

THE INTERACTION OF STELLAR OBJECTS WITHIN A COMMON ENVELOPE

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ABSTRACT

We use high-resolution, three-dimensional hydrodynamic simulations to study the hydrodynamic and gravitational interaction between stellar companions embedded within a differentially rotating common envelope. We evaluate the contributions of the nonaxisymmetric gravitational tides and ram pressure forces to the drag force and, hence, to the dissipation rate and the mass accumulated onto the stellar companion. We find that the gravitational drag dominates the hydrodynamic drag during the inspiral phase, implying that a simple prescription based on a gravitational capture radius significantly underestimates the dissipation rate and overestimates the inspiral decay timescale. Although the mass accretion rate fluctuates significantly, we observe a secular trend leading to an effective rate that is significantly less than the rate based on a gravitational capture radius. We discuss the implications of these results within the context of accretion by compact objects in the common-envelope phase.

Subject headings: binaries: close — hydrodynamics — stars: evolution

1. INTRODUCTION

To understand the evolution of binary star systems, it is essential to analyze the interactions between their stellar components. Examples of such influences include the spin-orbit tidal interaction and mass transfer, as well as interactions that result in the loss of mass and angular momentum. Equally important are the interactions of stars orbiting about their common center of mass within a differentially rotating common envelope. It is generally accepted that such an evolutionary stage is essential for the formation of short-period binary systems containing compact objects (see, e.g., Iben & Livio 1993 and Taam & Sandquist 2000). In this case, the interaction determines the orbital evolution of the system and the conditions under which the common envelope is ejected, leading to the survival of a remnant binary system or to a merger that forms a rapidly rotating single star. The amount of mass and angular momentum accreted by the inspiralling components during this phase also has direct implications for the properties of the compact object population in binary systems.

Lacking multidimensional hydrodynamical simulations of the common-envelope phase, the initial numerical and semi-analytical studies of the problem used simple prescriptions for the stellar interactions based on the pioneering work by Hoyle & Lyttleton (1939) and Bondi & Hoyle (1944), as generalized by Bondi (1952). These seminal studies focused on the idealized problem of the capture of matter by a gravitating point object moving supersonically with respect to a uniform medium. In this framework, a gravitational capture radius, R_{cap} , plays an important role in determining the rates of mass accretion and energy dissipation. R_{cap} is given by

$$R_{\text{cap}} = \frac{2GM}{v_{\text{rel}}^2 + c_s^2}, \quad (1)$$

where M is the mass of the gravitating object, v_{rel} is the velocity of the object with respect to the medium, and c_s is the local speed of sound. When a density gradient with scale height H

is present, the effective accretion radius R_{acc} is (Dodd & McCrea 1952)

$$R_{\text{acc}} = \frac{R_{\text{cap}}}{1 + (R_{\text{cap}}/2H)^2}. \quad (2)$$

The energy dissipation rate is then $L_d \approx \pi R_{\text{acc}}^2 \rho v_{\text{rel}}^3$, where ρ is the upstream density. To improve on these estimates, hydrodynamic effects were approximated analytically by Ruderman & Spiegel (1971), Wolfson (1977), and Bisnovatyi-Kogan et al. (1979) as well as numerically by Hunt (1971, 1979), Shara & Shaviv (1980), and Shima et al. (1985). These early multidimensional simulations considered axisymmetric flow, and their results have been used to calibrate the energy loss rate. In particular, the drag coefficients obtained from such simulations (see, e.g., Shima et al. 1985) have been used to estimate the rate of energy dissipation in the common envelope.

However, many of the simplifying assumptions underlying these studies are inadequate for direct application to common-envelope interactions. The flow is nonaxisymmetric and distinctly nonuniform, reflecting the existence of velocity or density gradients (the density may span several scale heights within R_{acc}). The effect of relaxing these assumptions has been studied in two dimensions by Fryxell & Taam (1988) and Taam & Fryxell (1989) and in three dimensions by Sawada et al. (1989) and Ruffert (1999). These studies could not encompass the full complexity of common-envelope interactions, since the envelope's self-gravity was ignored. Furthermore, because the companions move in elliptical orbits, their cores interact with matter that has already been affected in previous orbital phases. Thus, the state of the gas and its environment in these calculations must be regarded as highly idealized.

Within the past decade, three-dimensional numerical studies of the common-envelope phase that have relaxed earlier geometrical assumptions have been carried out by Sandquist et al. (1998, 2000), De Marco et al. (2003a, 2003b), and Taam & Ricker (2006). Recently, we have carried out high-resolution adaptive-mesh-refinement (AMR) simulations of common-envelope evolution with effective resolutions of 2048^3 (R. E. Taam & P. M. Ricker, in preparation), allowing the interaction of the stars within the common envelope to be examined and quantified.

In this Letter we report on some results of our numerical studies. We focus on analyzing a single high-resolution sim-

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ulation, to determine the hydrodynamic and gravitational contributions to the drag forces affecting the orbital motion of the stellar components during the early inspiral phase. We also analyze the accumulation of mass by the stellar components within the common envelope, to compare their magnitudes to estimates based on an accretion-radius formalism. In § 2, we briefly describe the numerical method and our assumed model for the binary system. Descriptions of the method of analysis and the numerical results are presented in § 3. Finally, we summarize our results and comment on their possible implications for applications involving compact objects in short-period binary systems.

2. NUMERICAL METHOD

For our simulations we use FLASH, a parallel AMR code that includes both grid- and particle-based numerical methods (Fryxell et al. 2000). The Euler equations describing the evolution of the common-envelope gas are solved using the piecewise parabolic method (Colella & Woodward 1984). We use the AMR to refine the mesh on the basis of the second derivative of gas density, for shock capturing, and on the basis of the positions of the stellar components, for high force resolution in their vicinities. This method significantly improves on stationary nested-grid techniques because the refined region can change to track the motion of both stars, thus allowing investigations of systems with components of similar mass.

However, even with AMR, the core of an evolved red giant star (the progenitor of a white dwarf) is too compact compared to the size of the computational domain (a factor of 40,000 smaller) given the large number of time steps that we require. Since the core has a much higher mean density than does the common envelope, its strongest interaction with the gas is gravitational. Therefore, we model the red giant core and the companion object using spherical particle clouds containing 2×10^5 particles each, obtaining forces on the clouds using cloud-in-cell (CIC) interpolation. (We use clouds rather than single particles to avoid problems related to CIC force anisotropy.) The cloud radii (i.e., $R \sim R_\odot$) are taken to be 3 times the size of the smallest zone spacing. The particles in each cloud move rigidly together with the cloud's center of mass. More details are given by Taam & Ricker (2006).

We considered the common-envelope evolution of a binary system consisting of a $1.05 M_\odot$ red giant having a $0.36 M_\odot$ core and a $0.6 M_\odot$ companion with an orbital period of 44.2 days. The initial red giant model was taken from a one-dimensional stellar evolution code developed by Eggleton (1971, 1972). We interpolated this model onto a three-dimensional Cartesian grid with nine levels of refinement. Since each block contained 8^3 zones, and since the coarsest level contained one block, this corresponded to a minimum-zone spacing of 2×10^{10} cm (a factor of 2048 smaller than the domain size). We artificially damped transient motions in the red giant envelope, evolving this model for a dynamical timescale. We then turned off the damping and added the particle cloud representing the companion star. At this point, the red giant core and envelope, as well as the companion, were given circular orbital velocities and a spin angular velocity equal to 95% of the synchronous value.

3. NUMERICAL RESULTS

After an initial phase of 12 days, the orbital separation decreased monotonically with time, with an increasing rate of orbital shrinkage. During the later phase (~ 43 days), the orbital separation decreased by a factor of 3, from 4.3×10^{12} to

1.4×10^{12} cm, as a result of the strong drag forces on the cores. In the physical system, the drag has both hydrodynamic and gravitational components, and, in our simulation, the gravitational component dominates.

This can be seen by defining a measured hydrodynamic drag force F_d on each core via

$$F_d = \int \rho \mathbf{v}_{\text{rel}} (\mathbf{v}_{\text{rel}} \cdot \mathbf{n}) dA, \quad (3)$$

where the integration is taken over a closed spherical surface centered on the core, and \mathbf{n} is the unit vector perpendicular to its surface. While the particle clouds have definite radii, these are determined by numerical constraints and do not correspond to the physical radii of the objects that the clouds represent. Moreover, the physical radii can be much smaller than the finest zone spacing, depending on the type of companion star. Hence, we studied the effect of choosing different (resolvable) radii for the spherical surfaces used to measure the drag, ranging from 3.5×10^{10} to 1.0×10^{12} cm. We found that the hydrodynamic drag on the companion reaches a characteristic early inspiral value (at 34 days) of $\sim 9 \times 10^{32}$ dynes at a radial scale of 3×10^{11} cm from its center, only $\sim 10\%$ of the gravitational drag on this object. At this radial scale, the hydrodynamic drag on the red giant core is at most 4×10^{32} dynes, about 1/4 of the gravitational drag on it. In general, the total gravitational drag force is nearly 2 orders of magnitude larger than the hydrodynamical drag. The drag force causes the orbital energy to dissipate at a rate given by the line integral of the drag force along the orbital path; in the case presented here, the gravitational drag corresponded to a dissipation rate $L_d \sim 8 \times 10^{39}$ ergs s^{-1} .

The flow near the inspiraling companion leads to the accretion of both mass and angular momentum. The flow pattern and density distribution in this region are illustrated in Figure 1. The density increases as one approaches the embedded object, revealing a high density contrast, with scale length corresponding to $\sim 10^{11}$ cm, which is the direct result of the companion's gravitational influence in the common envelope. In the immediate region surrounding the embedded companion, radiation pressure is comparable to gas pressure. Matter is shown to be significantly deflected by the companion. The velocity drops off more gradually than the density, with a scale length ~ 5 times larger. In this region, typical velocities are of the order of 150 km s^{-1} , whereas the velocity of the companion is of the order of $\sim 50 \text{ km s}^{-1}$. The gas flows are subsonic, and there is no evidence for the presence of shocks.

We note that our calculation does not include an explicit model for the embedded boundary condition imposed by the surface of the companion star. A realistic inner boundary condition would fall somewhere between two extreme types: a fully absorbing boundary, corresponding to accretion onto a black hole, and a fully reflecting boundary, perhaps corresponding to a star with a surface that retains very little accreted material. In the latter case, we would expect to see a clear accretion shock. Because we allow gas to accumulate in the companion's vicinity, it heats up due to compression and creates a back pressure that resists further accretion. Thus, our calculation is intermediate between the two cases, and the accretion rate that we measure should be regarded as only indicative.

Since the particle cloud does not provide a boundary for the gas on the grid, we computed the mass flux (using the respective equivalents to eq. [3]) for a range of assumed radii. (The ac-

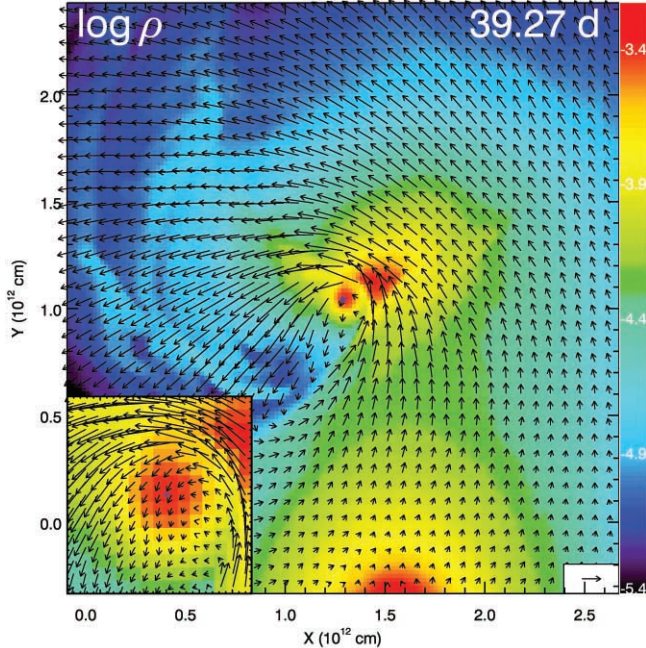


FIG. 1.—Flow field near the companion at 39.27 days. Colors denote the logarithm of gas density in the x - y plane, in units of g cm^{-3} . Arrows indicate the gas velocity in every fourth zone. The arrow in the lower right-hand corner corresponds to 106 km s^{-1} . The blue arrow at the center indicates the companion's velocity. The red giant core lies just off the bottom of the plot. The inset shows the region immediately surrounding the companion, enlarged $3 \times$ per dimension. The vector scale is the same as in the main image.

creted angular momentum rates could not be reliably calculated close to the companion due to insufficient numerical resolution. The length scale over which the pressure gradients would be calculated is comparable to the size of the control region.) We found that the mass accretion rate fluctuates significantly on timescales of less than a day. Thus, we integrated this rate to determine the accumulation of mass with time. In Figure 2, the accumulated mass is illustrated as a function of time for different control radii, measured from the companion's position, ranging from 3.5×10^{10} to 2.1×10^{11} cm. A secular trend is evident, although variations are found to be larger at greater distances from the companion. The numerical results show that the magnitude of the accreted mass is a function of the radius of the control surface, with lower accreted mass associated with smaller control surfaces. In particular, only about 1/3 of the matter at the outermost surface reached the particle cloud. It appears that the differences in the mass accumulation decrease with time at smaller control surfaces, whereas they increase with time at larger radii, suggesting that estimates for the mass accretion rate should be based on evaluations at small radii. After an evolution of 39 days, the companion object would have accreted $\sim 5 \times 10^{-4} M_{\odot}$, leading to an effective mass accretion rate of about $0.005 M_{\odot} \text{ yr}^{-1}$ (assuming that the angular momentum does not significantly impede the accretion process). During the first 35 days of evolution, instantaneous mass accretion rates rarely exceed $0.05 M_{\odot} \text{ yr}^{-1}$, and then they quickly rise to $\sim 0.1 M_{\odot} \text{ yr}^{-1}$.

Equations (1) and (2) can be used to infer an expected accretion rate $\dot{M}_{\text{BH}} = \pi R_{\text{acc}}^2 \rho v_{\text{rel}}$ on the basis of the gravitational capture radius formalism. Using the companion mass for M and taking ρ , v_{rel} , c_s , and H as observed in the simulation, we find $R_{\text{acc}} \approx 8.0 \times 10^{10}$ cm and $\dot{M}_{\text{BH}} \approx 3.2 M_{\odot} \text{ yr}^{-1}$. This esti-

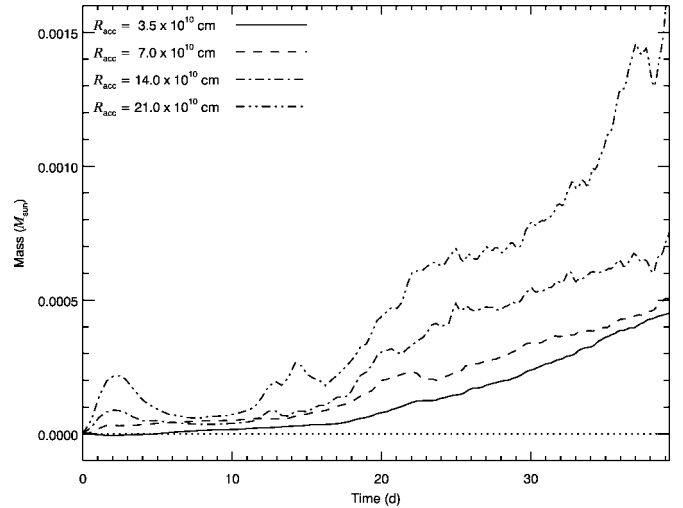


FIG. 2.—Variation of mass, in units of solar mass, accumulated within a range of radii for the control surface centered on the embedded companion as a function of time in days.

mate is significantly larger than the instantaneous or average mass accretion rate determined from the simulation. Although our lack of an embedded boundary condition implies that our measured accretion rates are only indicative, we note that the rate expected in the gravitational capture radius formalism would allow for the accretion of the entire red giant envelope onto the companion within less than 80 days. Thus, this formalism must significantly overestimate the real accretion rate. The physical reason for this discrepancy is evident in Figure 1. Whereas the capture radius formalism assumes a supersonic wind flowing directly past the accretor, the common-envelope system develops a relatively slow moving region near the companion, and matter accretes subsonically. Most of the gas is accelerated by the companion's potential and flung out to large radius, where it becomes unavailable for later accretion due to its angular momentum. In response, the companion spirals inward toward the red giant.

4. DISCUSSION

We have quantitatively described the interaction of stars within a common envelope on the basis of the analysis of the early inspiral stage of a $1.05 M_{\odot}$ red giant and a $0.6 M_{\odot}$ binary companion. The orbital decay is dominated by the nonaxisymmetric gravitational drag associated with the self-gravitating matter in the common envelope. On the basis of high-resolution three-dimensional hydrodynamic simulations, this drag is 1–2 orders of magnitude greater than the hydrodynamic drag. As a consequence, the orbital decay timescale is much shorter than that derived from analyses based on the Hoyle-Lyttleton-Bondi picture of accretion from a uniform medium by a gravitating point mass moving at supersonic speeds. In this latter description, the gravitational drag associated with an accretion wake is not significantly larger than the hydrodynamic drag. The effect of long-range gravitational interactions is critical for reliable estimates of the orbital decay timescale and energy dissipation rate. In this picture, the drag and the mass accretion rate are not as directly related as they are in the Hoyle-Lyttleton-Bondi-type description, because the dominant drag term is gravitational, rather than hydrodynamic, in origin. This decoupling is reflected in the much larger ratio $L_{\text{d}}/M \sim 10^{15} - 10^{16} \text{ cm}^2 \text{ s}^{-2}$ measured in the simulation than expected from the gravitational capture radius formalism ($\sim 10^{14} \text{ cm}^2 \text{ s}^{-2}$).

Although the mass accretion rates estimated from our simulation should be regarded as only indicative, due to the lack of a detailed inner boundary treatment for the companion, the structure of the flow (dominated as it is by tidal effects) strongly suggests that the true mass accretion rate should be much smaller than the rate expected in the gravitational capture radius formalism. The effective capture radius, on the basis of the observed ambient density and relative velocity, is almost an order of magnitude smaller than the expected value. In any case, the expected value would lead to an unrealistic level of accretion over the common-envelope period. Furthermore, the inspiral time suggested by the early evolution of our three-dimensional simulation is much shorter than that found in earlier one- and two-dimensional calculations that are based on the Bondi-Hoyle-Lyttleton picture. Assuming that the discrepancy in mass accretion rate continues into the deep inspiral phase, the total accumulation of matter onto the companion should be much smaller than previously expected.

These results would have little effect on the mass of an embedded main-sequence star, because of its tendency to expand as a result of the high entropy within the common envelope (see Hjellming & Taam 1991); however, it can significantly affect the outcome for neutron stars within a common envelope. In particular, the estimated accretion rates exceed $10^{-3} M_{\odot} \text{ yr}^{-1}$, for which steady state accretion flows with neutrino losses are possible (Chevalier 1989; Houck & Chevalier 1991). At these hypercritical mass accretion rates, photons are trapped in the flow, and the Eddington limit is not applicable. On the basis of this hypercritical accretion flow regime, Chevalier (1993) and Brown (1995) suggest that neutron stars embedded in the common envelope would accrete sufficient mass to form low-mass black holes (although see Chevalier 1996), and, hence, the formation of binary radio pulsars would require an evolutionary scenario involving progenitor stars of nearly equal mass (Brown 1995). However, the population synthesis of binary black holes and neutron stars by Belczynski et al. (2002), including hypercritical accretion, resulted in an average

accretion of $0.4 M_{\odot}$. Such a high rate of mass accretion is inconsistent with the observed masses of binary radio pulsars ($\sim 1.35 M_{\odot}$; Thorsett & Chakrabarty 1999) and indicates the need for a reduction in accreted matter during the common-envelope phase (see also Belczynski et al. 2007). Our calculations show that the necessary reduction may arise naturally as a result of a more realistic treatment of the common-envelope phase. Consequently, this reduction could also lead to a reduction in the number of low-mass black holes, depending on the maximum mass of neutron stars, resulting from the accretion induced collapse of massive accreting neutron stars in the common-envelope phase. Similarly, the mass accretion, which was found to be as large as several solar masses for black hole accretors, would also be reduced, thereby affecting the masses and spins of double black holes emerging from the common-envelope phase.

Further investigations are planned to examine the generality of these results regarding mass accretion and to quantify the importance of these processes for determining the properties (mass and spin) and ultimate fate of the compact components in short-period binary system populations. Such studies are not only important for determining the masses of binary neutron star and black hole systems resulting from the common-envelope phase (Belczynski et al. 2007), but also their orbital periods, which directly influence the expected merger rates of such binary populations as sources for gravitational wave detection in the advanced LIGO experiment.

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