Astronomy 465 — Problem Set 5

Prof. Kelly Douglass

Due Thursday, November 10 at 2PM EST

1. Show that the tidal force per unit mass in a high-speed encounter is

$$\vec{F}_{\rm tid}(\vec{r}) = \frac{GM_P}{R^3}(2x'\hat{x} - y'\hat{y} - z'\hat{z})$$

   in a coordinate system with its origin at the center of mass of the body being perturbed.

2. Use the age of the Milky Way's oldest globular clusters (∼13 Gyr) and the orbital velocity of the local standard of rest (220 km/s) to estimate the greatest distance from which globular clusters have spiraled into the nucleus due to dynamical friction. Use $5 \times 10^6 M_\odot$ for the cluster's mass. How does your answer compare with the size of the Milky Way's central bulge?

3. Estimate the tidal radius of the Large Magellanic Cloud (mass of $2 \times 10^{10} M_\odot$) as it orbits the Milky Way. Take the mass of the Milky Way's dark matter halo to be $5.4 \times 10^{11} M_\odot$. How does your answer compare with the size of the LMC, which has an angular diameter of 460'?

4. The Small Magellanic Cloud (mass of $2 \times 10^9 M_\odot$) is 60 kpc away and has one-tenth the mass of the LMC. The SMC has an angular diameter of 150', and its angular separation from the LMC is 21°.
   (a) What is the distance between the LMC and the SMC?
   (b) Ignoring the influence of the Milky Way, estimate the present tidal radii of both the LMC and the SMC due to each other.
   (c) The Magellanic Clouds are in eccentric orbits around one another, and during the last few hundred million years they have been moving apart with an average speed of about 110 km/s. How recently did the LMC extend beyond its tidal radius? How recently did the SMC extend beyond its tidal radius? Assuming that the Magellanic Stream was formed by tidal stripping when the LMC and SMC were close to each other, when would you estimate that the Magellanic Stream was formed?