

# Notes on potential energy and factor of 1/2

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May 17, 2020

Let us first speak of particles and gas as two separate and distinct entities, as they are separate and distinct in the simulation. The potential energy between particles and gas is

$$E_{\text{pot,par-gas}} = \int_{\text{gas}} \phi_{\text{par}}(\mathbf{r}) \rho_{\text{gas}}(\mathbf{r}) d^3\mathbf{r} = -G \sum_{i=1}^N m_i \int_{\text{gas}} \frac{\rho_{\text{gas}}(\mathbf{r})}{|\mathbf{r} - \mathbf{r}_i|} d^3\mathbf{r}, \quad (1)$$

where  $\phi_{\text{par}}$  is the potential of the particles,  $N$  is the number of particles,  $m_i$  and  $\mathbf{r}_i$  are the particle masses and positions, and  $\rho_{\text{gas}}$  is the density of gas only. Alternatively, we could calculate the potential energy between gas and particles using the gas potential:

$$E_{\text{pot,par-gas}} = \int_{\text{particles}} \phi_{\text{gas}}(\mathbf{r}) \rho_{\text{par}}(\mathbf{r}) d^3\mathbf{r} = \int_{\text{particles}} \phi_{\text{gas}}(\mathbf{r}) \sum_{i=1}^N m_i \delta^3(\mathbf{r} - \mathbf{r}_i) d^3\mathbf{r} = \sum_{i=1}^N m_i \phi_{\text{gas}}(\mathbf{r}_i). \quad (2)$$

Alternatively, we could apply the general formula for potential energy from Sec. 2.1 of Binney & Tremaine (2008), where now the integral is over the full volume and there is no distinction between particles and gas (all that matters is the mass, not how concentrated that mass is or whether it's gaseous or solid, etc.).

$$\begin{aligned} E_{\text{pot}} &= \frac{1}{2} \int_V \phi(\mathbf{r}) \rho(\mathbf{r}) d^3\mathbf{r} \\ &= \frac{1}{2} \int_{\text{gas}} [\phi_{\text{par}}(\mathbf{r}) + \phi_{\text{gas}}(\mathbf{r})] \rho_{\text{gas}}(\mathbf{r}) d^3\mathbf{r} + \frac{1}{2} \int_{\text{particles}} [\phi_{\text{par}}(\mathbf{r}) + \phi_{\text{gas}}(\mathbf{r})] \rho_{\text{par}}(\mathbf{r}) d^3\mathbf{r} \\ &= \frac{1}{2} \int_{\text{gas}} \phi_{\text{par}}(\mathbf{r}) \rho_{\text{gas}}(\mathbf{r}) d^3\mathbf{r} + \frac{1}{2} \int_{\text{gas}} \phi_{\text{gas}}(\mathbf{r}) \rho_{\text{gas}}(\mathbf{r}) d^3\mathbf{r} \\ &\quad + \frac{1}{2} \int_{\text{particles}} \phi_{\text{par}}(\mathbf{r}) \rho_{\text{par}}(\mathbf{r}) d^3\mathbf{r} + \frac{1}{2} \int_{\text{particles}} \phi_{\text{gas}}(\mathbf{r}) \rho_{\text{par}}(\mathbf{r}) d^3\mathbf{r} \\ &= \frac{1}{2} E_{\text{pot,par-gas}} + E_{\text{pot,gas-gas}} + E_{\text{pot,par-par}} + \frac{1}{2} E_{\text{pot,par-gas}}. \end{aligned} \quad (3)$$

where the last equality uses Eqs. (1) and (2).

If the integrand of Eq. (3) is interpreted as a potential energy density, then it is natural to include the 1/2 factor, and write the particle-gas potential energy density for the gas as  $\frac{1}{2} \phi_{\text{par}}(\mathbf{r}) \rho_{\text{gas}}(\mathbf{r})$ , and that for the particles as  $\frac{1}{2} \phi_{\text{gas}}(\mathbf{r}) \sum_{i=1}^N m_i \delta^3(\mathbf{r} - \mathbf{r}_i)$ . In this interpretation, half the particle-gas PE resides in the gas and half resides in the particles.

Note that Eq. 3 gives all the three potential energy terms computed in the simulation, particle-gas PE  $E_{\text{pot,par-gas}}$ , gas PE due to self-gravity  $E_{\text{pot,gas-gas}}$ , and particle-particle PE  $E_{\text{pot,par-par}}$ . The factor 1/2 in the latter two terms is needed to avoid double-counting. However the term  $E_{\text{pot,par-par}}$  also contains self-gravity of the particles, which is formally infinite for both particles. As we are not interested in the unbinding of the cores, we can ignore the self-gravity of individual particles.

## References

Binney, J., & Tremaine, S. 2008, *Galactic Dynamics*, 2nd ed. (Princeton University Press, Princeton)

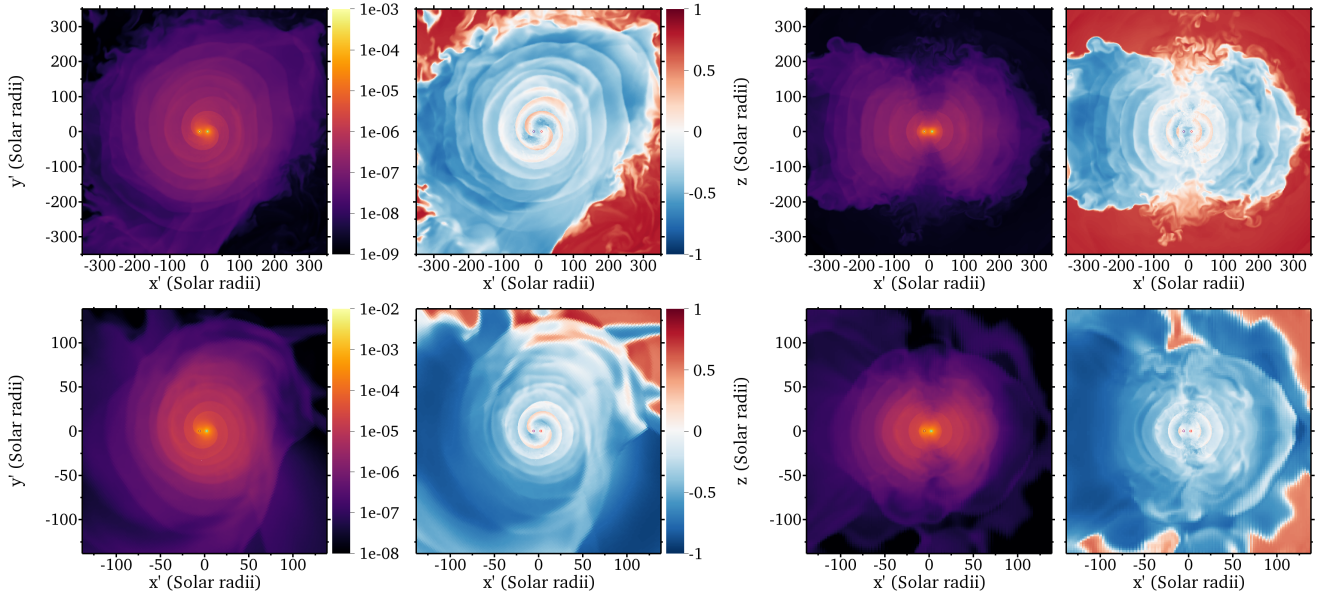


Figure 1: Comparison of AGB run (top row), and RGB run (bottom row), at 10 orbits, corresponding to  $t = 193$  d for the AGB run and to  $t = 40$  d for the RGB run. The left column shows snapshots of gas density  $\rho$  in the orbital plane. The primary core particle is located on the left, the secondary on the right, and the particle centre of mass at the origin. The second column shows the same slice but for the normalized gas energy density  $\mathcal{E}_{\text{gas}}/\max(\mathcal{E}_{\text{bulk,gas}} + \mathcal{E}_{\text{int,gas}}, -\mathcal{E}_{\text{pot,gas}})$ . Here blue denotes bound gas and red unbound. The gas-particle potential energy density has now been corrected by inserting the factor  $1/2$ . The third and fourth columns are similar but now for the orthogonal slice, also through the particles. The size of the field of view is proportional to the initial orbital separation  $a_i$ .

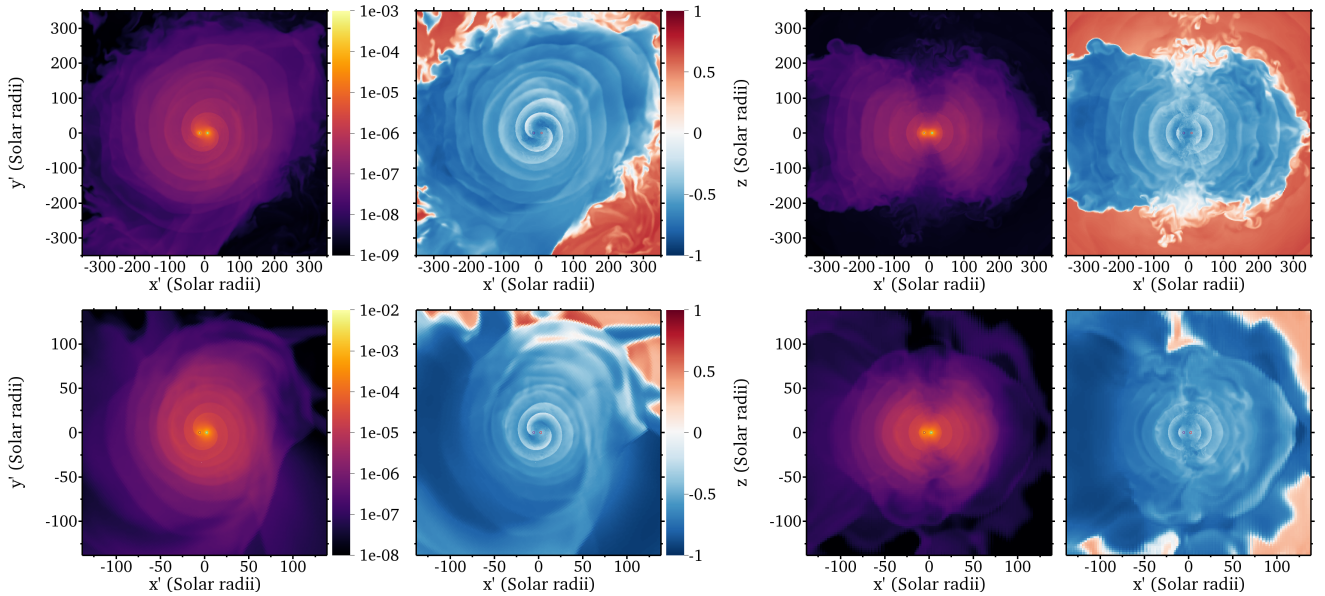


Figure 2: Similar to Fig. 1 but this is the original version: the gas-particle potential energy density has *not* been corrected by inserting the factor  $1/2$ .

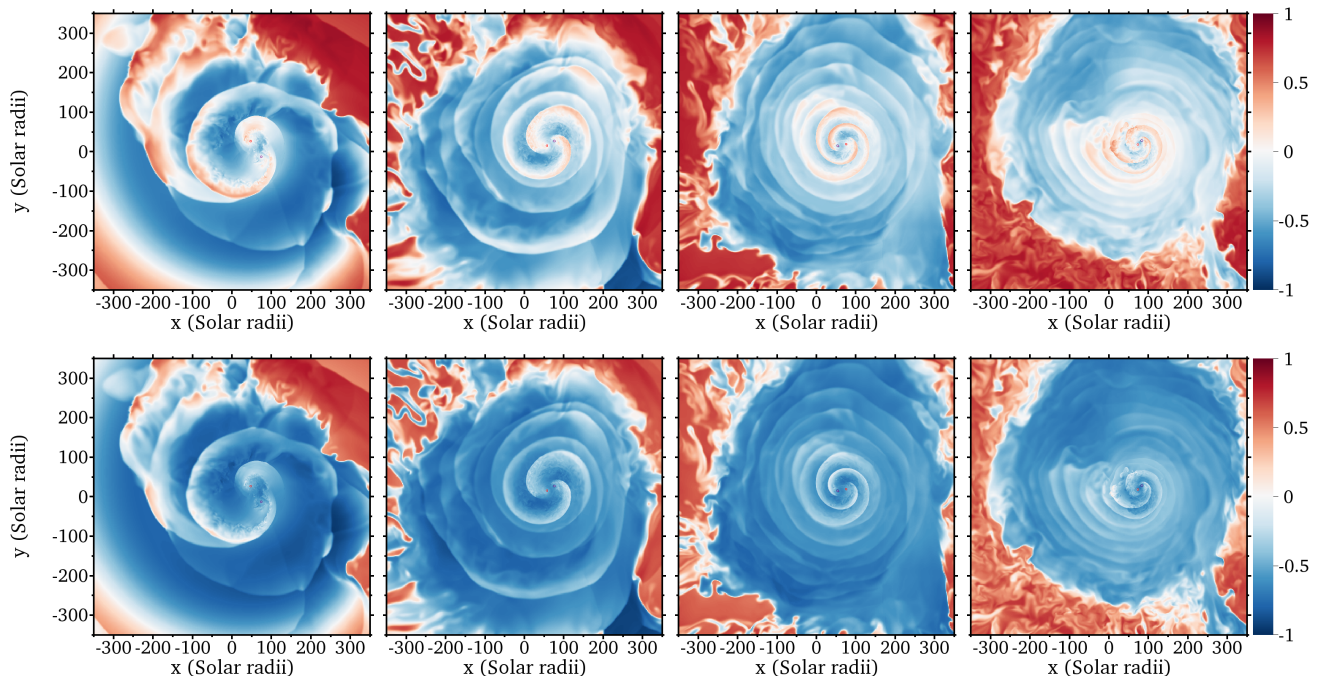


Figure 3: Snapshots at  $t = 100, 150, 200,$  and  $250$  d, showing the normalized gas energy density  $\mathcal{E}_{\text{gas}}/\max(\mathcal{E}_{\text{bulk,gas}} + \mathcal{E}_{\text{int,gas}}, -\mathcal{E}_{\text{pot,gas}})$ . Positive red values show unbound gas with  $\mathcal{E}_{\text{gas}} > 0$ , negative blue values show bound gas, and white shows marginally bound or unbound gas. **Top row:** with the gas-particle potential energy term now corrected to include the factor of  $1/2$ . **Bottom row:** Original version, *without*  $1/2$  factor in gas-particle potential energy density term.