Homework #1: Due 10/23/00

Physics 254, Fall 2000
Prof. Kevin McFarland

This homework covers the "introductory" material of this course, namely

- Relativity and Lorentz Transformations
- Fundamentals of Particle Physics: reactions, fermions and forces
- Symmetries and Conservation Laws
- Detectors and Accelerators

1. A charged pion at rest decays into a (massless) neutrino and a lepton. Find the speed, $v = \beta c$, of the final state lepton in terms of $m_{\pi^\pm}$ and m_ℓ.
   $m_{\pi^\pm} \approx 139.6$ MeV, $m_\mu \approx 105.7$ MeV, $m_e \approx 0.511$ MeV. Evaluate β numerically for $\pi^+ \to \mu^+\nu_\mu$ and $\pi^+ \to e^+\nu_e$.

2. A neutral pion ($m_{\pi^0} \approx 134.9$ MeV) with momentum p in the lab frame decays into two photons. Derive the photon energy in the lab frame as a function of the emission angle of the photon in the π^0 rest frame with respect to the π^0 direction of motion in the lab frame. Find the distribution of γ angles from the π^0 direction of motion in the lab frame.

3. A proton beam is incident on a stationary proton target. Find the *minimum* beam energy required to observe the reaction
   $$pp \to pp\pi^+\pi^-.$$
   ($m_p \approx 938.3$ MeV, $m_{\pi^\pm} \approx 139.6$ MeV)

4. (Ferbel & Das, I.4) What would be the approximate counting rate obsered in the Rutherford scattering of 10 MeV α particles off lead foil at an angle of $\theta = \pi/2$ in the lab? Assume 10^6 α per second incident on the foil, a 1 mm foil thickness and a square 1 cm^2 detector placed 1 m from the interaction point. ($\rho_{\text{Pb}} = 11.3$ g/cm^3) What is the counting rate at $\theta = \pi/4$?

5. For each of the following reactions or decays, indicate whether the reaction proceeds at the highest rate *via* the strong interaction, electromagnetic interactions, weak interactions or is forbidden in all. (Use the Particle Data Group web page to look up the quark content or properties of any particle you don't know.)

   (a) $pp \to \pi^+\pi^-$
   (b) $\mu^- \to e^-\overline{\nu}_e$
   (c) $\pi^0 \to e^+e^-$

(d) $K^- \to \pi^- \pi^0$

(e) $K^{*0} \to K^+ \pi^-$

(f) $\pi^- p \to n \pi^0$

(g) $\pi^- p \to \Lambda^0 K^0$

(h) $\Sigma^0 \to \Lambda \gamma$

(i) $\overline{\nu}_e p \to n e^+$

(j) $\overline{\nu}_e p \to \pi^0 e^+$

(k) $D_s^+ \to K^+ \overline{K}^0$

6. Draw all the lowest order weak diagrams leading to $e^+ e^- \to W^+ W^-$. (Remember that both the γ and the Z^0 couple to the W.)

7. The D^0 meson has quark content $c\overline{u}$. Which weak decay is more likely, $D^0 \to K^- \pi^+$ or $D^0 \to K^+ \pi^-$? Draw Feynman diagrams and explain.

8. The ρ^0 meson and ω^0 mesons are both made of up and down quarks and anti-quarks, are spin-1 with negative parity, and have similar masses $m_\rho \approx 770$ MeV, $m_\omega \approx 785$ MeV. However, the ρ is part of an isotriplet (ρ^+, ρ^0, ρ^-) whereas the ω is an isosinglet ($I = 0$). The dominant decay mode of the ρ^0 is $\pi^+ \pi^-$ (remember that $\pi^0 \pi^0$ is forbidden by spin statistics). Explain why the lifetime of the ρ is a factor of 10 shorter than the ω.

9. Use isospin symmetry to find the ratio of the cross-sections for the reactions $pd \to \pi^0\ {}^3He$ and $pd \to \pi^+\ {}^3H$.

10. The η^0 meson ($I = 0$, spin 0, negative parity) has a mass of 549 MeV and decays in roughly equal proportion to $\gamma\gamma$, $\pi^+ \pi^0 \pi^-$ and $\pi^0 \pi^0 \pi^0$. Explain why $\eta \to \pi^+ \pi^-$ is forbidden for strong and electromagnetic interactions. The fact that $\gamma\gamma$ and the pion decay modes compete says that the pion decay modes are electromagnetic and *not* strong decays. Explain why strong decays to three pions do not occur.

11. Estimate the range of a 20 GeV muon in lead, in water, in air. (Use the PDG web page to look up $(-dE/dx)_{\text{min}}$ and the density for each of these materials and ignore the rise in (-dE/dx) as $\gamma \gg 1$ or $\beta \ll 1$.)

12. Time-of-flight is sometimes used as an experimental technique to identify charged particles. Imagine that you want to distinguish between the decay at rest of the B^0 meson to $K^+ \pi^-$ from the decay to $\pi^+ \pi^-$ by observing a time of flight difference in the two final state particles. Explain what time of flight resolution over what distance would be required to distinguish between the two scenarios.