which something bad happened (clouds, tracking failure, dome lights inadvertently turned on, etc.)
3. Align the images so that the stars are all in the same pixels.
4. Normalize the images so that stars and blank sky are the same (corresponding) brightness in each.
5. Remove cosmic rays and satellite trails (and other artifacts).
6. Average all of the images in the same filter; convert to physical units.
7. Extract the quantities that you need (e.g. fluxes for each star) and proceed to scientific analysis.

For pretty pictures

After steps 1–6,

7. Make a composite RGB or false-color image from the images taken with different filters, with brightness scales (usually) chosen so that A0 stars look white.
8. Deconvolve (sharpen) your averaged L image, preferably using the maximum entropy method in CCDStack, IDL, or Python.
9. Process your deconvolved L image and composite RGB or false-color image in Photoshop, making a composite LRGB image with
   - High contrast and dynamic range (black blank sky, vibrant colors, reduced noise)
   - No flaws such as "dust donuts," etc.
   - Cropping to the desired size and aspect ratio.

How long of an exposure is needed?

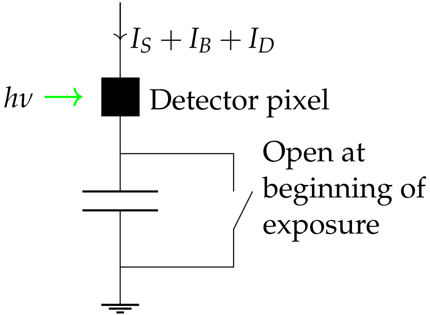
Before starting a project, the first thing to figure out is how long it will take. For astronomical images, the instrument sensitivity and the target flux both need to be known to estimate the necessary exposure time.

- Individual frames should be exposed long enough that the sky background is larger than the dark current but not so long that there are many cosmic-ray hits in the frame.
  - Usually 5–12 minutes per frame, depending on whether or not the moon is up.
- As many frames as it takes to fill the available time.
  - 4–5 hours is a good target for a high-quality color image of a Messier object.
- More is always better, but a signal-to-noise ratio of at least 10 on your target is needed.
  - An image barely showing objects half as bright takes four times as long.

Sensitivity and signal-to-noise ratio

To astronomers, “sensitive” means a large ratio of signal to noise.

Signal = current produced by the CCD’s absorption of light from the celestial object, which we will call I_S. For a small bandwidth — filter width $\Delta\lambda \ll \lambda$ — a typical CCD pixel draws



$$I_S = \tau\eta q_e \frac{\lambda P_S}{hc}$$

The other terms are

- τ Transmission of optics and atmosphere, within $\Delta\lambda$; range = 0–1
- η Quantum efficiency of detector; range = 0–1
- λ Wavelength; range = 3000–7000Å for visible light
- h, c, q_e Physical constants, by their usual symbols
- P_S (Signal) power incident from object, in watts or erg/s

Sensitivity and signal-to-noise ratio

Because of the finite, quantized electron charge and random arrival time of electrons at given points in a circuit, there are three statistically-independent noise sources in the CCD current. The general term for this effect is shot noise. In terms of the current drawn,

$$
\begin{aligned}
I_N^2 = \overline{(\Delta I^2)} = \frac{q_e I}{\Delta t} &= \frac{q_e}{\Delta t}(I_S + I_B + I_D) \\
&= \frac{q_e}{\Delta t}\left[\frac{\lambda\tau\eta q_e}{hc}(P_S + P_B) + I_D\right]
\end{aligned}
$$

Here, we refer to a single pixel or group of pixels in the CCD, and

- P_S Power from the object incident on the pixel
- P_B Incident background power (e.g. moonlight, city lights)
- I_D Dark current: Current drawn by the pixel even when no light shines; can be made negligible at sufficiently low CCD operating temperature
- Δt Exposure time: time over which the current is averaged

Sensitivity and signal-to-noise ratio

The reason for this form of the square of the noise current is that electrons in small currents pass given points in a circuit randomly and at discrete times. This is a process governed by Poisson statistics.

Reminders on Poisson:

- For a Poisson-distributed variable with mean value $\overline{N}$, the variance $\overline{\Delta N^2}$ is

$$\overline{\Delta N^2} \equiv \overline{(N - \overline{N})^2} = \overline{N^2} - \overline{N}^2 = \overline{N}$$

$$\Delta N_{\mathrm{rms}} = \sqrt{\overline{N}}$$

- If we are talking about N electrons counted in a time Δt, we can write this in terms of charge or current as

$$\overline{\Delta Q^2} = q_e \overline{Q} \qquad \overline{\Delta I^2} = \frac{q_e}{\Delta t}\overline{I}$$

$$\Delta Q_{\mathrm{rms}} = \sqrt{q_e \overline{Q}} \qquad I_N = \sqrt{\frac{q_e}{\Delta t}\overline{I}}$$

Sensitivity and signal-to-noise

So the signal-to-noise ratio is

$$\frac{S}{N} \equiv \frac{I_S}{I_N} = \tau\eta q_e \frac{\lambda P_S}{hc}\left(\frac{q_e}{\Delta t}\left[\frac{\lambda\tau\eta q_e}{hc}(P_S + P_B) + I_D + I_R\right]\right)^{-1/2}$$

$$= P_S\sqrt{\frac{\lambda\tau\eta}{hc\left(P_S + P_B + \frac{hc}{\lambda\tau\eta q_e}[I_D + I_R]\right)}\Delta t}$$

Usually, one of the terms in the denominator is much larger than the others, and the smaller ones can be ignored to good approximation:

Background-limited $P_B \gg P_S, hc(I_D + I_R)/\lambda\tau\eta q_e$

$$\left(\frac{S}{N}\right)_{\mathrm{BL}} = P_S\sqrt{\frac{\lambda\tau\eta}{hcP_B}\Delta t} \propto P_S\sqrt{\eta\Delta t}$$

Sensitivity and signal-to-noise

Source-limited $P_S \gg P_B, hc(I_D + I_R)/\lambda\tau\eta q_e$

$$\left(\frac{S}{N}\right)_{\rm SL} = \sqrt{\frac{\lambda\tau\eta P_S}{hc}\Delta t} \propto \sqrt{\eta P_S \Delta t}$$

Dark-current-limited $I_D \gg I_R, \lambda\tau\eta q_e(P_B + P_S)/hc$

$$\left(\frac{S}{N}\right)_{\rm DL} = \frac{\lambda\tau\eta P_S}{hc}\sqrt{\frac{q_e}{I_D}\Delta t} \propto \eta P_S \sqrt{\Delta t}$$

Note that, in all cases, the S/N scales with $\sqrt{\Delta t}$. **To increase the S/N by a factor of 2, the exposure time must increase by a factor of 4.**

Use these equations to estimate the exposure time necessary for a given project.

Sensitivity and signal-to-noise

Along with these noise sources related to the object, natural backgrounds, and detector, there is additional noise from the readout circuitry, called read noise, that appears as a random extra number of electrons added to each pixel in each image, **independent of exposure time**.

- Read noise is statistically independent of the other noise sources. That means the grand-total variance is the sum of the separate variances (the quadrature sum).
- Including read noise, the grand-total variance in the charge on each pixel is

$$\overline{\Delta Q^2} = I_N^2 \Delta t^2 + q_e^2 N_R^2$$

  where N_R is the root-mean-square (rms) number of electrons randomly added to each pixel.

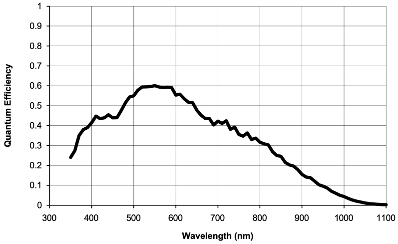
We strive to have sensitivity limited by natural sources of background. Thus, we

- Operate the CCD at low enough T that $I_D \ll I_B$
- Take long enough exposures that $I_N^2 \Delta t^2 \gg q_e^2 N_R^2$

Our imaging system

Santa Barbara Instrument Group (SBIG) STX 16803 CCD cameras. We have two (#1, #2).

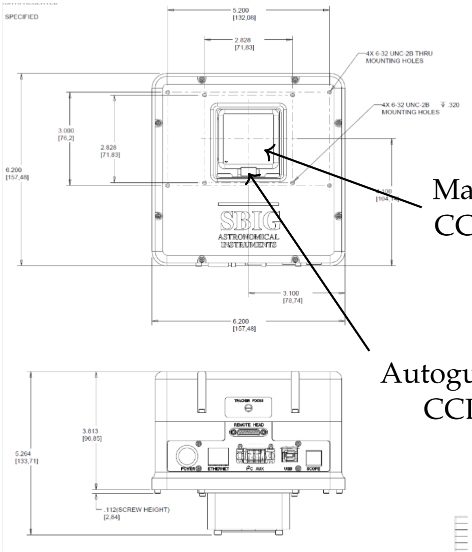
- Frame-transfer CCD, 4096×4096, 9 μm pixels, plus a separate interline CCD, 657×495, 7.4 μm pixels for autoguiding, next to the big imaging CCD. The latter is blocked by Filter Wheel #2.
- Quantum efficiency $\eta = 0.45 - 0.6$ across the visible band.
- 16-bit output; 1.27 electrons per data number (DN) for Camera #1, 1.35 electrons per DN for Camera #2. (Pixel readouts are recorded in DN.)
- Dark current $I_D / q_e = 0.009$ electrons per second per pixel at $T = -20°$C.



ON Semiconductor

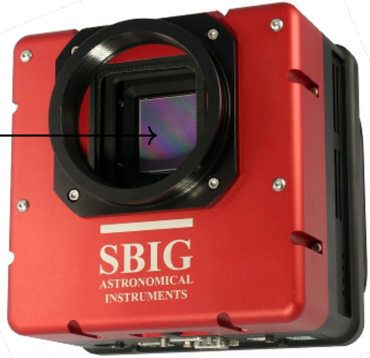
- Read noise $N_R = 9$ electrons (rms) per pixel at $T = -20°$C. Half that, if the image is binned into 2×2 pixel blocks, as we usually do except with our L filter.

Our imaging system



Main CCD

Autoguider CCD



Shutter open