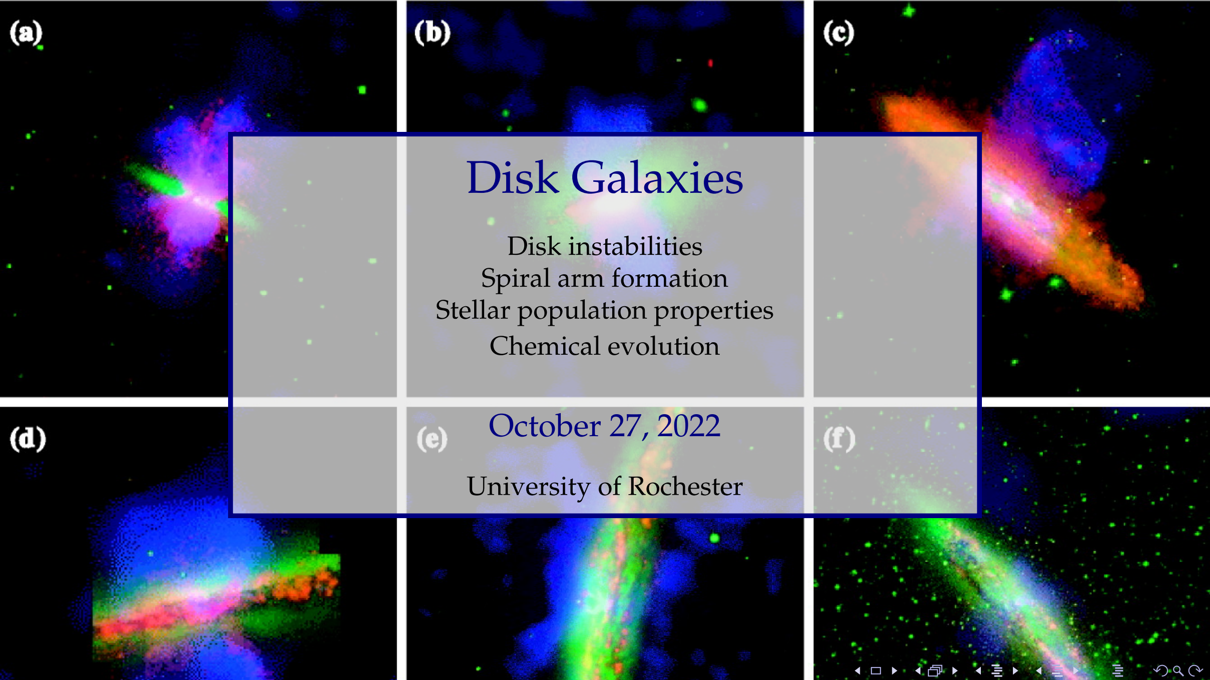

Disk Galaxies

Disk instabilities
Spiral arm formation
Stellar population properties
Chemical evolution

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Local instabilities

Local instabilities occur when the characteristic size of the perturbation is much smaller than the disk.

If we write the perturbation mode as

$$\Sigma_1(r, \phi, t) = A(r, t) e^{i[m\phi + f(r,t)]}$$

where

$f(r, t)$ is the shape function

$A(r, t)$ is the amplitude of the density wave that varies slowly in r

then, for fixed t, $m\phi + f(r, t) =$ constant, and peaks in the density waves occur when $m\phi + f(r, t) = 2\pi n$.

Local instabilities: Tightly-wound waves

A perturbation in the neighborhood of (R_0, ϕ_0) is then

$$\Sigma_1(r, \phi, t) \approx \Sigma_a e^{ik(R_0,t)(r-R_0)}$$

where

$$\begin{aligned} \Sigma_a &= A(R_0, t)e^{i(m\phi_0+f(R_0,t))} \\ k(R_0, t) &\equiv \left[\frac{\partial f(r,t)}{\partial r}\right]_{R_0} = \frac{2\pi}{\Delta r} \end{aligned}$$

Tightly-wound waves are essentially plane waves in the radial direction, with a wave-vector $k\hat{r}$ and wavelength Δr.

Local instabilities in a gaseous disk

If we define the dimensionless parameters

$$Q \equiv \frac{c_s \kappa}{\pi G \Sigma_0} \qquad \lambda_{\rm crit} \equiv \frac{4\pi^2 G \Sigma_0}{\kappa^2}$$

then the dispersion relation becomes

$$\omega^2 = \frac{4\pi^2 G\Sigma_0}{\lambda_{\rm crit}} \left[1 - \frac{\lambda_{\rm crit}}{\lambda} + \frac{Q^2}{4}\left(\frac{\lambda_{\rm crit}}{\lambda}\right)^2\right]$$

where $\lambda \equiv \frac{2\pi}{|k|}$.

The limiting value of Q is

$$Q(\lambda) = 2\sqrt{\frac{\lambda}{\lambda_{\rm crit}}\left(1 - \frac{\lambda}{\lambda_{\rm crit}}\right)}$$

Local instabilities in a stellar disk

With the pressure a result of random stellar motion, the dispersion relation is

$$(m\Omega - \omega)^2 = \kappa^2 - 2\pi G\Sigma_0 |k| F\left(\frac{\omega - m\Omega}{\kappa}, \frac{k^2\sigma_r^2}{\kappa^2}\right)$$

where

$$\begin{aligned} \sigma_r &= \text{radial velocity dispersion} \\ F(s,x) &= \tfrac{1-s^2}{\sin(s\pi)} \int_0^\pi e^{-(1+\cos\tau)x} \sin(s\tau)\sin\tau \, d\tau \end{aligned}$$

While λ's definition does not change, $Q \equiv \frac{\sigma_r \kappa}{\pi G \Sigma_0}$.

In the limiting case ($\omega = 0$),

$$\frac{\lambda_{\text{crit}}}{\lambda} F\left(0, Q^2\left(\frac{\lambda_{\text{crit}}}{\lambda}\right)^2\right) = 1$$

Toomre stability criterion

The limiting inequalities

$$Q \equiv \frac{c_s \kappa}{\pi G \Sigma_0} > 1 \qquad \text{(stable gaseous disk)}$$

$$Q \equiv \frac{\sigma_r \kappa}{3.36 G \Sigma_0} > 1 \qquad \text{(stable stellar disk)}$$

are known as Toomre's instability criterion, with Q sometimes called the Toomre parameter.

Note: The Toomre instability criterion is derived for axisymmetric perturbations with a wavelength much smaller than the disk size.

Global instability in the Maclaurin disk

The Maclauren disk is a fluid disk with an unperturbed surface density equal to

$$\Sigma_0(r) = \begin{cases} \Sigma_c\sqrt{1-\left(\frac{r}{a}\right)^2} & r \le a \\ 0 & r > a \end{cases}$$

where a is the radius of the disk. The corresponding potential in the disk is

$$\Phi_0(r) = \frac{1}{2}\Omega_0^2 r^2 + \text{constant} \qquad \Omega_0^2 = \frac{\pi^2 G\Sigma_c}{2a}$$

Global instability in the Maclaurin disk

The dispersion relation is, therefore,

$$(\omega - m\Omega)^3 - (\omega - m\Omega)\left\{4\Omega^2 + (\ell^2 + \ell - m^2)\left[\Omega_0^2(1 - g_{\ell m}) - \Omega^2\right]\right\}$$
$$+ 2m\Omega\left[\Omega_0^2(1 - g_{\ell m}) - \Omega^2\right] = 0$$

where

$$g_{\ell m} = \frac{(\ell + m)!(\ell - m)!}{2^{2\ell - 1}\left[\left(\frac{\ell + m}{2}\right)!\left(\frac{\ell - m}{2}\right)!\right]^2}$$

The $\ell = m = 0$ mode is unphysical. The $\ell = m = 1$ mode corresponds to the disk translation. The first perturbation modes are the $\ell = 2, m = 0, 2$ modes, where

$\ell = 2, m = 0$ Axisymmetric pulsation of the disk → "breathing" mode

$\ell = m = 2$ Rotating, elliptical disk deformation → "bar" mode

Global instability in the Kalnajs disk

The Kalnajs disk is a stellar disk with an unperturbed surface density and potential equal to that of the Maclauren disk.

The pressure is provided by the random motion of the stars, where

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \frac{a^2}{3}(\Omega_0^2 - \Omega^2)\left(1 - \frac{r^2}{a^2}\right)$$

The disk becomes unstable when

$$\left(\frac{\Omega}{\Omega_0}\right)^2 < \frac{125}{486}$$

Do galaxies exhibit bar instabilities?

Kinematic observations of the Milky Way indicate that the disk should be unstable to the bar mode, but the Milky Way disk is stable.

Alternate expression for the bar instability (based on observables):

$$\begin{aligned}\varepsilon_m \equiv V_{\max}\sqrt{\frac{R_d}{GM_d}} &\gtrsim 1.1 \qquad \text{(stable stellar disks)} \\ &\gtrsim 0.9 \qquad \text{(stable gaseous disks)}\end{aligned}$$

An isolated exponential disk has $\varepsilon_m \approx 0.63$.

Secular evolution of the disk

Secular evolution refers to the slow changes in mass and angular momentum that are independence of cosmology.

Main mechanisms for secular evolution include

- Resonance coupling
- Gas response
- Bending instability and formation of a bulge

Resonance coupling

Central bars that develop as a result of the bar mode have a highly flattened, triaxial structure that rotates as a solid body with an angular frequency Ω_p (pattern speed). The potential can then be written as

$$\Phi(r,\phi) = \Phi_0(r) + \Phi_1(r,\phi) \qquad \left|\frac{\Phi_1}{\Phi_0}\right| \ll 1$$

The motion of a star in the unperturbed potential is that of an epicycle with frequency $\kappa(r)$ around a guiding center that rotates around the galaxy center with frequency $\Omega(r)$.

Φ_1 results in a movement of the star in the bar's frame of reference of $\phi(t) = (\Omega(r) - \Omega_p)t$.

The m-fold symmetry of Φ_1 will cause the star to be in roughly the same location in the (r,ϕ) plane with a frequency $\Omega_d \equiv m(\Omega(r) - \Omega_p)$.

Resonance coupling

Resonances will occur between $\kappa(r)$ and Ω_d at several radii. For $m = 2$, the important resonances are

- Corotation resonance: $\Omega_d = 0 \rightarrow \Omega(r) = \Omega_p$
- Lindblad resonances: $\Omega_d = \pm\kappa(r)$
  - Inner Lindblad resonance: $\Omega(r) - \frac{1}{2}\kappa(r) = \Omega_p$
  - Outer Lindblad resonance: $\Omega(r) + \frac{1}{2}\kappa(r) = \Omega_p$

These resonances will delineate various regimes of orbital orientations, exchange angular momentum between the bar and the stars, and exchange angular momentum between the bar and the dark matter halo.