Astronomy 465 — Problem Set 3

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Due Tuesday, October 8 at 11:05AM EDT

1. The Q value of a reaction is the amount of energy released (or absorbed) during the reaction. Calculate the Q value for each step of the PPI reaction chain:

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\begin{aligned}
{}^1_1\mathrm{H} + {}^1_1\mathrm{H} &\rightarrow {}^2_1\mathrm{H} + e^+ + \nu_e \\
{}^2_1\mathrm{H} + {}^1_1\mathrm{H} &\rightarrow {}^3_2\mathrm{He} + \gamma \\
{}^3_2\mathrm{He} + {}^3_2\mathrm{He} &\rightarrow {}^4_2\mathrm{He} + 2\,{}^1_1\mathrm{H}
\end{aligned}
$$

   Express your answers in MeV. The masses of ${}^2_1$H and ${}^3_2$He are $2.0141u$ and $3.0160u$, respectively.

2. Calculate the amount of energy released or absorbed in each of the following reactions. (Express your answers in MeV.)

   a) ${}^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{24}_{12}\mathrm{Mg} + \gamma$

   b) ${}^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{16}_{8}\mathrm{O} + 2\ {}^{4}_{2}\mathrm{He}$

   c) ${}^{19}_{9}\mathrm{F} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{16}_{8}\mathrm{O} + {}^{4}_{2}\mathrm{He}$

   The mass of ${}^{12}_{6}$C is $12.0000u$, by definition, and the masses of ${}^{16}_{8}$O, ${}^{19}_{9}$F, and ${}^{24}_{12}$Mg are 15.99491 u, 18.99840 u, and 23.98504 u, respectively. Are these reactions exothermic or endothermic? Where might you find each of these reactions occurring?

3. The lower end of the main sequence occurs near $0.072M_\odot$, with $\log T_e = 3.23$ and $\log(L/L_\odot) = -4.3$. On the other hand, an $85M_\odot$ star near the upper end of the main sequence has an effective temperature and luminosity of $\log T_e = 4.705$ and $\log(L/L_\odot) = 6.006$, respectively.

   (a) Estimate the hydrogen-burning lifetimes of stars near the lower and upper ends of the main sequence. Assume that the $0.072M_\odot$ star is entirely convective so that, through convective mixing, all of its hydrogen, rather than just the inner 10%, becomes available for burning.

   (b) Calculate the radii of a $0.072M_\odot$ star and a $85M_\odot$ star. (Hint: Use the Stefan-Boltzmann law.) What is the ratio of their radii?