

Periodically-Variable Objects

Characterizing time variations

March 5, 2024

University of Rochester

Periodically-variable objects

This includes mostly pulsating stars, spotted stars, and eclipsing systems, but we will also discuss characterizing any sort of time variation.

One-stop shopping for periodogram calculations: the NStED periodogram tool, which calculates Lomb-Scargle, box-fitting least squares, and Playchan periodograms on your input data.



M15, LRGB, from Mees

Cadence

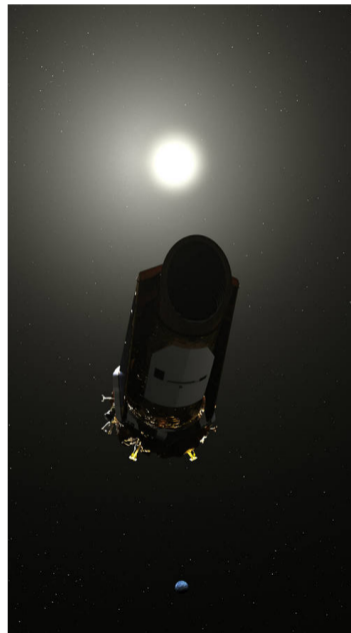
The rate at which photometric observations are made on potentially variable objects is called the **cadence**.

- ▶ The cadence has to be sufficiently fast to sample the variability thoroughly.
- ▶ If you know what the range of time durations is, a cadence can be designed.
- ▶ If the range is HUGE, from milisecond flares to years-long orbits, the design can be a challenge.
- ▶ The Vera Rubin Telescope (née LSST) will use many hybrid cadences, with a baseline of two back-to-back short exposures per filter per visit, and two visits per week to every object that is visible.



Cadence

- ▶ The main *Kepler* mission had roughly half-hour exposures every half hour for about 90 days at a time, for several years.
 - ▶ As a result, it was very good at detecting exoplanetary orbits and most forms of stellar pulsation.
 - ▶ We will use *Kepler* data in today's examples.



Finding variables: The simple way

In ASTR 244/444, we look for periodic variability in globular cluster members, and in transiting exoplanetary systems, and seek to measure periods.

The most practical way to spot the variation in globular clusters, since the variation is about half of a magnitude:

- ▶ For RR Lyr stars in globular clusters: blink fully calibrated and aligned images taken 1 to several hours apart.

Look hard.

Zoom in.



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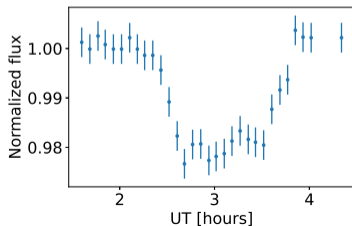
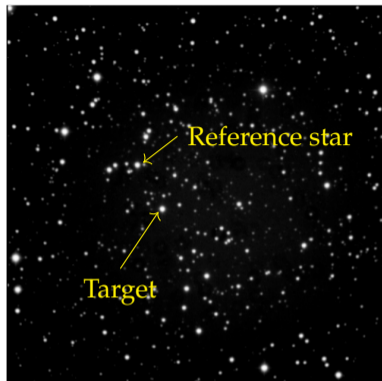


Finding variables: The simple way

For exoplanet transit light curves, good results can be obtained by

- ▶ Dark and bias subtracting, and aligning, all of the frames, but *not* normalizing them.
- ▶ Doing aperture photometry (Lesson 5) on the target and at least one unsaturated reference star of similar or greater brightness, in each frame.
- ▶ Dividing the target photometry by the reference and scaling the result to 1 outside the transit.
- ▶ Plotting v. time

At the right: transit of a hot Jupiter, Qatar-1b, from Mees. Blinking does not work here, as the variation is only about 0.02 magnitudes.



Finding variables in long-time-series data

What follows are the predominant precise and accurate ways to identify variables and measure periods.

Periodograms Power spectra derived from specialized forms of the Fourier transform, particularly the Lomb-Scargle periodogram ([Scargle 1982](#), [VanderPlas 2017](#)).

Phase-dispersion minimization Light curves folded and phase-shifted to minimize dispersion of brightness at a given phase, relative to a prior or to a smoothed version of the light curve ([Stellingwerf 1978](#), [Plavchan et al. 2008](#)).

Both of these improve with the photometric accuracy and the length of the continuous time series.

It is pointless to use either one unless the time series spans at least a few periods.

- ▶ But that sometimes does not matter, e.g. if all you are after is the magnitude and dispersion of a population of variables, as in the baseline globular cluster experiment.

Fourier transforms

A few reminders from MATH 281 or its equivalent:

- ▶ The Fourier transform of a continuous function $f(t)$ is the amplitude spectrum $F(\omega)$:

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt \quad \Leftrightarrow \quad f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega)e^{i\omega t} d\omega$$

- ▶ If the data $f(t)$ were a sinusoid with (angular) frequency ω_0 , the transformation $F(\omega)$ would be a pair of delta functions: infinitely narrow spikes at $\pm\omega_0$.
- ▶ If the time series were to last for only a time T , the amplitude spectrum for a pure sinusoid would not be infinitesimally narrow. For example,

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-T}^T e^{i\omega_0 t} e^{-i\omega t} dt = \sqrt{\frac{2}{\pi}} \frac{\sin((\omega - \omega_0)T)}{(\omega - \omega_0)T}$$

The sinc function, with central peaks at $\pm\omega_0$, $\sim 2\pi/T$ wide (FWHM).

Discrete Fourier transforms

In real life, transforms are discrete, as our time-series data are series of averages over intervals of time: **samples**.

If the sampling interval is uniform at Δt for $N + 1$ total samples, then the transform becomes

$$F(\omega_k) = \sum_{n=0}^N f_n e^{-i\omega_k n \Delta t}$$

or, with $\Delta\omega = 2\pi/N\Delta t$ and $\omega_k = k\Delta\omega$,

$$F_k = \sum_{n=0}^N f_n e^{-2\pi i k n / N}$$

Power spectra

“Periodogram” is another name for power spectrum, apart from an extra factor of the inverse of the number of samples:

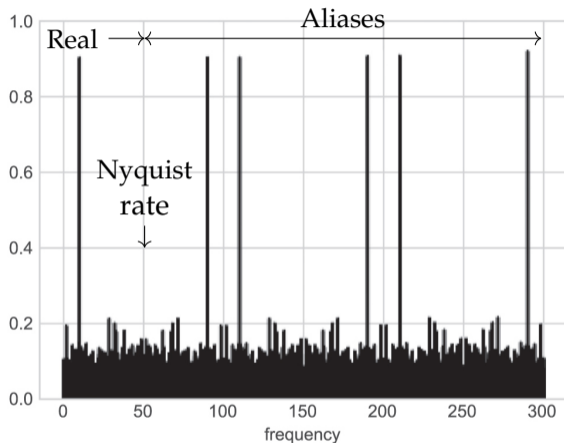
$$P_0(\omega) = |F(\omega)|^2 = \left| \sum_{n=0}^N f_n e^{-i\omega n \Delta t} \right|^2 \quad \text{(Discrete) power spectrum}$$

$$P_S(\omega) = \frac{1}{N} \left| \sum_{n=0}^N f_n e^{-i\omega n \Delta t} \right|^2 \quad \text{Classical (Schuster) periodogram}$$

To be precise, the periodogram is an estimate of the power spectrum, as the power spectrum is a continuous function of the parameters of the star or other emitting region, but this distinction is often ignored.

Aliasing

- ▶ With discrete sampling, say $N + 1$ samples at interval Δt , any frequency higher than $\omega_0 = \pi/\Delta t$ cannot be taken seriously, since the power is incorrectly mapped to these frequencies. This is called **aliasing**.
- ▶ In the frequency domain instead of the time domain: to represent the frequency content of a signal that is zero outside the range $\pm\omega_0$, the frequency data must be sampled at a rate of at least $\nu_0 = 2(\omega_0/2\pi)$.
- ▶ This is known as the **Nyquist theorem**, and the minimum sampling rate is known as the **Nyquist rate**.



A power spectrum extended to frequencies way too high, to show the aliasing. From VanderPlas (2017).

Nonuniform sampling and the Lomb-Scargle periodogram

Sampling intervals in astronomical observations are hardly ever uniform. For a set of samples at times t_n , and upon writing the classical periodogram as

$$P_S(\omega) = \frac{1}{N} \left(\left[\sum_{n=0}^N f_n \cos \omega t_n \right]^2 + \left[\sum_{n=0}^N f_n \sin \omega t_n \right]^2 \right)$$

Scargle (1982) generalized the expression to

$$P(\omega) = \frac{A^2}{2} \left[\sum_{n=0}^N f_n \cos \omega(t_n - \tau) \right]^2 + \frac{B^2}{2} \left[\sum_{n=0}^N f_n \sin \omega(t_n - \tau) \right]^2$$

and showed that there are unique forms for A , B , and τ such that the new periodogram reduces to the classical one in the limit of a uniform sampling interval, and gives other desirable properties as well, like analytically-computable statistics.

Nonuniform sampling and the Lomb-Scargle periodogram

It also turns out that this generalized periodogram is identical to what you get by fitting a sinusoid to the data at every frequency by χ^2 minimization, as was shown by [Lomb \(1976\)](#).

The result is called the **Lomb-Scargle periodogram**:

$$P_{LS}(\omega) = \frac{1}{2} \frac{\left[\sum_{n=0}^N f_n \cos \omega(t_n - \tau) \right]^2}{\sum_{n=0}^N \cos^2 \omega(t_n - \tau)} + \frac{1}{2} \frac{\left[\sum_{n=0}^N f_n \sin \omega(t_n - \tau) \right]^2}{\sum_{n=0}^N \sin^2 \omega(t_n - \tau)}$$

where

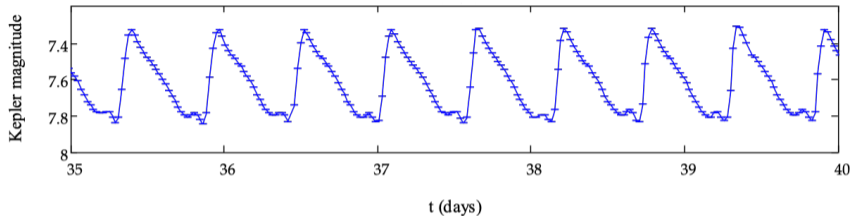
$$\tau = \frac{1}{2\omega} \arctan \left(\frac{\sum_{n=0}^N \sin 2\omega t_n}{\sum_{n=0}^N \cos 2\omega t_n} \right)$$

and is much easier to calculate than it looks.

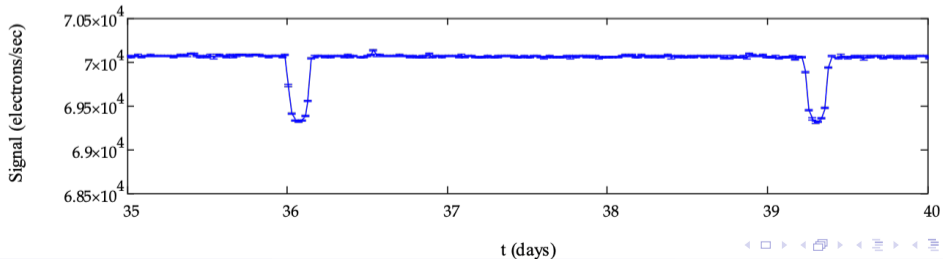
Its use is now universal for measuring periods from time-series data on objects with **large duty-cycle** variability, such as RR Lyr and Cepheid variable stars. Built into the [NStED periodogram tool](#).

Terminology

A large duty cycle oscillation:

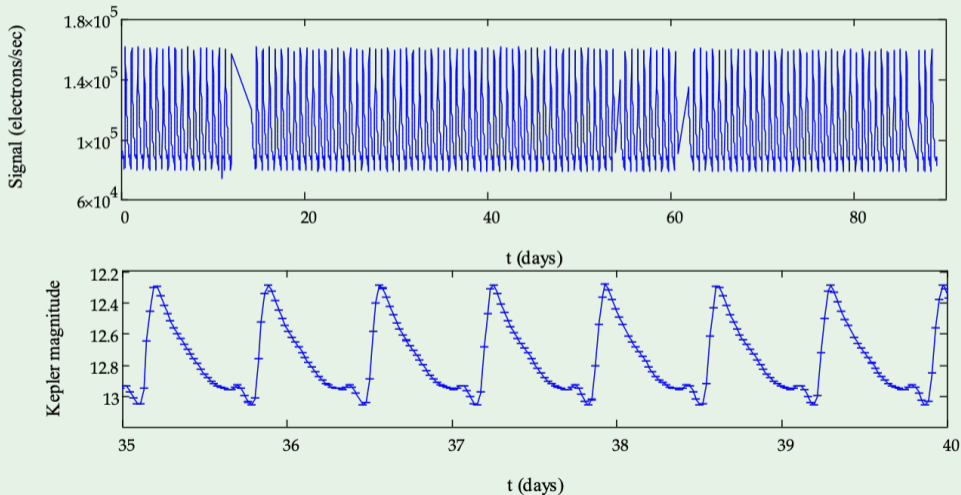


A small duty cycle oscillation:



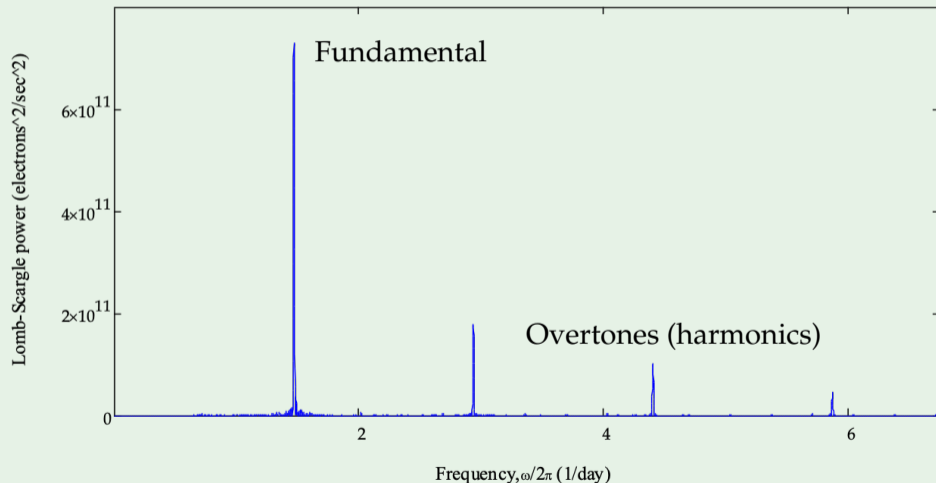
NR Lyrae, a perfect RR Lyrae star

Full light curve, and a close-up, from *Kepler* (Q2). *Kepler* gives really high quality data.



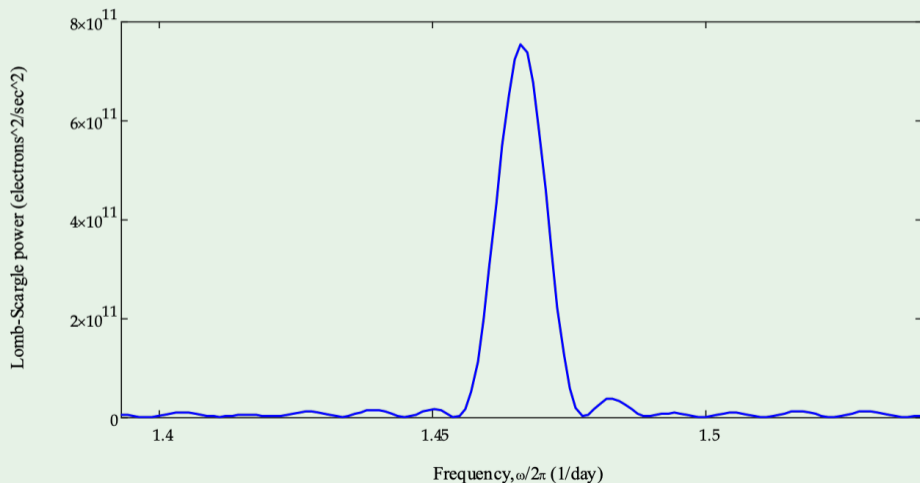
NR Lyrae, a perfect RR Lyrae star

L-S periodogram, after “prewhitening:” essentially filtering out low frequencies.



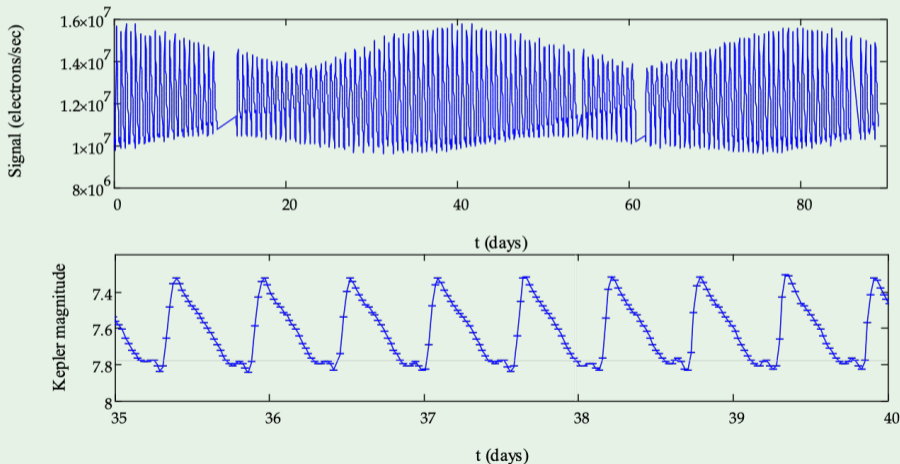
NR Lyrae, a perfect RR Lyrae star

Zoom-in on the fundamental mode. Note the square-sinc-function-like sidebands. The pulsation period is 16.369 hours.



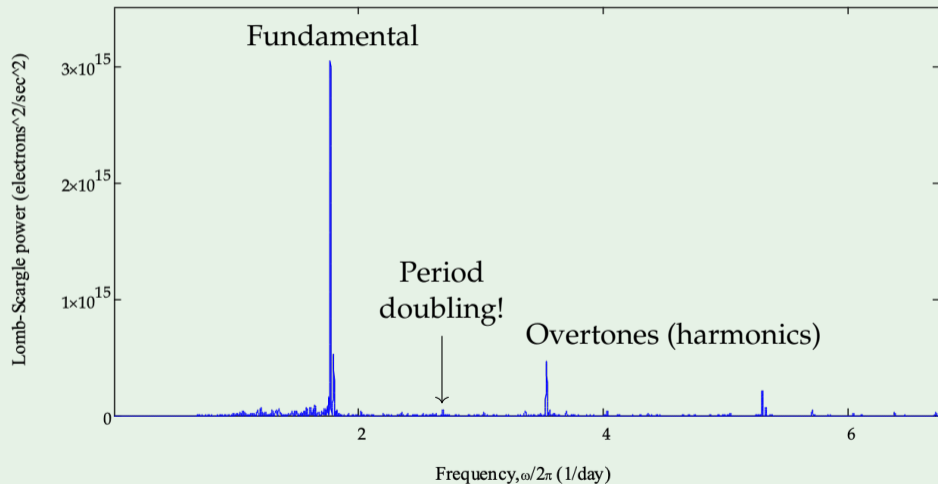
RR Lyrae, the archetype

Full light curve from *Kepler* (Q2). Note the periodic envelope, classically known as the Blazhko (1907) effect.



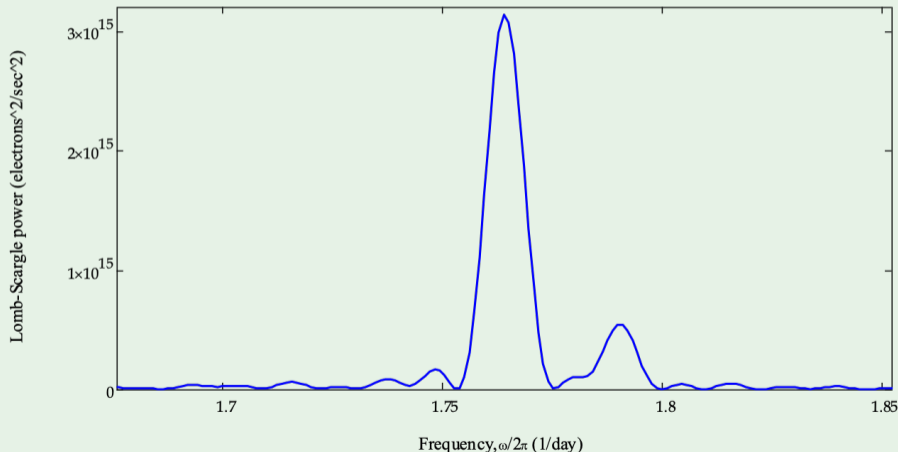
RR Lyrae, the archetype

L-S periodogram, after prewhitening. Power at half-integer “harmonics!”



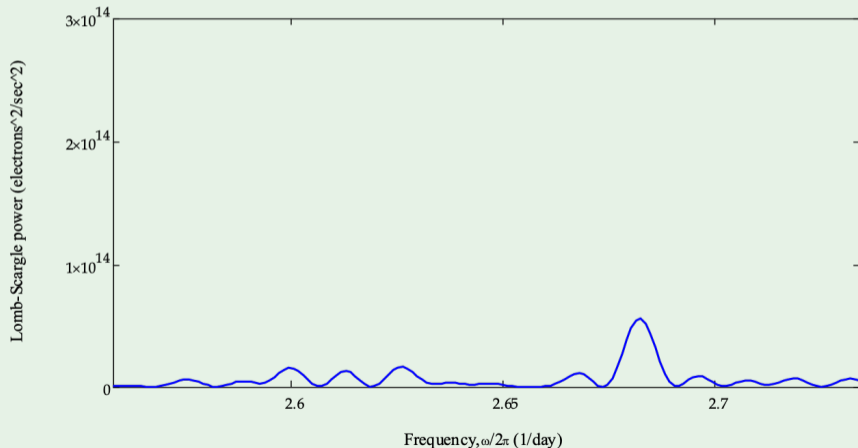
RR Lyrae, the archetype

Zoom-in on the fundamental mode. Note the lopsided sidebands, which are the power-spectrum counterpart of the Blazhko effect. The pulsation period is 13.605 hours.



RR Lyrae, the archetype

“1.5th harmonic:” sidebands present but not the main pulsation. Half-integer features in the power spectrum — a.k.a. period doubling — shows the origin of the Blazhko effect in nonlinear pulsation ([Szabo et al. 2010](#)). No feature like this in NR Lyrae.



How to make a phase-folded light curve

For long time-series observations, it is useful and customary to phase-fold the light curves so that all of the events in the series are plotted together according to their **phase** within the period.

- ▶ All programming languages have a built-in function called `mod` that can be used to map time t_n into phase ϕ_n :

$$\phi_n = \frac{2\pi}{\delta t} \text{mod}(t_n, \delta t) + \delta\phi$$

where

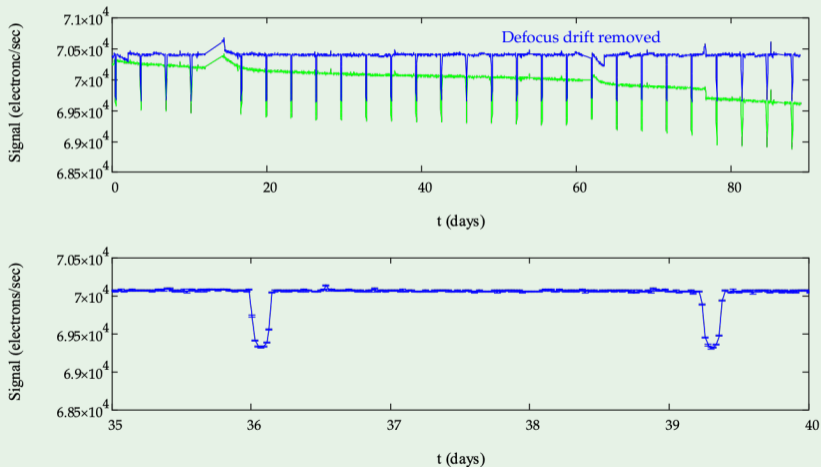
δt is the period of the variability (pulsation, orbit, ...)

$\delta\phi$ is the initial phase offset

- ▶ `mod` takes every δt segment of the time series and shifts it to earlier times by the amount δt . The other factors just convert the period to the phase scale, $0-2\pi$ radians, and shifts the scale overall to center it in plots as desired.

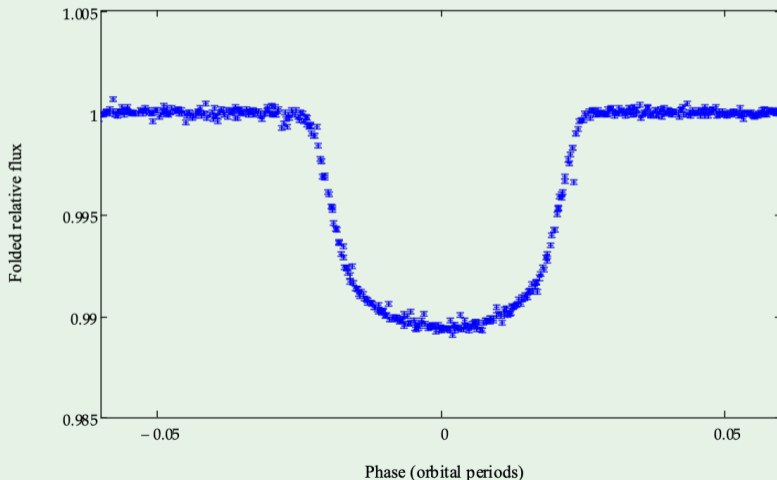
Kepler-6b

Kepler-6b is a classic hot Jupiter. The early *Kepler* (Q1) data are published in the *Kepler* special issue ([Dunham et al. 2010](#)); this is Q2.



Kepler-6b

Phase folded over $\delta t = 3.234723$ days, $\delta\phi = -0.3\pi$, phase values divided by 2π before plotting.



Phase-dispersion minimization

This procedure immediately suggests an automated, model-independent way to fold the light curve and determine the best value of the period δt :

- ▶ Start with a value for δt . Generate an initial folded light curve. Calculate its mean and variance,

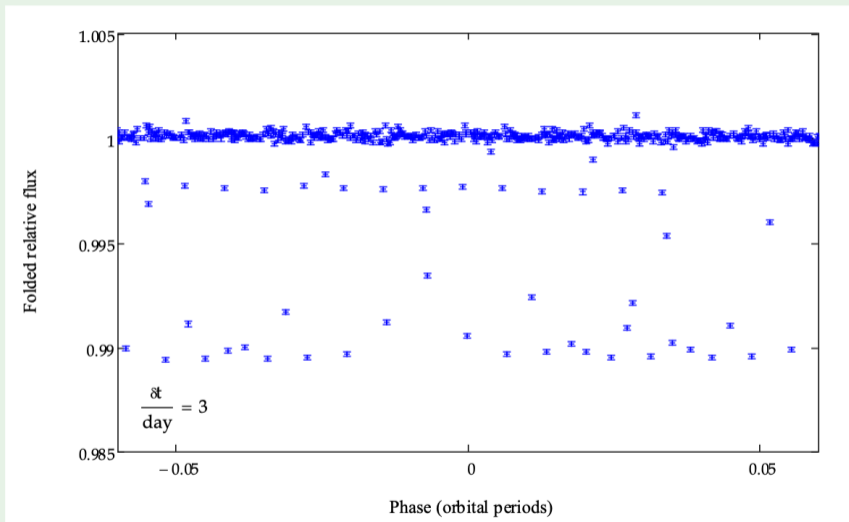
$$\bar{f} \text{ and } \sigma^2 = \sum_n \frac{(f_n - \bar{f})^2}{N - 1}$$

- ▶ Break the folded light curve into M bins, each containing m points. Within each bin j , calculate the mean and variance s_j^2 . Sum the variances to create

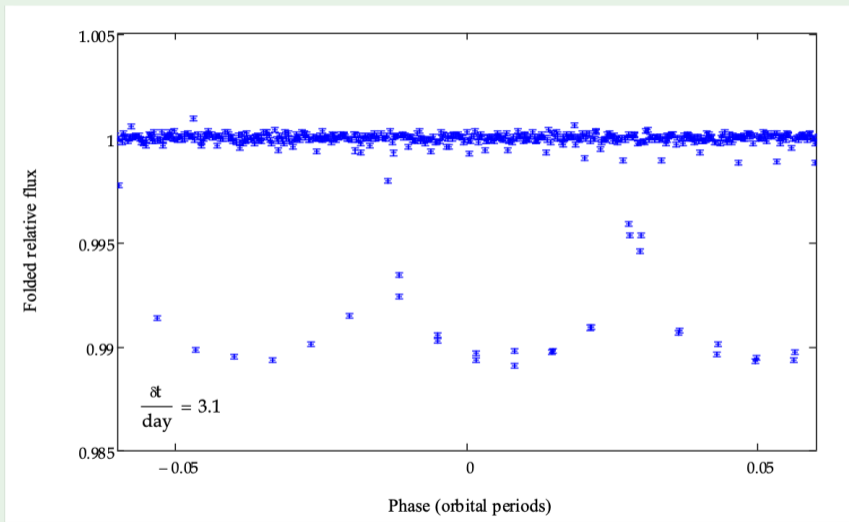
$$s^2 = \frac{\sum_j (m_j - 1) s_j^2}{\sum_j m_j - M} \quad \Theta = \frac{s^2}{\sigma^2}$$

- ▶ If δt is not the true period, then s^2 will be close to σ^2 and $\Theta \sim 1$.
- ▶ As δt approaches the true period, Θ approaches a minimum, with $\Theta \ll 1$.
- ▶ So, vary δt to find the minimum, or incorporate it all into a root-finding routine.

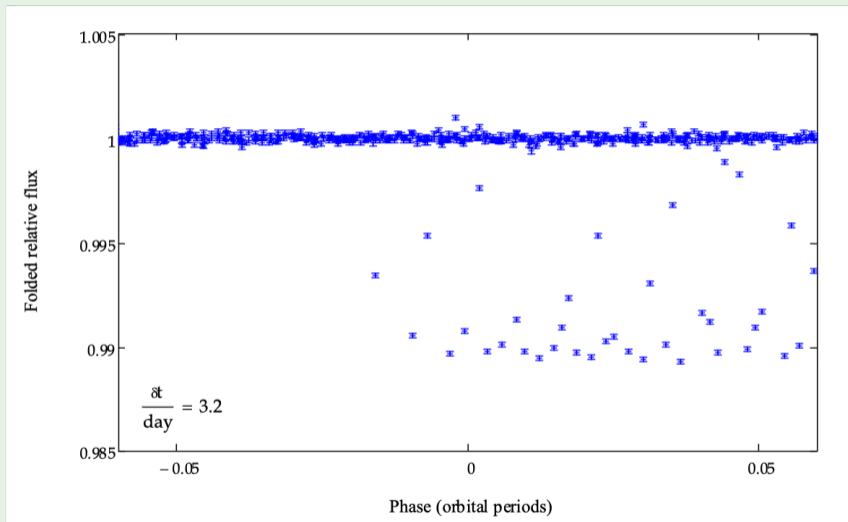
Minimizing the phase dispersion in Kepler-6b



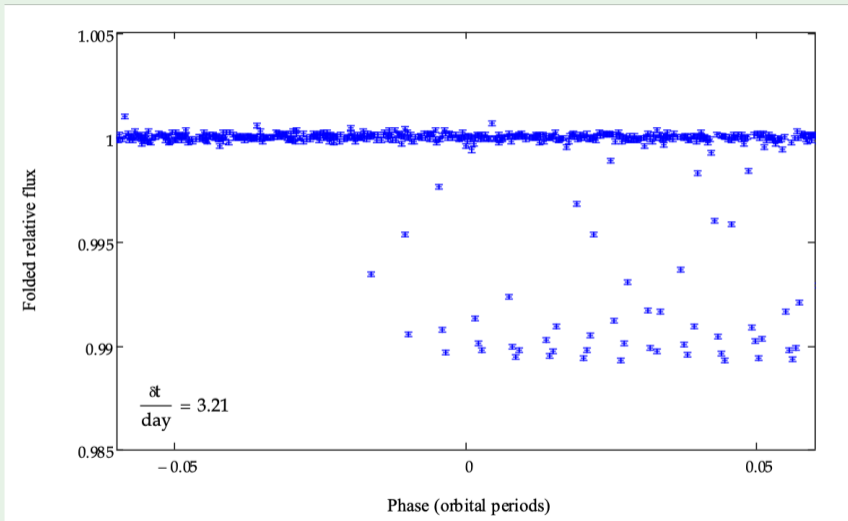
Minimizing the phase dispersion in Kepler-6b



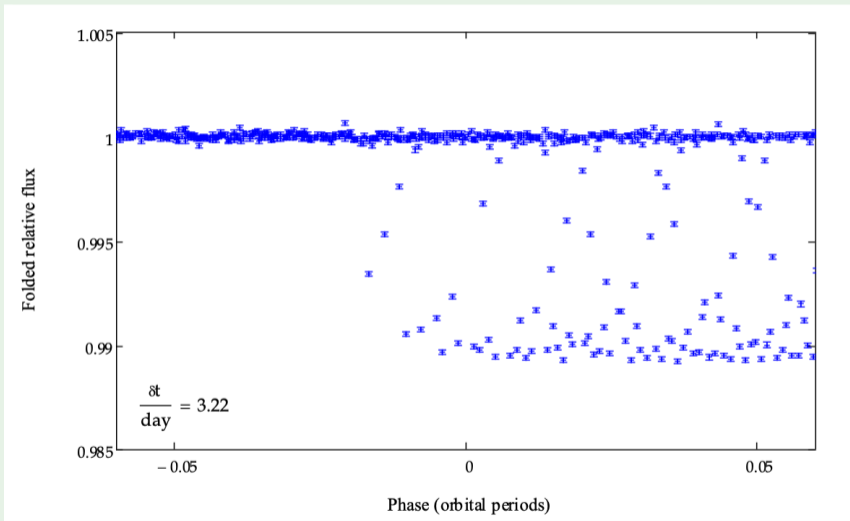
Minimizing the phase dispersion in Kepler-6b



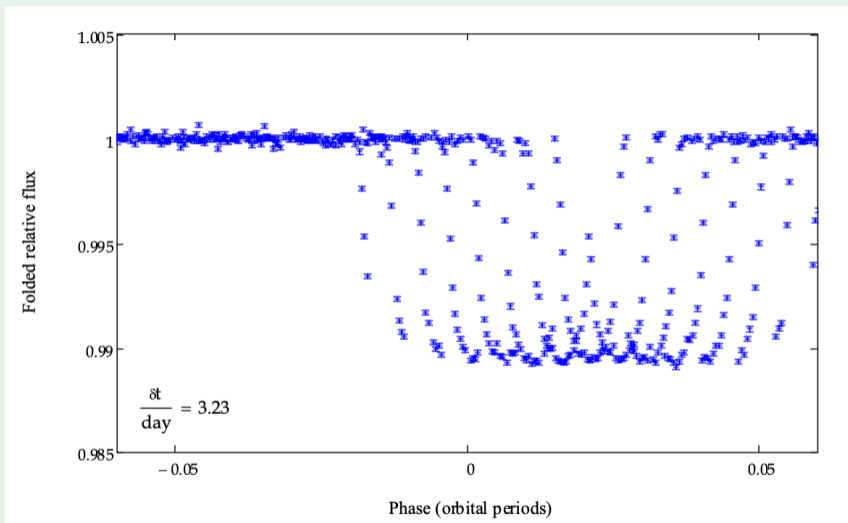
Minimizing the phase dispersion in Kepler-6b



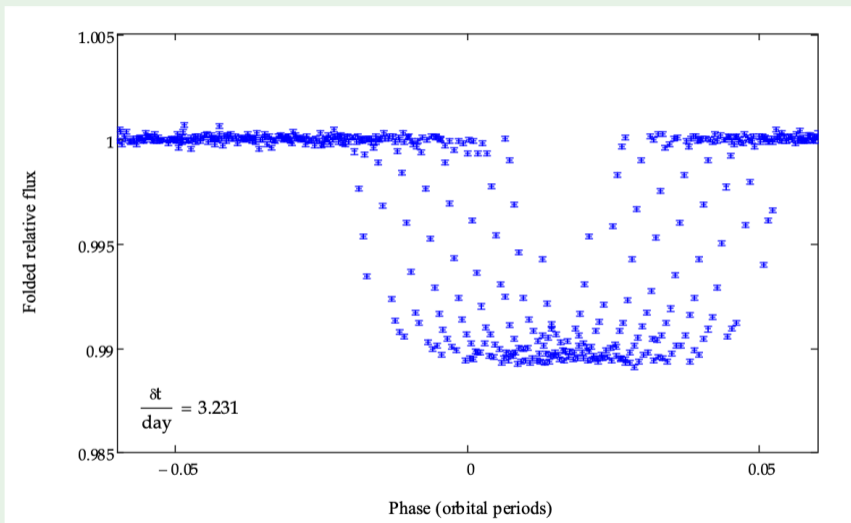
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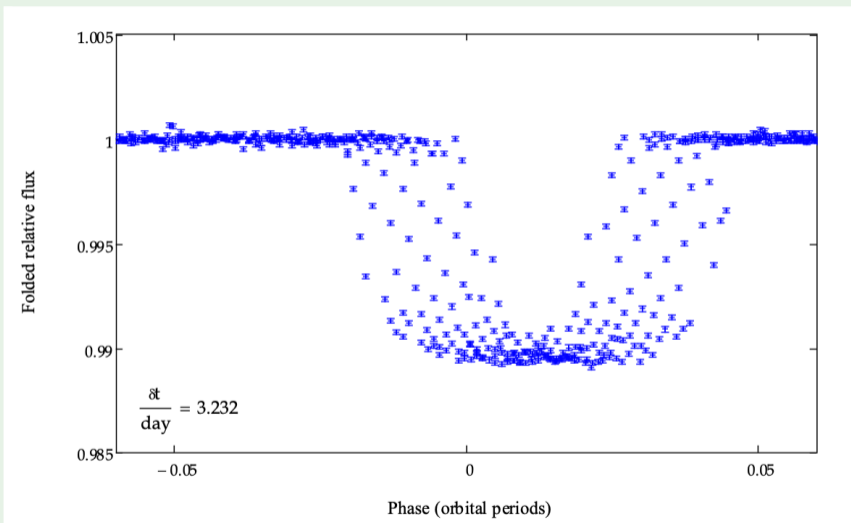
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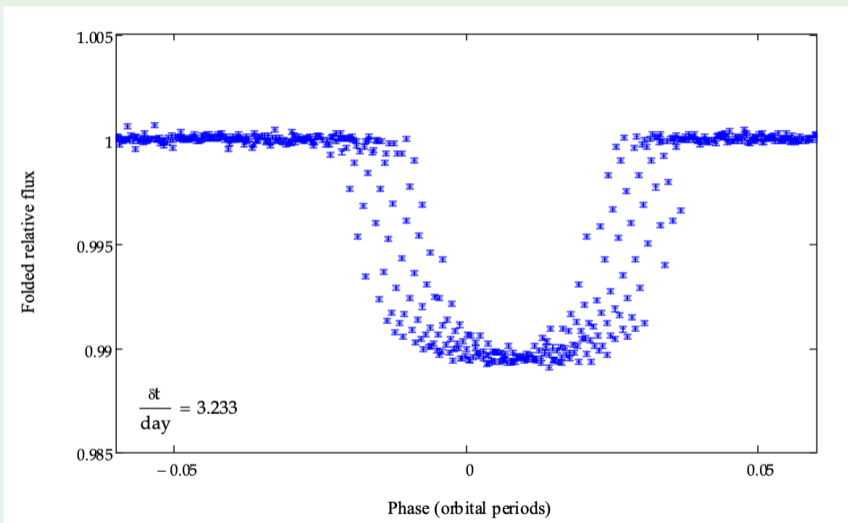
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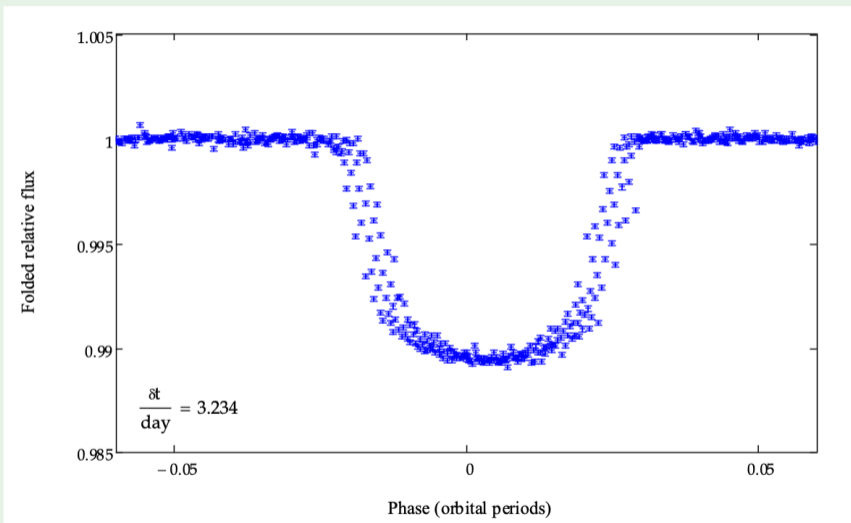
Minimizing the phase dispersion in Kepler-6b



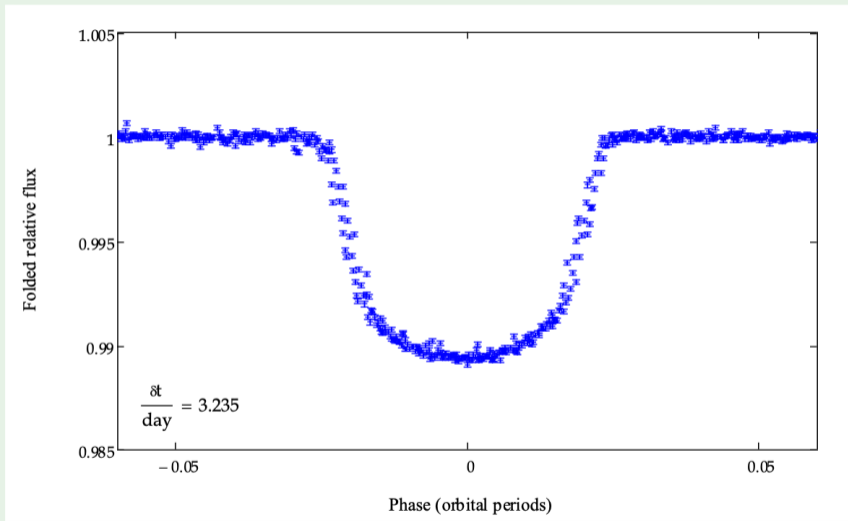
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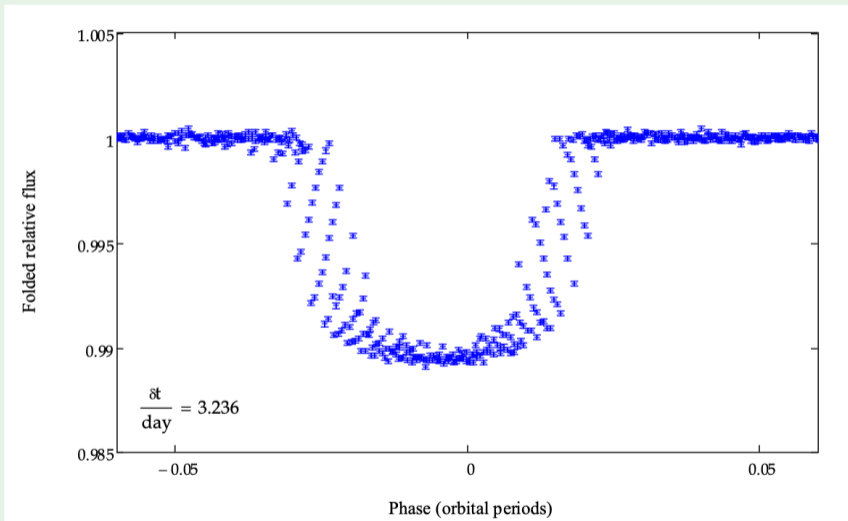
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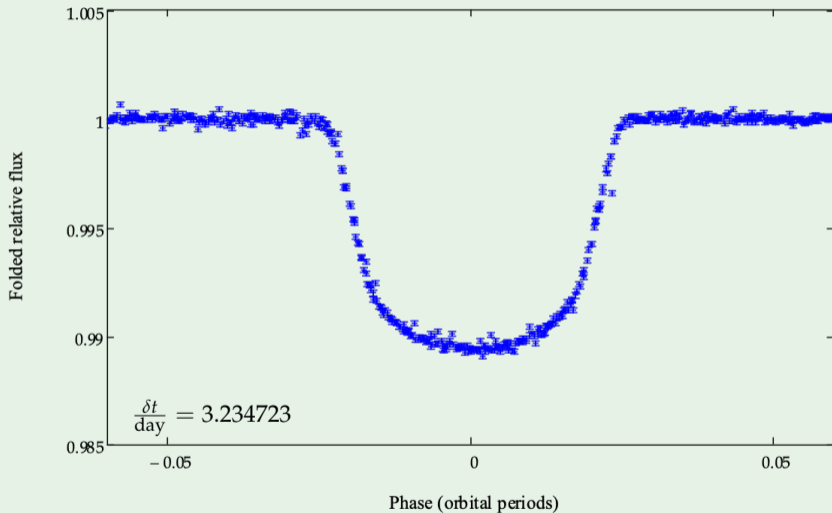
Minimizing the phase dispersion in Kepler-6b



Minimizing the phase dispersion in Kepler-6b



Minimizing the phase dispersion in Kepler-6b



Phase-dispersion minimization

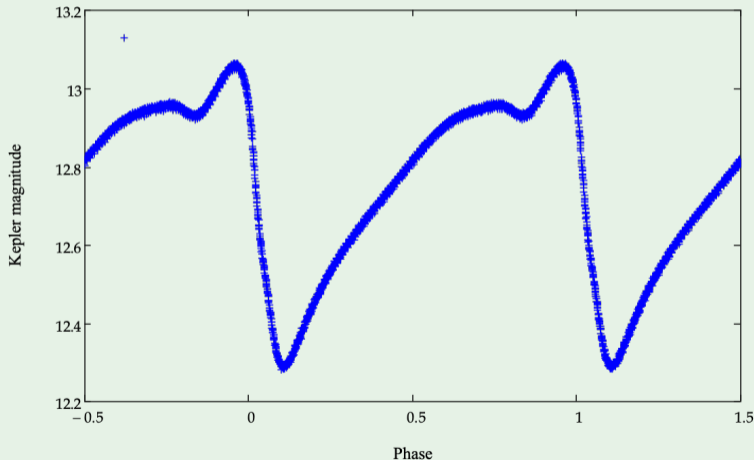
This means of folding light curves and determining periods was invented by [Stellingwerf \(1978\)](#) and has been in standard use ever since.

A particularly nice version by [Plavchan et al. \(2008\)](#) has been incorporated into the [NStED periodogram tool](#).

This is the way that we recommend folding light curves and measuring periods for those who are doing the exoplanet-transit project (if you have more than one transit imaged).

Phase-folded light curve for NR Lyrae

The smooth run of the points and error-bar ends probably indicates that the tiny uncertainties are still overestimated. (See next lesson.)



Phase-folded light curve for RR Lyrae

No error bars plotted, as the large Blazhko effect would make it too confusing.

