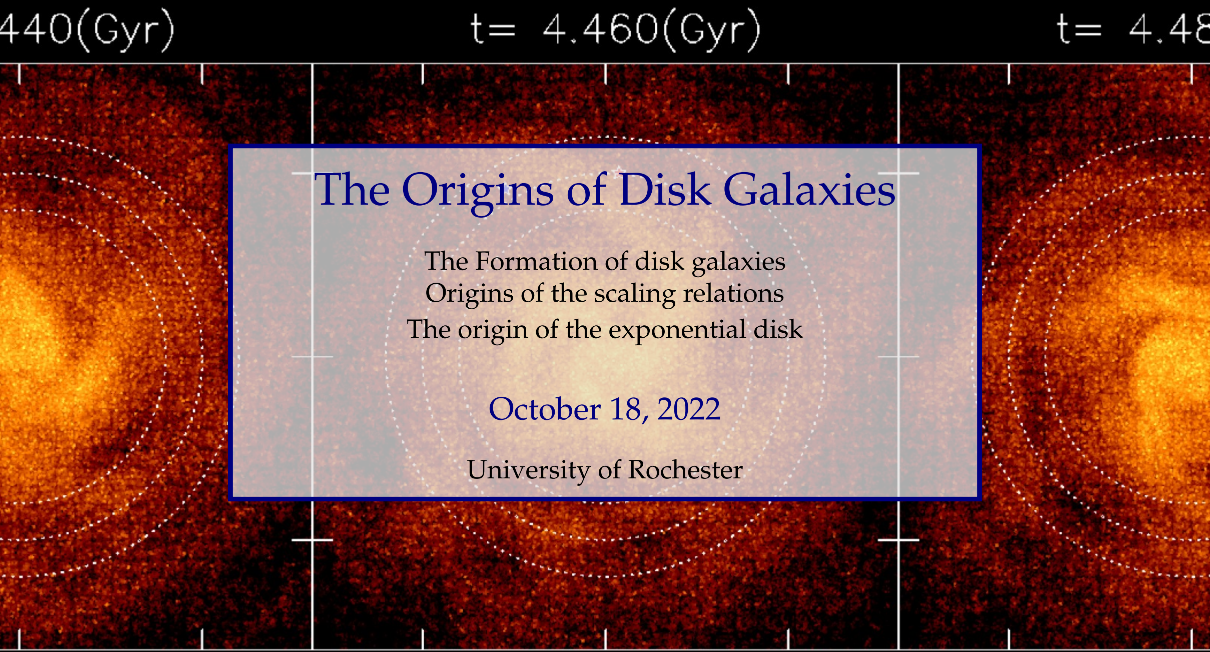

The Origins of Disk Galaxies

The Formation of disk galaxies
Origins of the scaling relations
The origin of the exponential disk

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Adiabatic contraction of the disk

If the potential of the system changes little while the disk forms, then these adjustments are adiabatic and so the final state is independent of how it got there.

In a spherically-symmetric system with all particles on spherical orbits, the product of a particle's orbital radius and the mass contained within that radius is a constant:

$$M_i(r_i)r_i = M_f(r_f)r_f$$

The final mass is

$$M_f(r_f) = M_d(r_f) + (1 - m_d)M_i(r_i)$$

where m_d is the fraction of the total mass contained in the disk, and $M_d(r_f)$ is the disk mass contained within radius r_f.

Self-gravitating disk in DM halo

Assume a halo with an unperturbed density profile

$$\rho(r) = \frac{1}{4\pi r^2}\frac{dM(r)}{dr}$$

The total energy of the halo is

$$E = -\frac{1}{2}M_{\rm vir}V_{\rm vir}^2 F_E$$

where (for a NFW profile)

$$F_E = \frac{c}{2}\frac{1-\frac{1}{(1+c)^2}-\frac{2\ln(1+c)}{1+c}}{\left[\frac{c}{1+c}-\ln(1+c)\right]^2}$$

Self-gravitating disk in DM halo

Assuming that all disk particles are moving on perfectly circular orbits (so $V_{\rm rot}(R) = V_c(R)$), the total angular momentum of the disk is

$$L_d = 2M_d R_d V_{\rm vir} F_R$$

where

$$F_R = \frac{1}{2}\int_0^{r_{\rm vir}/R_d} u^2 e^{-u} \frac{V_c(uR_d)}{V_{\rm vir}}\, du$$

The self-gravity of the disk results in

$$V_c^2(r) = V_{c,d}^2 + \frac{GM_{\rm h,ac}(r)}{r}$$

Including a bulge

If a galaxy contains both a bulge and disk, then

$$M_{\text{bary}} \equiv M_d + M_b \equiv m_{\text{bary}} M_{\text{vir}}$$

The total angular momentum of the baryons is

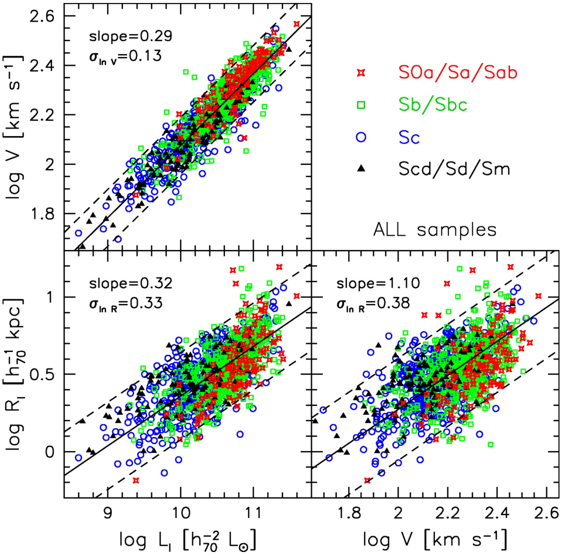
$$L_{\text{bary}} \equiv \ell_{\text{bary}} L_{\text{vir}}$$

If $L_b = \ell_b L_{\text{vir}}$ is the initial angular momentum of the baryons from which the bulge formed, and if all this angular momentum is transferred to the disk and halo (so that the bulge ends up with 0 angular momentum), then

$$L_d = \left[\ell_{\text{bary}} - (1 - f_t)\ell_b\right] L_{\text{vir}}$$

where f_t is the fraction of L_b that is transferred to the disk.

Disk galaxy scaling relations



Forming disks from relic angular momentum distribution

This model assumes that the angular momentum is not redistributed as the disk forms. Assuming that the gas has the same specific angular momentum distribution as the dark matter, and that the disk is non-self-gravitating in an isothermal sphere,

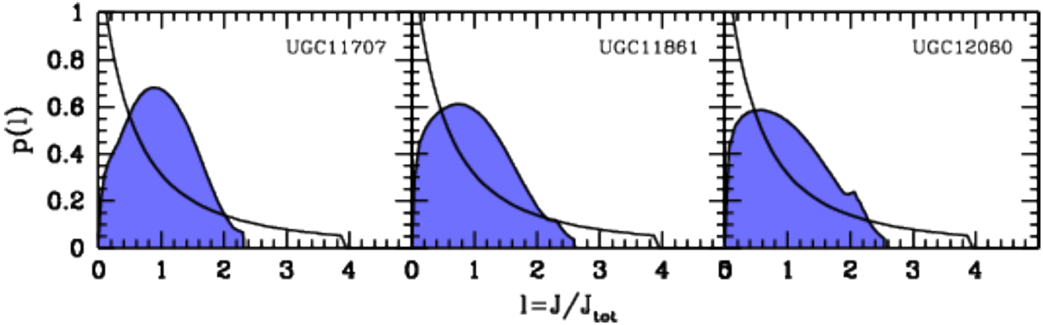
$$\Sigma_d(R) = \mu \frac{M_d}{2\pi R_d'^2} \left(\frac{R}{R_d'} \right)^{-1} \left(1 + \frac{R}{R_d'} \right)^{-2}$$

where μ is a free shape parameter and

$$R_d' \equiv \frac{\mathcal{L}_c}{V_c} = \sqrt{2}(\mu - 1)\xi^{-1}\lambda r_{\mathrm{vir}}$$

with $\xi = 1 - \mu \left[1 - (\mu - 1) \ln \left(\frac{\mu}{\mu - 1} \right) \right]$.

Forming disks from relic angular momentum distribution



A viscous disk

The gas surface density will evolve into the stellar surface density via the star formation law,

$$\frac{\partial \Sigma_*}{\partial t} = \frac{\Sigma(R, t)}{t_*}$$

where t_* is the star formation time scale. The evolution of the gas surface density is governed by

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}(\Sigma R v_R) = -\frac{\Sigma}{t_*}$$

and

$$\frac{\partial}{\partial t}(\Sigma R^2 \Omega) + \frac{1}{R}\frac{\partial}{\partial R}(\Sigma R^3 \Omega v_R) = \frac{1}{R}\frac{\partial}{\partial R}\left(\nu \Sigma R^3 \frac{\partial \Omega}{\partial R}\right) - R^2 \Omega \frac{\Sigma}{t_*}$$

By eliminating v_R and assuming that $\frac{\partial \Omega}{\partial t} = 0$,

$$\frac{\partial \Sigma}{\partial t} = -\frac{1}{R}\frac{\partial}{\partial R}\left\{\frac{\frac{\partial}{\partial R}\left(\nu \Sigma R^3 \frac{\partial \Omega}{\partial R}\right)}{\frac{\partial}{\partial R}(R^2 \Omega)}\right\} - \frac{\Sigma}{t_*}$$