Highlights of the New Material for Midterm #4

CAVEAT EMPTOR

I strongly advise that you do not take this to be a substitute for your own gathering of the material in your mind because that won't work. Rather, I suggest that you look over these topics and make sure you've thought about and understood these before the exam.

Also, please note that I am only attempting to summarize the most important points in the material. Nowhere am I implying that you need only know these items. You are still responsible for all that is covered in classes, workshops and assigned reading and problems.

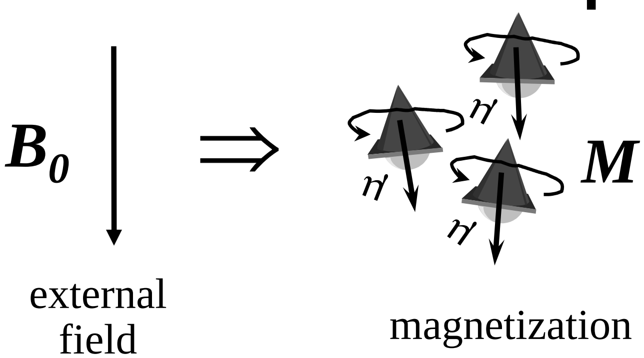
New Material

- Magnetic Materials
- Faraday's Law
- Mutual and Self Inductance
- LR Circuits
- AC Circuits

- **TA Review Before Next Midterm**
  - December 1st, 8:30pm, B&L 109

Magnetic Materials Review

- **In response to an external magnetic field, materials develop internal magnetization**



$$
\begin{aligned}
B_{total} &= B_0 + \mu_0 M \\
&= (1+\chi_m) B_0 \\
&= \kappa_m B_0
\end{aligned}
$$

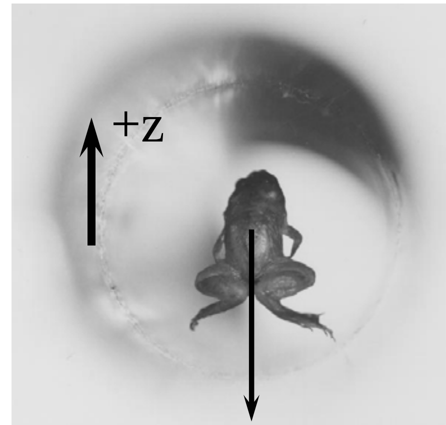
- **Paramagnetism**
  - **weak alignment of M and B, $0<\chi_m<<1$**
- **Diamagnetism**
  - **weak anti-alignment of M and B, $-1<<\chi_m<0$**

} these are way weak effects!

- **Ferromagnetism**
  - **strong alignment of M and B, $\chi_m>>1$**

} very strong. magnetization dominates field!

It’s the Floating Frog!



- A diamagnetic frog levitates in a strong magnetic field as shown at right
- If gravity causes a downwards force in the picture at right, where is the magnitude of the field greatest?
  At the bottom of the page.
- Better be able to explain why… this brings together a lot of key ideas.

$$U = -\vec{\mu} \cdot \vec{B}(z) = |\chi_m| |\vec{B}(z)|^2$$

$\chi_m < 0$

$$F_z = -\frac{dU}{dz} = -(-\chi_m) 2B \frac{dB}{dz}$$

$$\therefore F_z > 0 \text{ if } \frac{dB}{dz} < 0$$

If potential energy is highest at bottom of page, then the frog feels an upward force.

Therefore, B is largest at bottom of the page.

The Big Picture

which you should be able to explain…

- **Electrostatics**
  - **motion of “*q*” in external E-field**
  - **E-field generated by Σq_i**
- **Magnetostatics**
  - **motion of “*q*” and “*I*” in external B-field**
  - **B-field generated by “*I*”**
- **Electrodynamics**
  - **time dependent B-field generates E-field**
    - **ac circuits, inductors, transformers, etc**
  - **our last topic (not this exam)**
time dependent E-field generates B-field
    - **electromagnetic radiation - *light***

Faraday's and Lenz's Laws

- **a changing magnetic flux through a loop induces a current in that loop**

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

need to understand what magnetic flux is. (Like electric flux)

negative sign indicates that the induced EMF opposes the change in flux

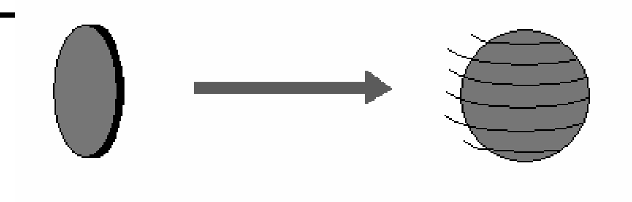
- **Need to also understand Faraday's Law in terms of Electric Field**
- **It does need a loop of wire to be true! This field can accelerate free charges not in a wire!**

$$\oint \vec{E} \bullet d\vec{l} = -\frac{d\Phi_B}{dt}$$

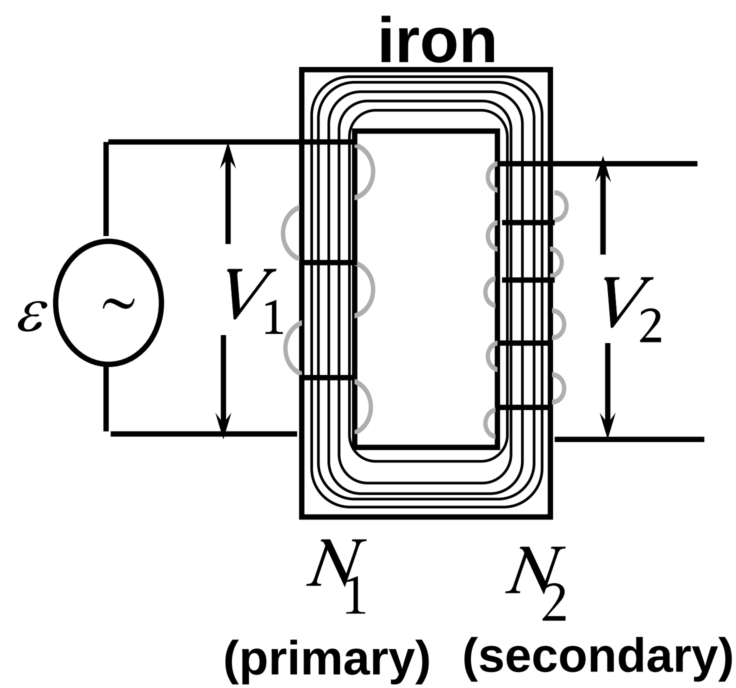
Applications

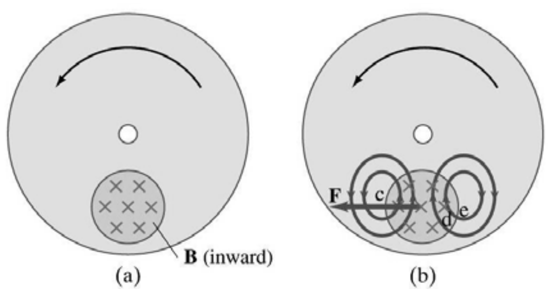
$$\mathcal{E} \equiv \frac{-d\Phi_B}{dt} = -\frac{d}{dt}\left[BA\cos\omega t\right] = BA\omega\sin\omega t$$

- should be able to explain how a generator works

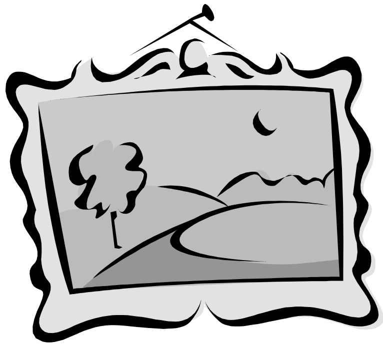


- should be able to explain how a transformer works

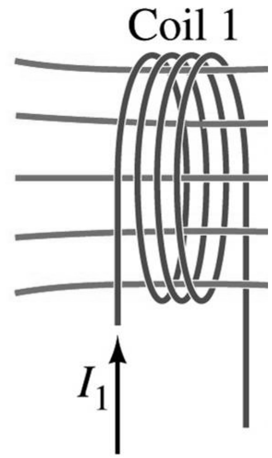




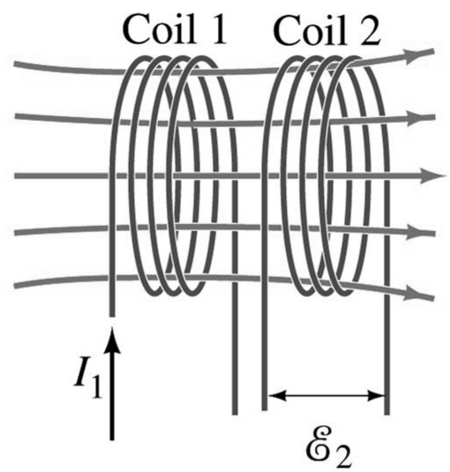
- should be able to explain our eddy current demonstrations done in class

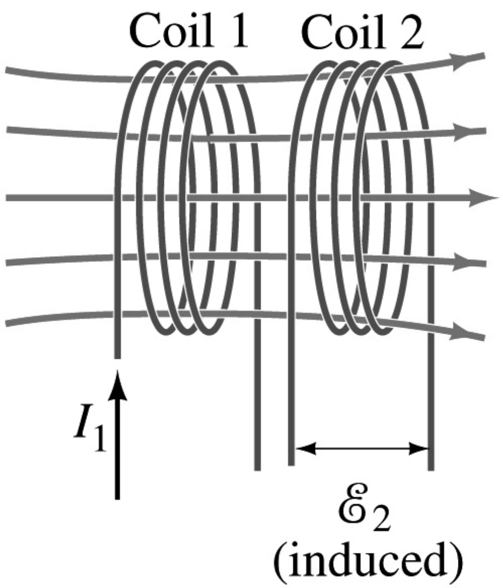


Big Picture of Inductance



- **A coil produces a magnetic field**
- **That magnetic field produces magnetic flux in that coil and adjacent coils**
- **If current changes in Coil 1, flux changes in Coils 1 and 2**
- **That change of flux causes an EMF which induces current!**
  - **in Coil 2 "Mutual Inductance"; in Coil 1, "Self Inductance"**





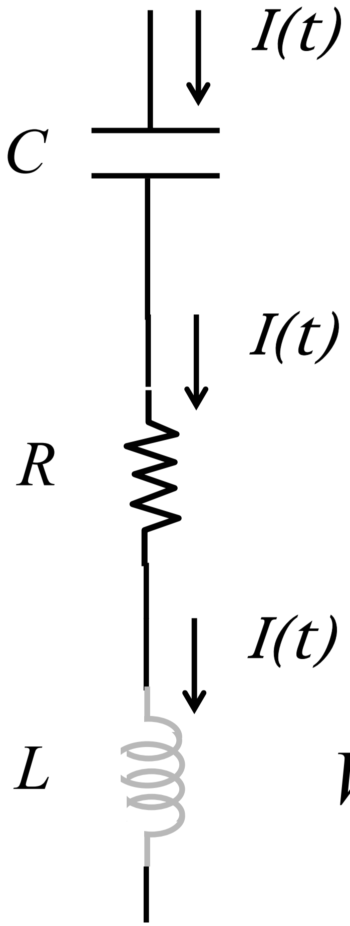
Inductors

- Current in Coil 1 causes
  - Magnetic flux in Coils 1 & 2
- *Changing current…*
  - Induces EMF in Coils 1 & 2
  - Self and Mutual Inductance
- Depends on geometry only!
  - Unit is *Henry*, T·m^2/A≡Ω·s

- Mutual Inductance $$M \equiv \frac{\Phi_B^{\text{through a from b}}}{I_b} = \frac{\Phi_B^{\text{through b from a}}}{I_a}$$
- Self-Inductance $$L \equiv \frac{\Phi_B}{I}$$
  - Self-Inductance for Solenoid $L = \mu_0 \frac{N^2}{l} \pi r^2$
  - L increases with Ferromagnetic core $\mu_0 \rightarrow \mu = \kappa_M \mu_0$

Summary so far…

Want to find voltage given a current, or find current if given a voltage.



$$V = \frac{Q}{C} = \frac{\int I\,dt}{C}$$

Voltage determined by *integral* of current and capacitance

capacitor "resists" integrated current

$$V = IR$$

Voltage determined by *current itself* and resistance

resistor "resists" current

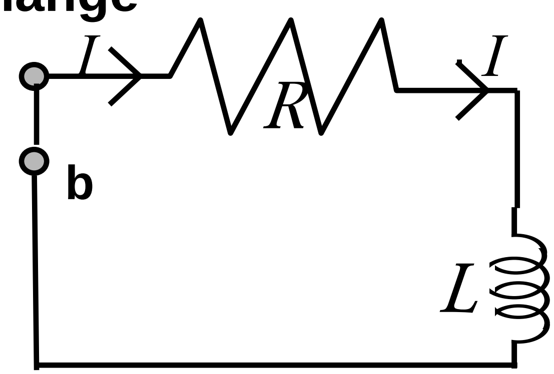
$$V = L\frac{dI}{dt} = L\frac{d^2Q}{dt^2}$$

Voltage determined by *derivative* of current and inductance

inductor "resists" change in current

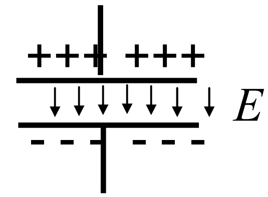
Rules of Thumb for Inductors in Circuits

- After circuit has had a long time to settle…
  - What is dI/dt? Zero
  - So the EMF across the inductor is? Zero
  - So it acts like a wire (no potential difference)
- When something changes in the circuit
  - How much should I be allowed to change instantaneously? Not a lick!
  - What is the mechanism for opposing change? Provide an EMF!
  - How much EMF? IR
  - Change is exponential in time, τ=L/R



Energy in the *Electric* and *Magnetic* Fields

Energy stored in a capacitor ...

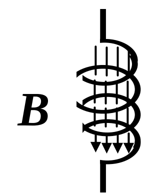


$$U = \frac{1}{2} C V^2$$

… energy density ...

$$u_{\text{electric}} = \frac{1}{2} \varepsilon_0 E^2$$

Energy stored in an inductor ….

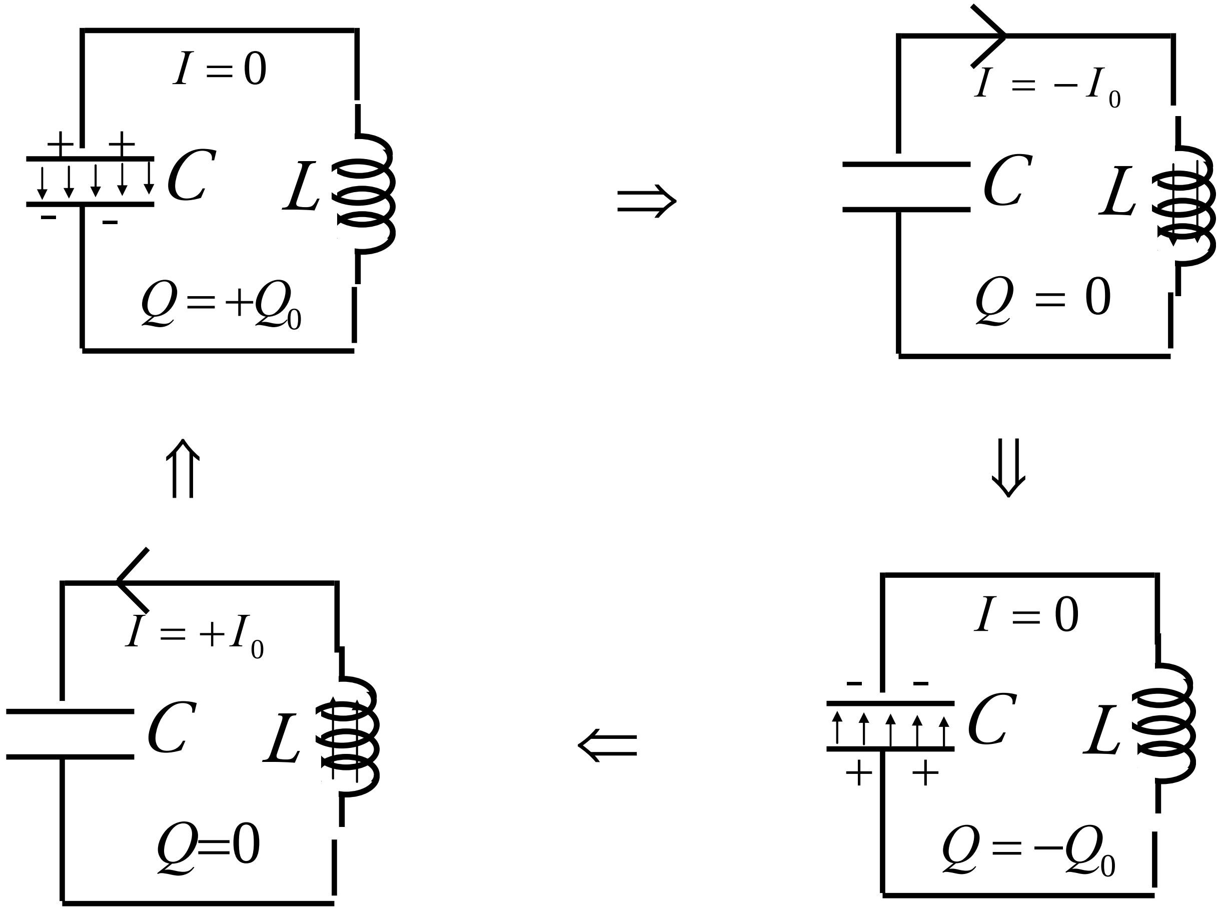


$$U = \frac{1}{2} L I^2$$

… energy density ...

$$u_{\text{magnetic}} = \frac{1}{2} \frac{B^2}{\mu_0}$$

LC Oscillations



$$L\frac{d^2Q}{dt^2}+\frac{Q}{C}=0$$

$$\omega_0=\frac{1}{\sqrt{LC}}$$

Response to an AC Voltage

- ***R*:** $V_R = RI_R = \varepsilon_m \sin\omega t \;\Rightarrow\; I_R = \dfrac{\varepsilon_m}{R}\sin\omega t$

  V **in phase with** ***I***

- ***C*:** $V_C = \dfrac{Q}{C} = \varepsilon_m \sin\omega t \;\Rightarrow\; I_C = \omega C\varepsilon_m \sin\left(\omega t + 90°\right)$

  V **lags** ***I*** **by 90°**

- ***L*:** $V_L = L\dfrac{dI_L}{dt} = \varepsilon_m \sin\omega t \;\Rightarrow\; I_L = \dfrac{\varepsilon_m}{\omega L}\sin\left(\omega t - 90°\right)$

  V **leads** ***I*** **by 90°**

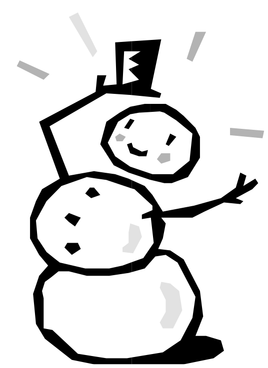
  - **Voltage/Current relationship across a single circuit element can be divided into:**
    - **magnitude**

      $$I = \frac{\varepsilon_m}{"X"}\sin\left(\omega t - \phi\right)$$

      $$X_L \equiv \omega L \qquad X_R \equiv R \qquad X_C \equiv \frac{1}{\omega C}$$

    - **relative phase**
      - » **leading, lagging**

ELI the ICE man

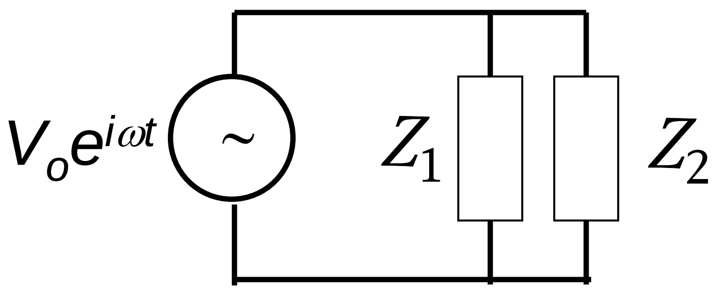


Impedance Networks

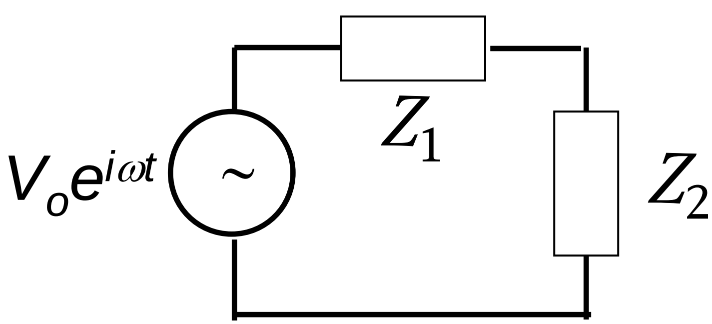
$$Z_R = R = R\left(e^{i0}\right) \qquad Z_C = \frac{1}{i\omega C} = \frac{-i}{\omega C} = \frac{e^{-i\pi/2}}{\omega C}$$

$$Z_L = i\omega L = \omega L e^{i\pi/2}$$

- **Combining impedances in series and parallel is just as simple as it was with resistors**
  - **But here impedances are complex numbers!**



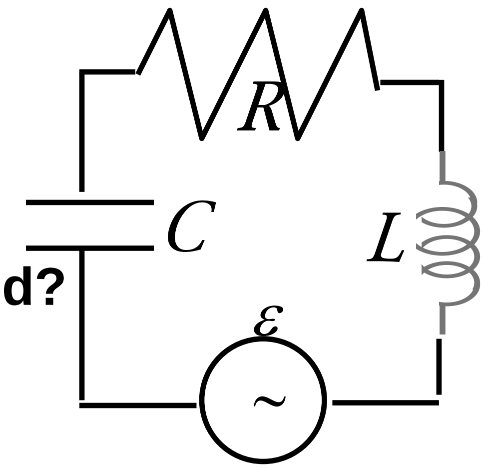
$$\frac{1}{Z_P} = \frac{1}{Z_1} + \frac{1}{Z_2}$$



$$Z_S = Z_1 + Z_2$$

Resonance in LRC Series Circuit

$$V_{\max} = I_{\max} \left| Z_{eq} \right|$$



- So when does the current reach a maximum if the voltage and R, L, C are fixed?

$$\left| Z_{eq} \right| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

- The current I_{max} will be a maximum at the resonant frequency ω_0 which makes the impedance Z purely real (R only)!

i.e.: when $\omega_0 L - \frac{1}{\omega_0 C} = 0$ or $\omega_0 = \frac{1}{\sqrt{LC}}$

Power in LRC Circuit

- The power supplied by the *emf* in a series LRC circuit depends on the frequency ω (maximum power is supplied at the resonant frequency ω_0).
- Can calculate from either power supplied by generator or power dissipated in resistor

$$P(t) = \varepsilon(t)I(t) = \left(\varepsilon_m \sin\omega t\right)\left(I_m \sin(\omega t - \phi)\right)$$

- average power delivered in a cycle.

$$\langle P(t)\rangle = \varepsilon_m I_m \langle \sin\omega t \sin(\omega t - \phi)\rangle$$

$$\varepsilon_{rms} \equiv \frac{1}{\sqrt{2}}\varepsilon_m \qquad I_{rms} \equiv \frac{1}{\sqrt{2}} I_m \qquad \Rightarrow \qquad \langle P(t)\rangle = \varepsilon_{rms} I_{rms} \cos\phi$$

- so power delivered also depends on relative phase of voltage and current in the generator