Notes on calculating velocities in common envelope simulations

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We have two coordinate systems. Box coordinates (x, y, z) and cylindrical coordinates (r, ϕ, z) with a common z axis but centered on particle 2. The gas velocity in the frame of particle 2 (the companion) is given by $\boldsymbol{v}_{\rm gas,2} = \boldsymbol{v} - \boldsymbol{v}_2$, i.e. the difference between the velocity of the gas and the velocity of particle 2. Likewise, the position in the frame of particle 2 is given by $\boldsymbol{r}_{\rm gas,2} = \boldsymbol{r} - \boldsymbol{r}_2$. The azimuthal component of the gas velocity in the frame of particle 2 is given by

$$v_{\phi,2} = \frac{(\boldsymbol{r}_{\rm gas,2} \times \boldsymbol{v}_{\rm gas,2})_z}{|\hat{z} \times \boldsymbol{r}_{\rm gas,2}|}, \tag{1}$$

where the denominator is just equal to the square-root of the sum of squares of the first two components of $\boldsymbol{r}_{\rm gas,2}$ (that is neglecting the z-component, since we want the distance from particle 2 in the orbital plane). Now, to go into the frame that is corotating with the orbit of particles 1 and 2, we need the orbital angular velocity, $\boldsymbol{\Omega}$. This is given by

$$\boldsymbol{\Omega} = \frac{(0, 0, (\boldsymbol{r}_{1,2} \times \boldsymbol{v}_{1,2})_z)}{|\hat{z} \times \boldsymbol{r}_{1,2}|}, \tag{2}$$

Then the velocity in the frame rotating about particle 2 with the orbital angular frequency is

$$\boldsymbol{v}^{\Omega}_{\rm gas,2} = \boldsymbol{v}_{\rm gas,2} - \boldsymbol{\Omega} \times \boldsymbol{r}_{\rm gas,2} \tag{3}$$

so that the azimuthal component is given by

$$v^{\Omega}_{\phi,2} = \frac{(\boldsymbol{r}_{\rm gas,2} \times \boldsymbol{v}^{\Omega}_{\rm gas,2})_z}{|\hat{z} \times \boldsymbol{r}_{\rm gas,2}|}. \tag{4}$$

Now, in the frame corotating with the orbit but centered on the center of mass of the two particles, we have the same value of $\boldsymbol{\Omega}$ and

$$\boldsymbol{v}^{\Omega}_{\rm gas,CM} = \boldsymbol{v}_{\rm gas,CM} - \boldsymbol{\Omega} \times \boldsymbol{r}_{\rm gas,CM}. \tag{5}$$

How does this compare to our result for the frame rotating about particle 2? Write down the velocity of particle 2 in the frame rotating about the center of mass,

$$\boldsymbol{v}^{\Omega}_{\rm 2,CM} = \boldsymbol{v}_{\rm 2,CM} - \boldsymbol{\Omega} \times \boldsymbol{r}_{\rm 2,CM}, \tag{6}$$

Now we can write

$$\boldsymbol{v}^{\Omega}_{\rm gas,2} = \boldsymbol{v}^{\Omega}_{\rm gas,CM} - \boldsymbol{v}^{\Omega}_{\rm 2,CM} = \boldsymbol{v}_{\rm gas,CM} - \boldsymbol{\Omega} \times \boldsymbol{r}_{\rm gas,CM} - \boldsymbol{v}_{\rm 2,CM} + \boldsymbol{\Omega} \times \boldsymbol{r}_{\rm 2,CM} = \boldsymbol{v}_{\rm gas,2} - \boldsymbol{\Omega} \times \boldsymbol{r}_{\rm gas,2}, \tag{7}$$

which is the same as equation (3). This could be generalized from the CM to any point. Therefore, the velocity of the gas in the corotating frame is the same, no matter where the center of rotation is. However, the ϕ-component would change. For example, for a gas particle situated on the straight line connecting the CM and particle 2, and situated in between the CM and particle 2, the ϕ-component would reverse when going from the rotating frame centered on the CM to the one centered on particle 2.