Astronomy 465 — Problem Set 7

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Due Tuesday, December 13 at 2PM EST

1. For a temperature of 7.3×10^5 K, make a graph of the Planck function, plotting $\log \nu B_\nu(T)$ v. $\log \nu$ for $\log \nu$ between 15.5 and 17.5. How does the behavior of your graph of a blackbody compare with that of the continuous spectrum for 3C 273 (see Fig. 2 of Perry et al., 1987)?

2. Consider material that is ejected from an AGN directly toward Earth.

   (a) If the redshift of the AGN is z_{AGN} and the redshift of the ejecta is z_{ej}, show that the speed of the ejecta relative to the AGN is given by

   $$\frac{v}{c} = \frac{(1 + z_{AGN})^2 - (1 + z_{ej})^2}{(1 + z_{AGN})^2 + (1 + z_{ej})^2}$$

   (b) Consider a radio-emitting knot ejected from the quasar 3C 273 directly toward Earth. If the speed of the approaching knot is measured to be $v = 0.9842c$, what is the speed of the knot relative to the AGN? From the reference frame of the AGN, what is the value of the knot's Lorentz factor?

3. Consider a relativistic ($\gamma \gg 1$) blazar jet that is coming directly toward the observer. If there is a time variation Δt_{rest} in the rest frame of the jet, use Special Relativity to show that the variation observed at Earth is approximately

   $$\Delta t_{\text{obs}} \simeq \frac{\Delta t_{\text{rest}}}{2\gamma}$$

4. The Roche limit is defined to the be distance of closest approach to a massive body before a small body is tidally disrupted. If the small body is a star and the large body is a supermassive black hole, the Roche limit is given by

   $$r_R = 2.4 \left(\frac{\bar{\rho}_{BH}}{\bar{\rho}_*} \right)^{1/3} R_S$$

   where R_S is the Schwarzschild radius, $\bar{\rho}_{BH}$ is the density of the black hole, and $\bar{\rho}_*$ is the average density of the star.

   (a) Setting the average density of the supermassive black hole equal to its mass divided by the volume contained within the Schwarzschild radius, derive an expression for the mass of a black hole that would have $r_R = R_S$.

   (b) If the Sun were to fall into a supermassive black hole, what maximum mass could the black hole have if the Sun would be tidally disrupted before crossing the event horizon? Compare your answer to the mass estimates of typical supermassive black holes in galactic nuclei.

   (c) If the supermassive black hole exceeded the mass found in the previous part, what would be the implications in terms of liberating the gravitational potential energy of the infalling star? Could infalling stars effectively power AGNs in this case?