Astronomy 465 — Problem Set 6

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Due Tuesday, November 12 at the beginning of class

1. If the surface brightness of an elliptical galaxy follows the $r^{1/4}$ law given by

$$\log\left(\frac{I(r)}{I_e}\right) = -3.3307\left(\left(\frac{r}{r_e}\right)^{1/4} - 1\right)$$

show that the average surface brightness over the area of a circular disk of radius r_e is given by

$$\langle I\rangle = 3.607 I_e$$

2. According to the virial theorem, the central radial-velocity dispersion is related to the mass and size of a galaxy by

$$\sigma_r^2 \propto \frac{M}{R}$$

Show that

$$L \propto \sigma_r^4$$

which is the Faber-Jackson relation.

3. To quantify the deviation from an ellipse in the cores of galaxies (to describe it as either "boxy" or "disky"), the shape of an isophotal contour (defined for some specific value of μ) is written in polar coordinates as a Fourier series of the form

$$a(\theta) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\theta + \sum_{n=1}^{\infty} b_n \sin n\theta$$

where a_0 represents the shape of a perfect circle, a_2 corresponds to the amount of ellipticity, and a_4 is associated with the degree of boxiness. If $a_4 < 0$, the isophotal surface tends towards a "boxy" appearance, and if $a_4 > 0$, the surface tends toward being "disky."

If the coefficient a_3 is greater than 0, what does this tell us about the shape of the isophote?

4. One way of fitting an ellipse to an isophote is to perform a least-squares fit of the polar equation for an ellipse,

$$R^{-2} = C - A\cos 2\theta - B\sin 2\theta$$

to the points (R_i, θ_i) that lie on the isophote. Express the semi-major axis length, a, the ellipticity, ε, and the position-angle of the major axis, θ_0, in terms of A, B, and C.