# **Disk Galaxies**

Mass components Angular momentum Formation of disk galaxies

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# Disk galaxies

Components of a disk galaxy include

- ▶ Disk of stars, dust, and cold gas with spiral arms and, sometimes, a central bar
- Central bulge
- Stellar halo
- Dark halo

Observations of disk galaxies indicate that

- Brighter disk galaxies are larger, redder, rotate faster, and have smaller gas fractions
- Disk galaxies have flat rotation curves
- ▶ The surface brightness profile of the disk is exponential
- There is a color and metallicity gradient in the disks, from redder, higher metallicities towards the center to bluer, lower metallicities at the edges.

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### Infinitesimally-thin exponential disk

Surface mass density:

$$\Sigma(R) = \Sigma_0 e^{-R/R_d}$$

where  $R_d \equiv$  the scale radius of the disk.

Total mass:

$$M_d = 2\pi\Sigma_0 R_d^2$$

Gravitational potential:

$$\Phi(R,z) = -2\pi G \Sigma_0 R_d^2 \int_0^\infty \frac{J_0(kR)e^{-k|z|}}{\left[1 + (kR_d)^2\right]^{3/2}} dk$$

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### Exponential disk with exponential thickness

Mass density distribution:

$$\rho(R,z) = \rho_0 e^{-R/R_d} e^{-|z|/z_d}$$

so that the gravitational potential is then

$$\Phi(R,z) = -2\pi G \Sigma_0 R_d^2 \int_0^\infty \frac{J_0(kR)}{[1+(kR_d)^2]^{3/2}} \frac{e^{-k|z|} - (kz_d)e^{-|z|/z_d}}{1-(kz_d)^2} dk$$

Problem:  $\frac{\partial \rho}{\partial z}$  is not continuous at z = 0.

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#### Exponential spheroid Mass distribution:

$$\rho(r) = \rho_0 K_0 \left(\frac{r}{R_d}\right)$$

(derived using the Abel integral equations), with  $\rho_0 = \frac{\Sigma_0}{\pi R_d}$ .

Gravitational potential:

$$\Phi(R,z) = \frac{GM_d}{\pi R_d^2} \int_0^\infty \frac{tK_1\left(\frac{t}{R_d}\right)}{(\tau+1)\sqrt{\tau+q_d^2}} d\tau$$

where  $q_d$  is the intrinsic flattening of the disk and

$$t = \sqrt{\frac{R^2}{\tau+1} + \frac{z^2}{\tau+q_d^2}}$$

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#### Isothermal sheet

Assume that the vertical density distribution is locally isothermal, so that the stellar distribution in the vertical direction is

$$f = \frac{1}{\sqrt{2\pi}\sigma_z} e^{-E_z/\sigma_z^2}$$

where

$$\sigma_z$$
 = velocity dispersion of the stars in the vertical direction  
 $E_z = \frac{1}{2}v_z^2 + \Phi(R, z)$ 

The mass density of the disk is then

$$\rho(R,z) = \rho(R,0) \operatorname{sech}^2\left(\frac{z}{2z_d}\right)$$

where  $\rho(R, 0) = \frac{\Sigma(R)}{4z_d}$  and  $z_d = \frac{\sigma_z^2(R)}{2\pi G\Sigma(R)}$ .

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#### Disk rotation curves

Model: axisymmetric disk with a spherical dark matter halo

For spherical systems,

$$V_c^2(r) = r \frac{d\Phi}{dr} = \frac{GM(r)}{r}$$

$$V_c(r) \propto \begin{cases} r^{-1/2} & \text{for point mass} \\ \text{constant} & \text{for } \rho(r) \propto r^{-2} \\ r & \text{for uniform sphere} \end{cases}$$

When the disk is axisymmetric,  $V_c^2(R) = R \left(\frac{\partial \Phi}{\partial R}\right)_{z=0}$ 

The circular velocities from each of the mass components are additive, so

$$V_c^2 = V_{c,b}^2 + V_{c,d}^2 + V_{c,h}^2$$

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# CDM cosmology: NFW profile

Predicted based on cosmological simulations of cold dark matter, the resulting density of dark matter halos follows the Navarro-Frank-White (NFW) profile,

$$ho(r) = 
ho_{
m crit} rac{\delta_{
m char}}{\left(rac{r}{r_s}
ight) \left(1+rac{r}{r_s}
ight)^2}$$

where  $r_s \equiv$  the scale radius and  $\delta_{char} \equiv$  the characteristic overdensity.

The corresponding circular velocity is

$$\frac{V_{c,h}(r)}{V_{\rm vir}} = \sqrt{\frac{1}{x} \frac{\ln(1+cx) - \frac{cx}{1+cx}}{\ln(1+c) - \frac{c}{1+c}}}$$

where  $x = r/r_{\text{vir}}$ ,  $c \equiv$  the halo concentration parameter, and  $V_{\text{vir}} = \sqrt{GM_{\text{vir}}/r_{\text{vir}}}$ .

Rotational velocity of a thin exponential disk

$$V_{c,d}^{2}(R) = 4\pi G \Sigma_{0} R_{d} y^{2} [I_{0}(y) K_{0}(y) - I_{1}(y) K_{1}(y)]$$

where

 $y = R/2R_d$ 

 $I_n, K_n$  are the modified Bessel functions of the first and second kinds, respectively

If we assume that the disk is composed of only stars, then

 $R_d$  = the scale length of the stellar light

 $\Sigma_0 = YI_0$ , where  $Y \equiv$  the stellar mass-to-light ratio and  $I_0 \equiv$  the central surface brightness.

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# Disk-halo mass degeneracy



Blue: stellar disk; Green: gas disk; Red: dark matter halo

#### Adiabatic contraction of the disk

If the potential of the system changes little while the disk forms, then these adjustments are adiabatic and so the final state is independent of how it got there.

In a spherically-symmetric system with all particles on spherical orbits, the product of a particle's orbital radius and the mass contained within that radius is a constant:

$$M_i(r_i)r_i = M_f(r_f)r_f$$

The final mass is

$$M_f(r_f) = M_d(r_f) + (1 - m_d)M_i(r_i)$$

### Self-gravitating disk in DM halo

Assume a halo with an unperturbed density profile

$$\rho(r) = \frac{1}{4\pi r^2} \frac{dM(r)}{dr}$$

The total energy of the halo is

$$E = -\frac{1}{2}M_{\rm vir}V_{\rm vir}^2F_E$$

where (for a NFW profile)

$$F_E = \frac{c}{2} \frac{1 - \frac{1}{(1+c)^2} - \frac{2\ln(1+c)}{1+c}}{\left[\frac{c}{1+c} - \ln(1+c)\right]^2}$$

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### Self-gravitating disk in DM halo

Assuming that all disk particles are moving on perfectly circular orbits (so  $V_{\text{rot}}(R) = V_c(R)$ ), the total angular momentum of the disk is

$$L_d = 2M_d R_d V_{\rm vir} F_R$$

where

$$F_{R} = \frac{1}{2} \int_{0}^{r_{\rm vir}/R_{d}} u^{2} e^{-u} \frac{V_{c}(uR_{d})}{V_{\rm vir}} du$$

The self-gravity of the disk results in

$$V_c^2(r) = V_{c,d}^2 + \frac{GM_{\mathrm{h,ac}}(r)}{r}$$

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# Including a bulge

If a galaxy contains both a bulge and disk, then

$$M_{
m bary} \equiv M_d + M_b \equiv m_{
m bary} M_{
m vir}$$

The total angular momentum of the baryons is

$$L_{\rm bary} \equiv \ell_{\rm bary} L_{\rm vir}$$

If  $L_b = \ell_b L_{vir}$  is the initial angular momentum of the baryons from which the bulge formed, and if all this angular momentum is transferred to the disk and halo (so that the bulge ends up with 0 angular momentum), then

$$L_d = [\ell_{\text{bary}} - (1 - f_t)\ell_b]L_{\text{vir}}$$

where  $f_t$  is the fraction of  $L_b$  that is transferred to the disk.

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