Elliptical galaxies

Dynamics
Formation
The fundamental plane
Stellar population properties
Bulges
Dwarf ellipticals

November 15, 2022

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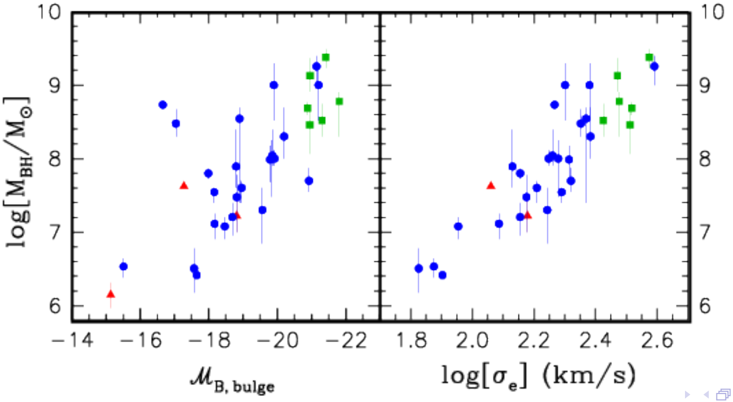
Central supermassive black holes

A massive black hole at a galaxy's center will only significantly influence the dynamics within a radius

$$r_{BH} = \frac{GM_{BH}}{\sigma^2} = 10.8\ \mathrm{kpc}\left(\frac{M}{10^8 M_\odot}\right)\left(\frac{\sigma}{200\ \mathrm{km/s}}\right)^{-2}$$



Kinetic energy tensor

The dynamics of elliptical galaxies are governed by the collisionless Boltzmann equation

$$\frac{df}{dt} = 0$$

Multiplying by the velocity and integrating over the velocity-space yields the Jeans equations

$$\frac{\partial(\rho\langle v_j\rangle)}{\partial t} + \frac{\partial(\rho\langle v_i v_j\rangle)}{\partial x_i} + \rho\frac{\partial\Phi}{\partial x_j} = 0$$

Multiplying all terms by x_k and integrating over all of space yields

$$\frac{\partial}{\partial t}\int \rho x_k\langle v_j\rangle\, d^3\vec{x} = -\int x_k\frac{\partial(\rho\langle v_i v_j\rangle)}{\partial x_i}\, d^3\vec{x} - \int \partial\rho x_k\frac{\partial\Phi}{\partial x_j}\, d^3\vec{x}$$

Kinetic energy tensor

We can use integration by parts to rewrite the first integral on the right-hand side as

$$\int x_k \frac{\partial(\rho\langle v_i v_j\rangle)}{\partial x_i}\, d^3\vec{x} = -\int \rho\langle v_k v_j\rangle\, d^3\vec{x} \equiv -2\mathcal{K}_{kj}$$

where $\mathcal{K}_{kj}$ is the kinetic energy tensor.

We can split the kinetic energy tensor into the contributions from the ordered and random motions:

$$\mathcal{K}_{ij} \equiv \mathcal{T}_{ij} + \frac{1}{2}\Pi_{ij}$$

where

$$\mathcal{T}_{ij} \equiv \frac{1}{2}\int \rho\langle v_i\rangle\langle v_j\rangle\, d^3\vec{x} \qquad \Pi_{ij} \equiv \int \rho\sigma_{ij}^2\, d^3\vec{x}$$

Potential energy tensor

We can also define the potential energy tensor

$$\mathcal{U}_{ij} \equiv -\int \rho x_i \frac{\partial \Phi}{\partial x_j} \, d^3\vec{x}$$

Combining these two energy tensors, and using that both $\mathcal{K}$ and $\mathcal{U}$ are symmetric,

$$\frac{1}{2}\frac{d}{dt}\int \rho \left[x_k \langle v_j \rangle + x_j \langle v_k \rangle \right] \, d^3\vec{x} = 2\mathcal{K}_{jk} + \mathcal{U}_{jk}$$

Moment of inertia tensor

We define the moment of inertia tensor

$$\mathcal{I}_{ij} \equiv \int \rho x_i x_j \, d^3\vec{x}$$

Differentiating with respect to time, and using the equation of continuity,

$$\begin{aligned} \frac{d\mathcal{I}_{jk}}{dt} &= \int \frac{d\rho}{dt} x_j x_k \, d^3\vec{x} \\ &= -\int \frac{\partial(\rho \langle v_i \rangle)}{\partial x_i} x_j x_k \, d^3\vec{x} \\ &= \int \rho [x_j \langle v_k \rangle + x_k \langle v_j \rangle] \, d^3\vec{x} \end{aligned}$$

Therefore,

$$\frac{1}{2}\frac{d}{dt}\int \rho [x_j \langle v_k \rangle + x_k \langle v_j \rangle] \, d^3\vec{x} = \frac{1}{2}\frac{d^2\mathcal{I}_{jk}}{dt^2}$$

Tensor Virial Theorem

We can then write the tensor virial theorem as

$$\frac{1}{2}\frac{d^2\mathcal{I}_{jk}}{dt^2} = 2\mathcal{K}_{jk} + \mathcal{U}_{jk}$$

which relates the gross kinetic and structural properties of gravitational systems.

If the system is in a steady state, then $\frac{d^2\mathcal{I}_{ij}}{dt^2} = 0$ and the tensor virial theorem reduces to

$$2\mathcal{K}_{jk} + \mathcal{U}_{jk} = 0$$

The common (scalar) virial theorem $2K + U = 0$ is simply the trace of this tensor equation.

Tensor virial theorem for an axisymmetric system

We can use the tensor virial theorem to relate the flattening of an elliptical galaxy to its kinematics.

Consider an oblate system with its symmetry axis along the z-direction. Exploiting this symmetry,

$$\langle v_r \rangle = \langle v_z \rangle = 0 \qquad \langle v_r v_\phi \rangle = \langle v_z v_\phi \rangle = 0$$

If we write $\langle v_x \rangle = \langle v_\phi \rangle \sin\phi$ and $\langle v_y \rangle = \langle v_\phi \rangle \cos\phi$, then

$$\begin{aligned}
\mathcal{T}_{xy} &= \frac{1}{2} \int \rho \langle v_x \rangle \langle v_y \rangle \, d^3\vec{x} \\
&= \frac{1}{2} \int_0^{2\pi} \sin\phi \cos\phi \, d\phi \int_0^\infty dr \int_{-\infty}^\infty \rho(r,z) \langle v_\phi \rangle^2 (r,z) \, dz \\
&= 0
\end{aligned}$$

A similar analysis shows that all other non-diagonal elements of $\mathcal{T}$, Π, and $\mathcal{U}$ are also 0. In addition, symmetry also gives $\mathcal{T}_{xx} = \mathcal{T}_{yy}$, and similar for Π and $\mathcal{U}$.

Tensor virial theorem

Taking all of these symmetries into account, the only independent, non-trivial virial equations are

$$2\mathcal{T}_{xx} + \Pi_{xx} + \mathcal{U}_{xx} = 0 \qquad 2\mathcal{T}_{zz} + \Pi_{zz} + \mathcal{U}_{zz} = 0$$

If the only streaming motion is rotation about the z-axis, then $\mathcal{T}_{zz} = 0$ and

$$2\mathcal{T}_{xx} = \frac{1}{2}\int \rho \langle v_\phi \rangle^2 \, d^3\vec{x} = \frac{1}{2} M v_0^2$$

where v_0 is the mass-weighted rotation velocity. Similarly,

$$\Pi_{xx} = M\sigma_0^2 \qquad \Pi_{zz} = (1-\delta) M\sigma_0^2$$

where $\sigma_0^2 \equiv \frac{1}{M}\int \rho \sigma_{xx}^2 \, d^3\vec{x}$ is the mass-weighted velocity dispersion along the line of sight, and $\delta \equiv 1 - \frac{\Pi_{zz}}{\Pi_{xx}} < 1$ is a measure of the anisotropy of the velocity dispersion.

Tensor virial theorem

Taking the ratio between the two non-trivial virial equations yields

$$\frac{\mathcal{U}_{xx}}{\mathcal{U}_{zz}} = \frac{1}{1-\delta}\left(1 + \frac{1}{2}\frac{v_0^2}{\sigma_0^2}\right)$$

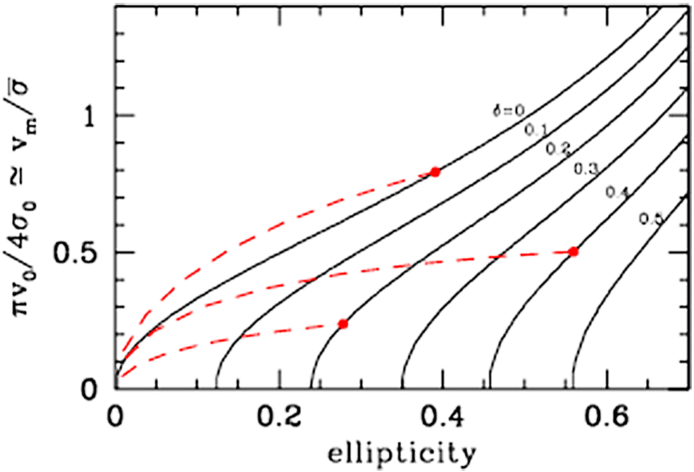
Roberts (1962) showed that this ratio depends only on the ellipticity, e, for systems stratified on similar coaxial oblate spheroids. Therefore, stellar systems can be flattened only by either rotation ($v_0^2 > 0$) or anisotropic velocity dispersion ($\delta > 0$).

It is common to identify σ_0 with $\overline{\sigma}$ (the average velocity interior to $0.5R_e$) and v_0 with $\frac{4v_m}{\pi}$ (where v_m is the maximum rotation velocity).

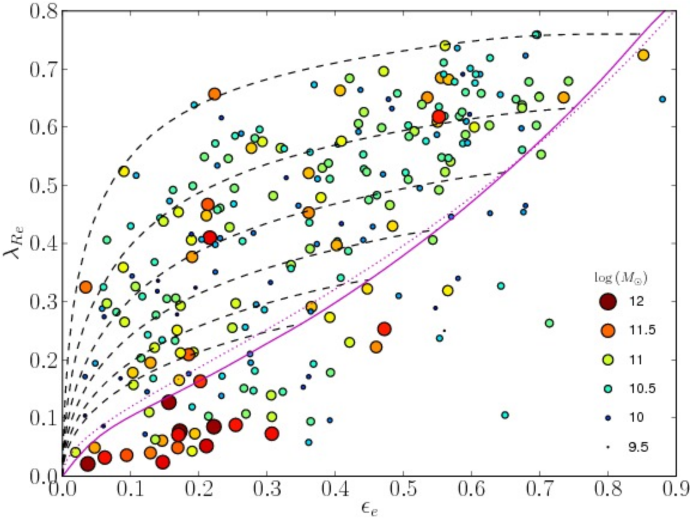
Tensor virial theorem

For the isotropic case ($\delta = 0$),

$$\frac{v_m}{\overline{\sigma}} \approx \sqrt{\frac{e}{1-e}}$$

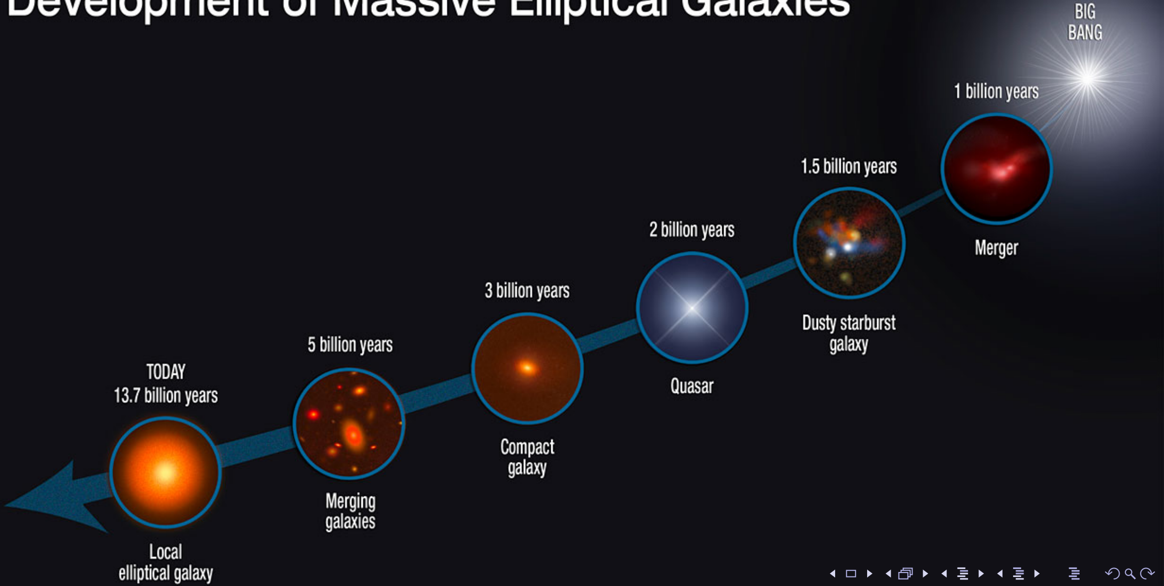


Comparison to observations



Emsellem et al. (2011)

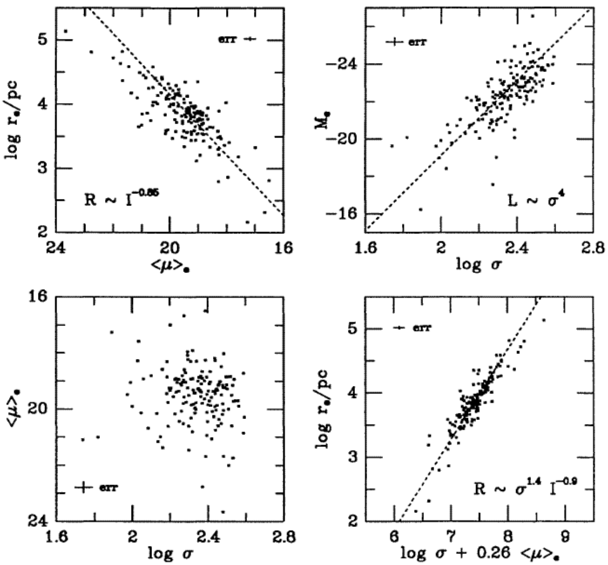
Development of Massive Elliptical Galaxies



The Fundamental Plane of elliptical galaxies

Similar to the Tully-Fisher relation for disk galaxies, ellipticals reveal a scaling relation between the luminosity and velocity dispersion, known as the Faber-Jackson relation.

Unlike the Tully-Fisher relation, the scatter in the Faber-Jackson relation is correlated with size, giving rise to a three-parameter Fundamental Plane relation.



The Fundamental Plane

The Fundamental Plane relation is generally written in the form

$$\log R_e = a \log \sigma_0 + b \log \langle I \rangle_e + \text{constant}$$

where $a \sim 1.2 - 1.5$ (depending on the waveband) and $b \sim -0.8$.

The Fundamental Plane relation is usually interpreted in terms of the Virial Theorem,

$$\frac{GM}{\langle R \rangle} = \langle v^2 \rangle$$

where

$\langle R \rangle \equiv$ average radius (so that $\frac{GM}{\langle R \rangle}$ is the absolute value of the average potential energy per unit mass)

$\langle v^2 \rangle \equiv$ average rms velocity (so that $\frac{1}{2}\langle v^2 \rangle$ is the average kinetic energy per unit mass)

The Fundamental Plane

We can express the virial theorem in terms of observables. Let

$$R_e = k_R \langle R \rangle \qquad \sigma_0 = k_v \sqrt{\langle v^2 \rangle}$$

where k_R and k_v are dimensionless quantities that depend on the density profile and orbital structure of the galaxy, respectively.

Since the luminosity of a galaxy is

$$L = 2\pi \langle I \rangle_e R_e^2$$

we find that

$$R_e = \frac{1}{2\pi G k_R k_v^2} \sigma_0^2 \langle I \rangle_e^{-1} \left(\frac{M}{L} \right)^{-1}$$

The Fundamental Plane

If elliptical galaxies are homologous (i.e. k_R and k_v are constant), and M/L is constant, then the virial theorem predicts a Fundamental Plane relation with $a = 2$ and $b = -1$.

The deviation from this prediction is known as the "tilt" of the Fundamental Plane, and reflects that ellipticals are not homologous and/or that

$$\left(\frac{M}{L}\right) \propto L^{\alpha} \langle I \rangle_e^{\beta}$$

with $(\alpha, \beta) \neq 0$.

The relationship between color and a (and the color-magnitude diagram) indicates a systematic variation in the stellar population, and therefore in M/L.

Observations indicate that M/M_* increases with luminosity, so the majority of the tilt is likely a result of a variation in M/L.

The Fundamental Plane in κ-space

It is more useful to use an orthogonal combination of σ_0, $\langle I \rangle_e$, and R_e (the observables):

$$\kappa_1 \equiv \frac{\log \sigma_0^2 + \log R_e}{\sqrt{2}}$$

$$\kappa_2 \equiv \frac{\log \sigma_0^2 + 2 \log \langle I \rangle_e - \log R_e}{\sqrt{6}}$$

$$\kappa_3 \equiv \frac{\log \sigma_0^2 - \log \langle I \rangle_e - \log R_e}{\sqrt{3}}$$

This implies that

$$\kappa_1 \propto \log(\sigma_0^2 R_e) \propto \log M$$

$$\kappa_3 \propto \log \left(\frac{\sigma_0^2 R_e}{\langle I \rangle_e R_e^2} \right) \propto \log \left(\frac{M}{L} \right)$$

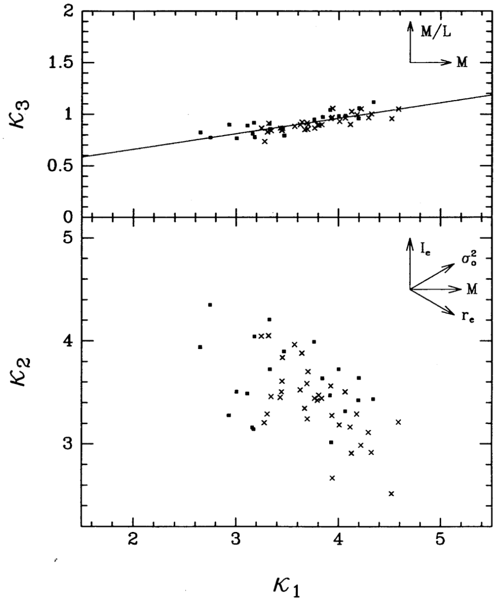
The Fundamental Plane in κ-space

In this "κ-space," the κ_1–κ_2 projection is very close to a face-on projection of the Fundamental Plane, while the κ_1–κ_3 projection shows the Fundamental Plane nearly edge-on.

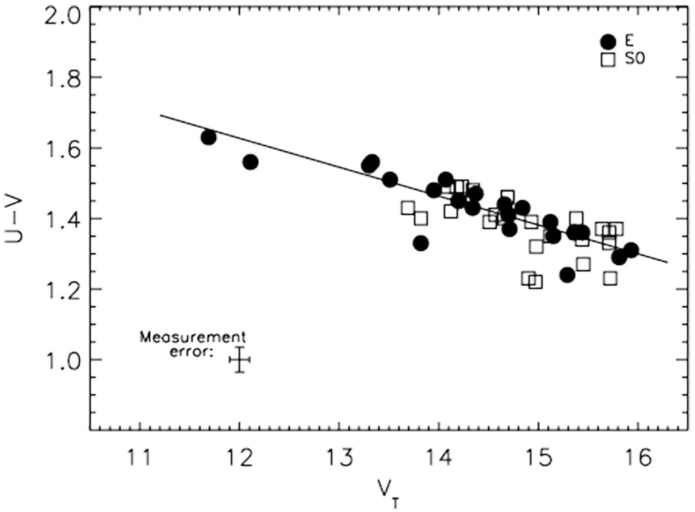
If ellipticals are homologous and $(M/L) \propto M^\gamma$, the virial theorem implies that

$$\kappa_3 = \sqrt{\frac{2}{3}}\gamma\kappa_1 + \text{ constant}$$

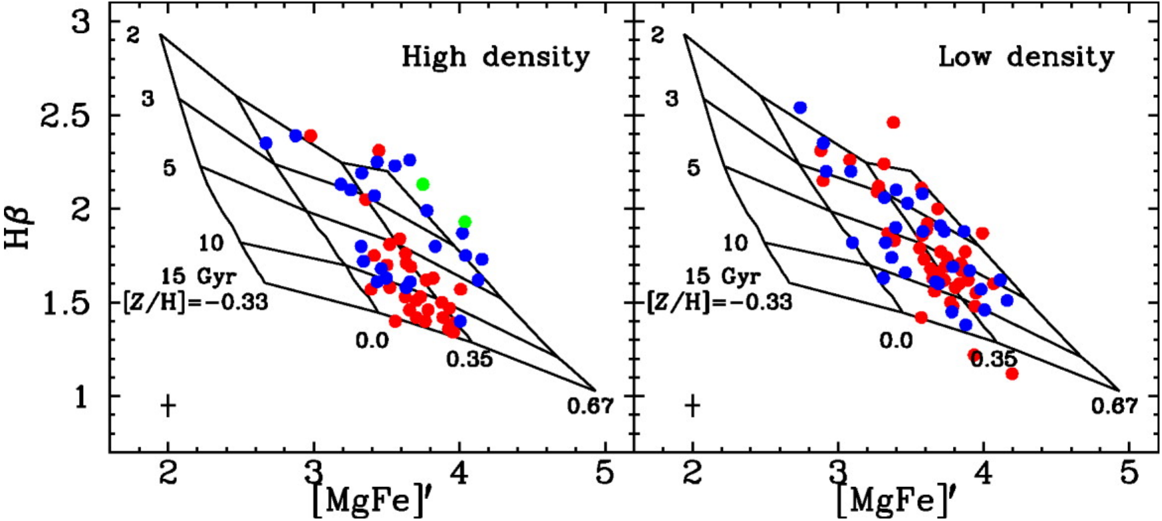
Bender et al. (1992)



Color-magnitude relation of elliptical galaxies



Breaking the age-metallicity degeneracy with absorption line indices



Elliptical – Lenticular – cD

Evolution of elliptical galaxies with redshift

Observations indicate that

- The absolute magntiudes of ellipticals of a fixed size increase with redshift
- High redshift ellipticals are bluer, but they still obey a tight color-magnitude relation.
- The slope of the color-magnitude relation does not change with redshift
- The slope of the Fundamental Plane also does not evolve with redshift, but the zero-point does. This indicates an evolution of M/L such that

$$\Delta \log \left(\frac{M}{L_B} \right) \propto -0.5z$$