# Astronomy 465 — Problem Set 1

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Due Tuesday, September 20 at 2PM EDT

1. (a) Show that the frequency shift observed along a long-slit spectrum is

$$\nu_{\rm obs}(R) - \nu_{12} = \pm\frac{\nu_{12}}{c} V_{\rm rot}(R)\sin i \tag{1}$$

where $V_{\rm rot}(R)$ is the rotational velocity of the galaxy a distance $R$ from the center, and $i$ is the angle of inclination of the galaxy's plane with respect to the line of sight.

(b) What is the wavelength shift observed?

(c) Now work the other way: by measuring a frequency/wavelength shift along the line of sight at some point on the surface of a galaxy, what is the corresponding rotational velocity at that location? Assume that you know the inclination angle and the position angle (east of north) of the galaxy on the sky.

2. Show that the total luminosity for an elliptical galaxy whose surface brightness profile follows a Sérsic profile is

$$L = \frac{2\pi n\Gamma(2n)}{(\beta_n)^{2n}} I_0 R_e^2 \tag{2}$$

3. (a) The luminosity density $j(\mathbf{r})$ of a stellar system is the luminosity per unit volume at position $\mathbf{r}$. For a spherical galaxy, show that the surface brightness $I(R)$ and luminosity density $j(r)$ are related by

$$I(R) = 2\int_R^\infty \frac{rj(r)}{\sqrt{r^2 - R^2}}\,dr \tag{3}$$

(b) What is the surface brightness of a spherical galaxy with luminosity density $j(r) = j_0(1 + \frac{r^2}{b^2})^{-5/2}$ (this is the Plummer model)?

(c) Invert Eqn. 3 using Abel's formula,

$$f(x) = \int_x^\infty \frac{g(t)}{(t-x)^\alpha}\,dt \qquad (0 < \alpha < 1)$$

where

$$\begin{aligned} g(t) &= -\frac{\sin\pi\alpha}{\pi}\frac{d}{dt}\int_t^\infty \frac{f(x)}{(x-t)^{1-\alpha}}\,dx \\ &= -\frac{\sin\pi\alpha}{\pi}\int_t^\infty \frac{dx}{(x-t)^{1-\alpha}}\frac{df}{dx} \end{aligned}$$

to obtain

$$j(r) = -\frac{1}{\pi}\int_r^\infty \frac{dR}{\sqrt{R^2 - r^2}}\frac{dI}{dR}$$

(d) Numerically determine the luminosity density in a spherical galaxy that follows the $R^{1/4}$ surface-brightness law. Plot $\log j(r)$ versus $\log \frac{r}{R_e}$, where $R_e$ is the effective radius.

4. The strip brightness $S(x)$ is defined so that $S(x)$ is the total luminosity in a strip of width $dx$ that passes a distance $x$ from the projected center of the system.
   (a) Show that in a spherical system,
   $$S(x) = 2\int_x^\infty \frac{RI(R)}{\sqrt{R^2 - x^2}}\,dR$$
   where $I(R)$ is the surface brightness at radius $R$.
   (b) Show that the luminosity density and the total luminosity interior to $r$ are related to the strip brightness by
   $$j(x) = -\frac{1}{2\pi x}\frac{dS}{dx} \qquad L(r) = -2\int_0^r x\frac{dS}{dx}\,dx$$
5. An axisymmetric galaxy has luminosity density that is constant on spheroids $R^2 + z^2/q^2$ with an axis ratio $q$. A distant observer located on the symmetry axis of the galaxy sees an image with circular isophotes and central surface brightness $I_n$. A second distant observer, observing the galaxy from a line of sight that is inclined by an angle $i$ to the symmetry axis, sees an image with elliptical isophotes with axis ratio $Q < 1$ and central surface brightness $I_0$.
   (a) What is the relation between $I_0$, $I_n$, and $Q$? Hint: The answers are different for oblate ($q < 1$) and prolate ($q > 1$) galaxies.
   (b) What is the relation between $q$, $Q$, and $i$?
   (c) Assuming that galaxies are oriented randomly, what fraction are seen from a line of sight that lies within 10° of the symmetry axis? From within 10° of the equitorial plane?