Astronomy 465 — Problem Set 2

Prof. Kelly Douglass

Due Thursday, September 26 in class

1. Show that the free-fall time, t_{ff}, for an isothermal spherical cloud of gas with uniform initial density ρ_0 is
$$t_{ff} = \sqrt{\frac{3\pi}{32G\rho_0}}$$
(Hint: What is the final velocity of a test particle that started at the surface of the cloud?)

2. The equations
$$\mathrm{O} + \mathrm{H} \rightarrow \mathrm{O}^* + \mathrm{H}$$
$$\mathrm{O}^* \rightarrow \mathrm{O} + \gamma$$
where O^* is an excited state of O, illustrate a cooling mechanism for a molecular cloud accomplished through the excitation of oxygen. Explain why the excitation of hydrogen rather than oxygen is not an effective cooling mechanism.

3. Explain why dense cores of molecular clouds are generally cooler than the surrounding giant molecular clouds, and why GMCs are cooler than diffuse molecular clouds.

4. By invoking the requirements of hydrostatic equilibrium, explain why the assumption of a constant gas pressure P_0 in our equation for the Jeans mass cannot be correct for a static cloud without magnetic fields. What does that imply about the assumptions of constant mass density in an isothermal molecular cloud having a constant composition throughout?

5. (a) Beginning with
$$\frac{d^2r}{dt^2} = -\frac{GM(r)}{r^2}$$
add a centripetal acceleration term and use conservation of angular momentum to show that the collapse of a cloud will stop in the plane perpendicular to its axis of rotation when the radius reaches
$$r_f = \frac{\omega_0^2 r_0^4}{2GM(r)}$$
where $M(r)$ is the mass interior to radius r, and ω_0 and r_0 are the original angular velocity and radius of the surface of the cloud, respectively. Assume that the initial radial velocity of the cloud is zero and that $r_f \ll r_0$. You may also assume (incorrectly) that the cloud rotates as a rigid body during the entire collapse.

   (b) Assume that the original cloud had a mass of $1M_\odot$ and an initial radius of 0.5 pc. If collapse is halted at approximately 100 AU, what is the initial angular velocity of the cloud?

   (c) What was the original rotational velocity of the edge of the cloud?

   (d) Assuming that the moment of inertia is approximately that of a solid sphere, $I_{\text{sphere}} = \frac{2}{5}MR^2$, when the collapse begins, and that of a uniform disk, $I_{\text{disk}} = 0.5MR^2$, when it stops, what is the rotational velocity at 100 AU?

   (e) After the collapse has stopped, what is the time required for a piece of mass to make one complete revolution around the central protostar? Compare your answer with the orbital period at 100 AU expected from Kepler's third law. Why would you expect the two to not be identical?