Astronomy 465 — Problem Set 1

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Due Thursday, September 12 at 11:05AM EDT

1. (a) Show that the frequency shift observed along a long-slit spectrum is

$$\nu_{\rm obs}(R) - \nu_{12} = \pm \frac{\nu_{12}}{c} V_{\rm rot}(R) \sin i \tag{1}$$

where $V_{\text{rot}}(R)$ is the rotational velocity of the galaxy a distance R from the center, and i is the angle of inclination of the galaxy's plane with respect to the line of sight.

- (b) What is the wavelength shift observed?
- (c) Now work the other way: by measuring a frequency/wavelength shift along the line of sight at some point on the surface of a galaxy, what is the corresponding rotational velocity at that location? Assume that you know the inclination angle and the position angle (east of north) of the galaxy on the sky.
- 2. Show that the total luminosity for an elliptical galaxy whose surface brightness profile follows a Sérsic profile is

$$L = \frac{2\pi n\Gamma(2n)}{(\beta_n)^{2n}} I_0 R_e^2 \tag{2}$$

3. (a) The luminosity density $j(\mathbf{r})$ of a stellar system is the luminosity per unit volume at position \mathbf{r} . For a spherical galaxy, show that the surface brightness I(R) and luminosity density j(r) are related by

$$I(R) = 2 \int_{R}^{\infty} \frac{rj(r)}{\sqrt{r^2 - R^2}} dr$$
(3)

- (b) What is the surface brightness of a spherical galaxy with luminosity density $j(r) = j_0 (1 + \frac{r^2}{b^2})^{-5/2}$ (this is the Plummer model)?
- (c) Invert Eqn. 3 using Abel's formula,

$$f(x) = \int_x^\infty \frac{g(t)}{(t-x)^\alpha} dt \qquad (0 < \alpha < 1)$$

where

$$g(t) = -\frac{\sin \pi \alpha}{\pi} \frac{d}{dt} \int_{t}^{\infty} \frac{f(x)}{(x-t)^{1-\alpha}} dx$$
$$= -\frac{\sin \pi \alpha}{\pi} \int_{t}^{\infty} \frac{dx}{(x-t)^{1-\alpha}} \frac{df}{dx}$$

to obtain

$$j(r) = -\frac{1}{\pi} \int_r^\infty \frac{dR}{\sqrt{R^2 - r^2}} \frac{dI}{dR}$$

(d) Numerically determine the luminosity density in a spherical galaxy that follows the $R^{1/4}$ surfacebrightness law. Plot $\log j(r)$ versus $\log \frac{r}{R_e}$, where R_e is the effective radius.

- 4. The strip brightness S(x) is defined so that S(x) is the total luminosity in a strip of width dx that passes a distance x from the projected center of the system.
 - (a) Show that in a spherical system,

$$S(x) = 2 \int_x^\infty \frac{RI(R)}{\sqrt{R^2 - x^2}} \, dR$$

where I(R) is the surface brightness at radius R.

(b) Show that the luminosity density and the total luminosity interior to r are related to the strip brightness by

$$j(x) = -\frac{1}{2\pi x} \frac{dS}{dx} \qquad L(r) = -2 \int_0^r x \frac{dS}{dx} dx$$

- 5. An axisymmetric galaxy has luminosity density that is constant on spheroids $R^2 + z^2/q^2$ with an axis ratio q. A distant observer located on the symmetry axis of the galaxy sees an image with circular isophotes and central surface brightness I_n . A second distant observer, observing the galaxy from a line of sight that is inclined by an angle i to the symmetry axis, sees an image with elliptical isophotes with axis ratio Q < 1 and central surface brightness I_0 .
 - (a) What is the relation between I_0 , I_n , and Q? Hint: The answers are different for oblate (q < 1) and prolate (q > 1) galaxies.
 - (b) What is the relation between q, Q, and i?
 - (c) Assuming that galaxies are oriented randomly, what fraction are seen from a line of sight that lies within 10° of the symmetry axis? From within 10° of the equitorial plane?