Observational uncertainties

Noise and systematic error
Propagation of uncertainty
Upper limits

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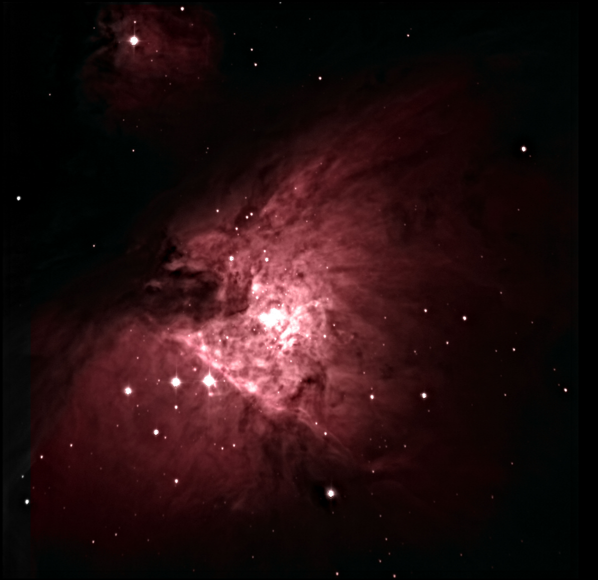
- Noise and systematic error
- Propagation of uncertainty
- Upper limits

Good references:

- Feigelson & Babu, 2012, *Modern statistical methods for astronomy* (New York: Cambridge)
- R



M42, LRGBHα, from Mees

Systematic uncertainty and error

Noise is reduced as more and more samples are averaged, as we have seen:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

but not all of the variation that you see in your signal is noise.

- Systematic uncertainty can loom as well, and be much larger than noise.
- Different origins from the noise, and best bookkept separately.
- Calibration is often (usually?) the leading cause of systematic uncertainty, but there are other correlated sources of variation to meet.
- Terminology: **Uncertainty** indicates a range of values, any of which is consistent with the measurement. **Error** is a mistake that should be fixed. “Error propagation” and “error bars” really refer to uncertainty and should be rephrased.

Calibration

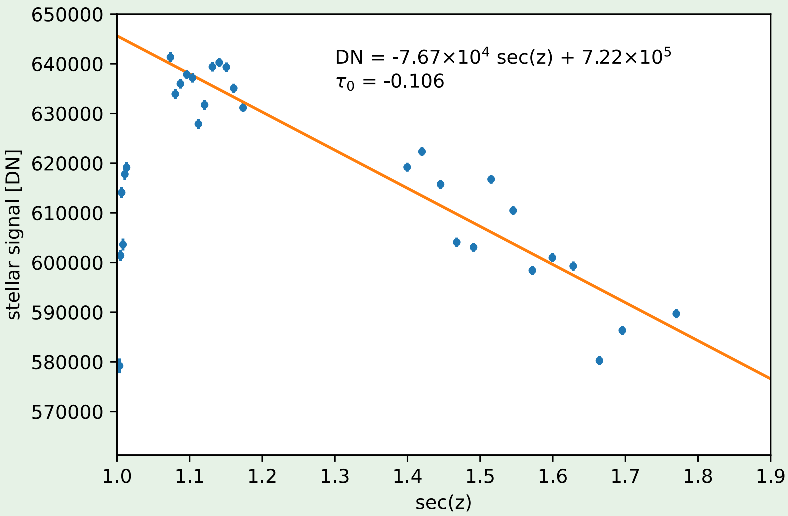
Suppose this star, last seen in Lesson 3, is a flux calibrator. We want to use the ratio of its standard flux, and our measured signal, to establish a conversion factor by which to multiply our images. What is the noise, and the systematic uncertainty, in the measurements?

- Aperture photometry, from ATV
- The S/N ratio is consistent with that expected from the sensitivity of the camera (Lesson 1), so the right-hand column does deserve the title "noise."
- Typically, noise is about 1000 DN.
- In Lesson 3, we showed that the signal exhibited the expected trend with $\sec z$...

$\sec z$	Signal [$\times 10^5$ DN]	Noise, rms [DN]
1.7701	5.90	921
1.6958	5.86	909
1.6642	5.80	912
1.6281	5.99	932
1.5994	6.01	915
1.5719	5.98	911
1.5454	6.10	924
1.5151	6.17	895
1.4910	6.03	895
1.4678	6.04	937
1.4455	6.16	909
1.4199	6.22	923
1.3996	6.19	906
1.1736	6.31	901
1.1608	6.35	923
1.1505	6.39	947
1.1407	6.40	936
1.1313	6.39	940
1.1206	6.32	981
1.1121	6.28	968
1.1039	6.37	953
1.0961	6.38	950
1.0873	6.36	977
1.0803	6.34	979
1.0736	6.41	1000
1.0132	6.19	1130
1.0108	6.18	1170
1.0083	6.04	1200
1.0064	6.14	1090

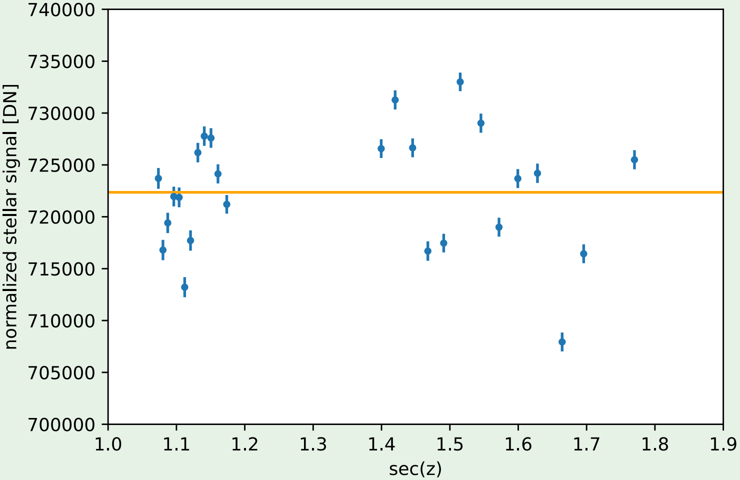
Calibration

Remove the trend, from this and all photometry subject to the same calibration.



Calibration

The standard deviation of the corrected points is 5733 DN. So the signal and the uncertainties can be written here as $\boxed{S = (7.22 \pm 0.01 \pm 0.06) \times 10^5 \text{ DN}}$



Interpreting the results

That the noise in a calibrated image is so much less than the systematic uncertainty indicates something important: that the **relative** brightness of objects in the same image is subject to quite a bit less uncertainty that the brightness in different images.

This has multiple origins: different PSFs in different images is often the leading effect.

So, for the best precision and smallest uncertainties in a sequence of images, take advantage of the smaller noise in each:

- In each image, measure the ratio of signal for every star relative to one or a few bright stars in that image.
- In the stack of images, determine the mean ratio of this *ratio* for each star.
- Correct each image in the stack by this ratio of ratios. Now, there will be less scatter from image to image of all of the stellar signals. The more stars that there are in an image, this strategy will result in the scatter approaching the noise. The overall calibration uncertainty still reflects the systematics, but that is less important than being able to find 1–2% deep transits.

Propagating uncertainties

The fundamental way to propagate uncertainties, or to combine uncertainties from a set of independent measurements, is already built into

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

That is, add the square deviations of each element in the set, and divide by one less than the number in the set. This is called **adding variances in quadrature**.

This presupposes that the members of the set have been established to be truly independent measurements.

Propagating uncertainties

For propagating uncertainties into results that are not linearly related to the signal — magnitudes, for example — we need one more result. Suppose that our signal is x and we are interested in the uncertainty in a function f that depends on x. Expand f in a Taylor series about the average value of x:

$$f(x) = f(\bar{x}) + \left.\frac{\delta f}{\delta x}\right|_{x=\bar{x}} (x - \bar{x}) + \ldots$$

so $\bar{f} = f(\bar{x})$. Furthermore, $\bar{f} \pm \sigma_f = f(\bar{x} \pm \sigma_x)$, so

$$\begin{aligned} \pm\sigma_f &= f(x \pm \sigma_x) - \bar{f} \\ &= \bar{f} + \frac{\delta f}{\delta x} f(\bar{x} \pm \sigma_x - \bar{x}) - \bar{f} \end{aligned}$$

$$\boxed{\pm\sigma_f = \pm\sigma_x \frac{\delta f}{\delta x}}$$

Since we have kept only first order, we presume that $\sigma_x \ll \bar{x}$.

The uncertainty in flux for a certain star is σ_f. What is the uncertainty in its magnitude?

With f_0 as the zero-magnitude flux, the magnitude m is

$$m = -2.5 \log\left(\frac{f}{f_0}\right) = -\frac{2.5}{\ln 10}(\ln f - \ln f_0)$$

so

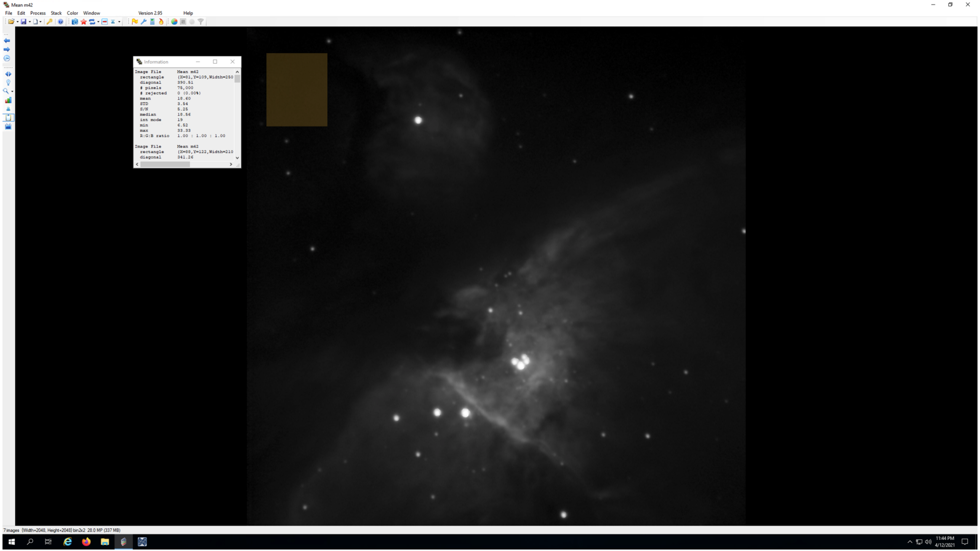
$$\sigma_m = \sigma_f \frac{d}{df}\left(-\frac{2.5}{\ln 10}(\ln f - \ln f_0)\right)$$
$$= -\frac{2.5}{\ln 10}\frac{\sigma_f}{f}$$
$$\boxed{\sigma_m \cong 1.09\frac{\sigma_f}{f}}$$

If you know your fluxes within 1% — which is doing pretty well — then you know your magnitudes within about 0.01.

Measuring noise from your images

- Using ATV for aperture photometry gives you signal and noise automatically, as we have seen.
- You can use ATV for photometry on objects besides stars.
- In its Information window, CCDStack presents the mean, median, and standard deviation of any rectangle you have just clicked-and-dragged in the image. With this, you can measure the signal and noise for objects of any size.
- **Upper limits**: If the average signal in an aperture is significantly less than the noise measured by the square root of the variance ($\sigma = \sqrt{\sigma^2}$), then the object is not detected. An upper limit is then reported.
  - In spectra, an upper limit is usually given as 3σ.
  - In images, where 3σ bumps are not uncommon, it is better to report 5σ upper limits.
  - Beware of basing a lot of science on 5.01σ “detections.”

Measuring noise from your images

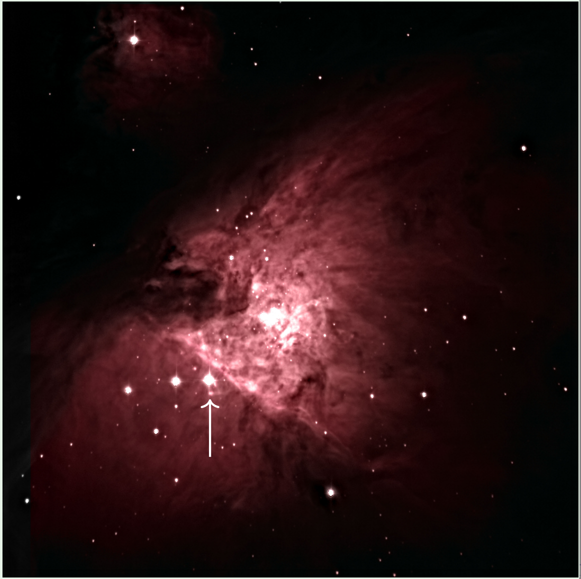


A compact, nonstellar object

From [SII], Hα, and [OIII] images, measure the signal and noise from the compact nonstellar object just south of θ^2 Ori A.

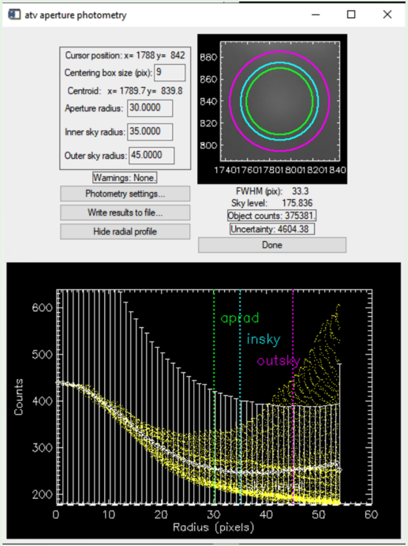
See the next pages for ATV photometry, and note that the aperture size is set a little larger than the object size. The sky annulus is kept small enough that it has no stars, nor much in the way of light belonging to the bright θ^2 Ori A.

Note: The image to the right has better resolution than the images that were measured.



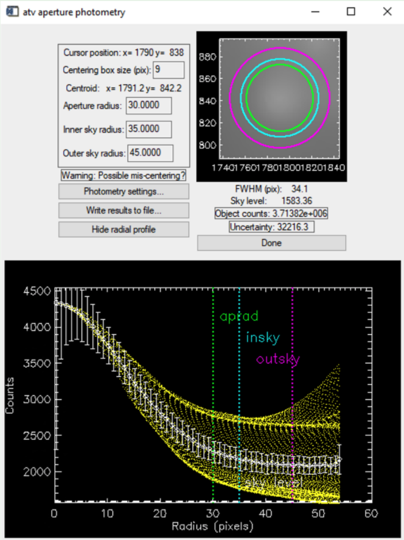
A compact, nonstellar object

[SII]: $S = (3.75 \pm 0.05) \times 10^5$ DN



A compact, nonstellar object

Hα: $S = (3.71 \pm 0.03) \times 10^6$ DN



A compact, nonstellar object

[OIII]: $S = (2.2 \pm 0.2) \times 10^5$ DN

It looks like the object was not detected in [OIII], despite a signal greater than the noise. The reason is that the spatial variation in the nebular emission is so large as to be much greater than the noise. In this case, it is better to report the "detection" as an upper limit:

$$S < 2 \times 10^5 \text{ DN}$$