Notes on new scenario to explain the WD 1856+534 system

Luke Chamandy

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1 Problem

The final separation is probably too large to explain by way of CE evolution because the loss of orbital energy would not have been sufficient to eject the envelope. The authors give a detailed analysis using the energy formalism along with stellar modeling of the WD progenitor. While they cannot completely exclude the CE scenario, they conclude that it is unlikely because the value of $\alpha_{\rm CE}$ would need to be very large. They propose a different scenario that does not involve CE (and mention a few other possible scenarios that also do not involve CE).

2 Observational constraints

Vanderburg et al. (2020) find that the mass of the WD is $M_{\rm WD} = (0.518 \pm 0.055)\,{\rm M}_\odot$ (they suggest that considerations about the system age might imply that the actual value could be slightly higher, maybe 0.6, and assuming more hydrogen in the composition would bring the mass down, resulting in a range $\sim (0.4\text{–}0.6)\,{\rm M}_\odot$). The planet mass is $M \lesssim 13.8\,{\rm M_J}$ and probably $\lesssim 11.7\,{\rm M_J}$, where ${\rm M_J} = 9.54 \times 10^{-4}\,{\rm M}_\odot$ is a Jupiter mass. The observed period is 1.4 d. Using Kepler's third law and the masses for the WD and planet, I get a mean separation $a \approx 4.2\text{–}4.5\,{\rm R}_\odot$. The planet radius is $(R = 10.4 \pm 1.0)\,{\rm R_E}$ if the eccentricity $e = 0$, and $R = (15.4^{+5.5}_{-3.7})\,{\rm R_E}$ if e is allowed to differ from zero, where ${\rm R_E} = 9.15 \times 10^{-3}\,{\rm R}_\odot$ is an Earth radius. For comparison, Jupiter's radius is ${\rm R_J} = 0.100\,{\rm R}_\odot$, so the planet radius $R \approx {\rm R_J}$. In some of their expamples they assume $R = {\rm R_J}$ and $M = {\rm M_J}$.

3 Scenario

In our proposed scenario, the primary star undergoes a CE phase involving another object (or objects) that merge with the core. (These objects are probably planets though could be a brown dwarf, etc.) This is insufficient for ejecting the envelope but causes the envelope to expand/bloat/partially eject. This ejecta quickly reaches the original orbit of the observed planet WD 1856 b. WD 1856 b then experiences a drag force from the surrounding ejecta, which causes the orbital separation to decrease. A CE phase results, but at the time of its onset the envelope has already undergone partial ejection/unbinding, so the transfer of orbital energy needed to eject the envelope during this (second) CE stage can be smaller, compared to what it would have had to be if no other CE had occurred previously.

4 Questions

At first glance, this scenario seems to be sensitive to the mass of the inner object involved in the first CE phase; it seems to require this mass to be tuned to eject the envelope partially but not completely. Even if this were the case, the scenario could still be viable. However, it is also possible that there were multiple planets and that this scenario repeats itself starting with the innermost planet, which initiates a CE phase which pulls in the second planet, etc. until the envelope is energetic enough that one planet in the sequence can eject what remains of the envelope, halting its inspiral. Then this planet still exists in a fairly tight orbit while the interior planets ave long-since merged with the stellar core (or become tidally shredded within the envelope).

Wouldn't there be evidence of past CE phases leading to merger/tidal shredding in the properties of the WD? Probably, but they would not be obvious. The planets would have a small mass as compared to the WD so the WD mass would not be affected much. Changes to its composition could perhaps be detectable.

5 Other implications

Since planets are common, this scenario may have general importance. It could help to explain why ejecting the envelope in nature is apparently easier than in simulations: before the main CE interaction involving a binary stellar companion, there are, typically, less energetic CE interactions involving planets that "loosen" the envelope.

This scenario also implies that we would not expect low mass planets to be as common around WDs as high mass planets (at least compared to their respective incidence frequencies around main sequence stars). This is because the low mass planets would be less likely to survive the CE stage compared to the high mass planets. On the other hand there would be a dependence on the initial separation, which may be larger for the higher mass planets, so one has to be careful.

6 Alternative versions

- Could it be that this happens much more slowly, in the sense that the giant first swallows the innermost planet, leading to a slightly less bound envelope. And then the primary expands on stellar evolution time scales, swallowing the next planet, further energizing the envelope, etc.? In other words, do we need to invoke drag from the ejecta or can we just invoke the extra energy input to the envelope from previous CE phases? Maybe the star puffs up a little after each CE phase, but for the next CE phase to occur, one has to wait for the star to evolve further…so rather than having each CE phase separated by a few dynamical times, they will be separated by much longer junctures. However, the envelope would tend to relax on the thermal timescale by radiating away the excess energy gained during the first CE stage. This timescale is expected to be short compared to the evolutionary timescale. So probably, this version of the scenario would not work.

- Maybe a PN occurs and the wind from the PN extends to the planet and causes it to fall in, perhaps followed by a CE which ejects whatever part of the envelope is left over and halts the inspiral. WRLOF may be involved in this scenario (Chen et al., 2018). This would happen on much longer timescales, and would require a PN but would not require other planets (unless they were somehow required for the PN to happen).

7 Proposed methods

7.1 Analytics

Assume that only two CE events occured, for simplicity, one with an inner planet since destroyed, and one with WD 1856 b. Free parameters are initial separation of WD 1856 b a_i, and mass and initial separation of the innermost planet involved in the first CE. Vanderburg et al. (2020) argue that $a_\mathrm{i} \gtrsim 1\,\mathrm{au}$ in order to have avoided destruction during the red giant phase, and they use 1–2 au in their estimates. The mass M_1 and radius R_1 of the initial primary are not well constrained; Vanderburg et al. (2020) explore masses between $\sim$ (1–3) $\mathrm{M}_\odot$, and radii between $\sim$ (100–250) $\mathrm{R}_\odot$. Assuming all orbits to be circular is probably okay. The parameter α_CE is not well constrained. And it is not constant in general. Also, λ_CE is not well constrained unless one adopts a detailed model.

We need to first show that the timescale for the planet to plunge into the envelope is small compard to the thermal timescale of the envelope. Otherwise, the envelope could radiate away the extra energy it received from the first CE phase before undergoing the second CE phase.

1) Independently from the above calculations, compute the mass of the first planet to undergo CE, in terms of the parameters of the observed system (there will some dependence on intial separations, but it will be weak). We can copy the method of Vanderburg et al. (2020) here (or simplify it). One can use a_f as the tidal shredding radius. This is the mass of the secondary that should be used to estimate the ejecta properties. If it is too small, the ejecta will be too diffuse. If it is too big then the initial system would not be plausible and also the WD properties might rule it out.

2) So we can produce a range of masses for the secondary that undergoes the first CE phase where the scenario would work, and argue that this range is reasonable. We can

do so by making comparisons with other known solar systems, and using our model to (very roughly) predict what they would look like in the aftermath of such a scenario.

3) Estimate CE ejecta properties from simulations, scaling appropriately with secondary mass. Mainly, we require the outflow speed and density as a function of separation, in the orbital plane.

4) Compute the ram pressure force of the wind on the planet and make sure that it is small compared to the star's gravity.

5) Compute BHL drag and hydro drag as a function of separation.

6) Convert this drag force to an $\dot{a}(a)$.

7) First of all ensure that $|\dot{a}|$ from this effect would be greater than (outward) $\dot{a}$ owing to reduced gravity owing to mass loss of the primary. (Tidal forces would also play a role but could probably be neglected in a first pass.)

8) Estimate the plunge timescale by computing $t_{\rm plunge} \sim \int_{a_{\rm i}}^{R_1} da/\dot{a}(a)$

9) Compute the thermal timescale $t_{\rm thermal}$ and check that $t_{\rm plunge} \ll t_{\rm thermal}$

7.2 Simulations

1) It would be worth simulating at least the initial stages of a few CE scenarios involving planetary mass companions to better constrain the density and velocity of the ejecta (say 3 simulations using secondary masses of 0.1, 1 and $10\,{\rm M_J}$ for the first CE phase).

2) It would then be interesting to actually have our planet (probably use $M = 1\,{\rm M_J}$) orbiting somewhere farther out and see whether the drag from the ejecta can cause it to migrate inward as fast as predicted (probably extending one of the above 3 simulations would suffice).

Remarks:

- Since we are most interested here in the initial stages of the first CE event, I think we could make the primary core quite large, which would allow us to get away with low resolution there.

- Since the ejecta may be of low density and outward velocity, one has to pay attention to the ambient pressure and density. If they are too large, this would lead to a reduction in the ejecta density and pressure. It would also produce its own (ambient) drag on the planet. We could avoid the latter by only inserting the planet once the ejecta has reached its initial orbital separation. On the other hand, as we would probably start the secondary just outside the surface of the primary, the ejecta density and velocity would be somewhat overestimated. So these two effects would offset one another to some extent.

- The results will also depend somewhat on the azimuthal location of the planet, as the ejecta is not axisymmetric.
- Initial rotation of the primary will be neglected (initiated with zero rotation) and hence angular momentum of the ejecta would also not be accurate.
- In any case, the goal of the simulation would be to confirm or refute our order of magnitude estimates, so high accuracy would not be required.

7.3 Overall strategy

Ideally, we would combine the analytics and simulations in one short paper (a letter).

8 Estimating the mass of the secondary involved in the first CE interaction

Suppose there is a prior CE phase that precedes the CE phase involving the observed planet. Let us call this CE1, whereas the second CE phase involving the observed planet will be called CE2. Let the binding energy (>0) of the primary at the start of mass transfer leading to this first CE interaction be given by

$$E_0 = \frac{GM'_\mathrm{p}M'_\mathrm{env}}{\lambda' R'_\mathrm{p}} \tag{1}$$

where primes are used to specify that we are referring to the state of the system during the first (in time) CE interaction. Suppose that a fraction β' of the energy needed to unbind the envelope is supplied by CE1, which means that CE2 need supply only a fraction $\beta = 1-\beta'$. Then, since $\beta' = 1-\beta$, the energy formalism for CE1 gives,

$$(1-\beta)E_0 \simeq \alpha' \frac{GM_\mathrm{wd}M'_\mathrm{com}}{2a'_\mathrm{f}}, \tag{2}$$

where the primary core mass is assumed equal to the observed mass of the WD, the final mass of the envelope interior to the orbit has been neglected (though we may absorb it in the factor $1-\beta$), and the term $-\alpha' GM'_\mathrm{p}M'_\mathrm{com}/2a'_\mathrm{i}$ on the RHS is neglected, as also neglected by Vanderburg et al. (2020); the validity of this last assumption can be checked for self-consistency later. Likewise, for CE2 we have

$$\beta E_0 \simeq \alpha \frac{GM_\mathrm{wd}M_\mathrm{com}}{2a_\mathrm{f}} \tag{3}$$

where the term $-\alpha G M_{\rm p} M_{\rm com}/2a_{\rm i}$ and the envelope mass interior to the final orbit are neglected (though we may absorb it in the factor β). Dividing equation (2) by equation (3), and solving for the companion mass for CE1, we obtain

$$m'_{\rm com} = \frac{a'_{\rm f}}{a_{\rm f}} \frac{\alpha}{\alpha'} \left(\frac{1}{\beta} - 1 \right) m_{\rm com}, \tag{4}$$

where the notation $m_{\rm i} \equiv M_{\rm i}/\,{\rm M}_{\odot}$ has been used. Here, $m_{\rm com}$ is not known precisely but was constrained by the observations to have an upper limit of $13.8\,{\rm M_J}$, and, since the radius of the observed planet is $\approx$ ${\rm R_J}$, below we scale $m_{\rm com}$ to ${\rm M_J}$.

The quantity $a_{\rm f}$ is related to the observed period by Kepler's third law:

$$a_{\rm f} = \left[\frac{G\,{\rm M}_{\odot}}{4\pi^2} (m_{\rm wd} + m_{\rm com}) P_{\rm f}^2 \right]^{1/3}, \tag{5}$$

where $P_{\rm f} = 1.4\,{\rm d}$ is the observed period.

Next, we divide equation (1) by equation (3) to obtain

$$\frac{1}{\beta} = \frac{2(m'_{\rm p} - m_{\rm wd}) m'_{\rm p}}{m_{\rm wd} m_{\rm com}} \frac{a_{\rm f}}{\alpha' \lambda' R'_{\rm p}} \frac{\alpha'}{\alpha}, \tag{6}$$

where we have substituted $m_{\rm env} = m'_{\rm p} - m_{\rm wd}$. Taking the reciprocal of equation (7), we obtain

$$\beta = \frac{m_{\rm wd} m_{\rm com}}{2(m'_{\rm p} - m_{\rm wd}) m'_{\rm p}} \frac{\alpha' \lambda' R'_{\rm p}}{a_{\rm f}} \frac{\alpha}{\alpha'}. \tag{7}$$

Following Vanderburg et al. (2020), we can assume the radius of the primary $R'_{\rm p}$ to be equal to the radius of its Roche lobe at the time that mass transfer is initiated, and then make use of the fitting formula they obtain for $R'_{\rm p}$ in terms of the core mass $m_{\rm wd}$. *Note: we should look at their method more carefully, as it may be somewhat sensitive to assumptions that may be questionable.* This gives

$$R'_{\rm p} = 5.56 \times 10^4 f(m_{\rm wd})\,{\rm R}_{\odot}, \tag{8}$$

where

$$f(m) \equiv \frac{m^{19/3}}{1 + 20m^3 + 10m^6} + f_0,$$

with $f_0 = 7.2 \times 10^{-5}$.

Equation (4), with equations (5), (6) and (8) can be used to obtain $m'_{\rm com}$ in terms of $\alpha'\lambda'$, $a'_{\rm f}$, $m_{\rm com}$, $P_{\rm f}$, $m_{\rm wd}$, $m'_{\rm p}$ and the ratio α/α'. Writing this out in full we have

$$\begin{aligned} m'_{\rm com} &= \frac{2(m'_{\rm p} - m_{\rm wd}) m'_{\rm p}}{m_{\rm wd}} \frac{1}{\alpha'\lambda'} \frac{a'_{\rm f}}{R'_{\rm p}} - \frac{\alpha}{\alpha'} \frac{a'_{\rm f}}{a_{\rm f}} m_{\rm com}. \\ &= \frac{2(m'_{\rm p} - m_{\rm wd}) m'_{\rm p}}{m_{\rm wd}} \frac{1}{\alpha'\lambda'} \frac{a'_{\rm f}}{5.56 \times 10^4 f(m_{\rm wd})\,{\rm R}_{\odot}} - \frac{\alpha}{\alpha'} \left(\frac{4\pi^2}{G\,{\rm M}_{\odot}} \frac{1}{(m_{\rm wd} + m_{\rm com}) P_{\rm f}^2} \right)^{1/3} a'_{\rm f} m_{\rm com}. \end{aligned} \tag{9}$$

Now, to estimate $a'_{\rm f}$, consider two limiting cases:

1) the planet merges with the white dwarf, with all of the liberated orbital energy released to the envelope. In this case, we can adopt

$$a'_{\rm f} = R_{\rm wd}; \tag{10}$$

2) after the planet gets tidally shredded, its orbital energy is no longer used to unbind the envelope. In this case, we can estimate

$$a'_{\rm f} = r'_{\rm s} \simeq \left(\frac{2m_{\rm wd}}{m'_{\rm com}}\right)^{1/3} R'_{\rm com}, \tag{11}$$

where $R'_{\rm com}$ is the radius of the companion in CE1.

Neither of these cases is completely realistic but they serve to bracket the range of possibilities. For case (2) ($a'_{\rm f} = r'_{\rm s}$), we must substitute equation (11) into equation (4) or (9), and solve for $m'_{\rm com}$. We obtain

$$m'_{\rm com} = (2m_{\rm wd})^{1/4}\left[\frac{R'_{\rm com}}{a_{\rm f}}\frac{\alpha}{\alpha'}\left(\frac{1}{\beta}-1\right)m_{\rm com}\right]^{3/4} \qquad ;\ \text{if } a'_{\rm f} = r'_{\rm s}, \tag{12}$$

or

$$\begin{aligned}
m'_{\rm com} &= (2m_{\rm wd})^{1/4}\left[\frac{R'_{\rm com}}{a_{\rm f}}\frac{\alpha}{\alpha'}\left(\frac{2(m'_{\rm p}-m_{\rm wd})m'_{\rm p}}{m_{\rm wd}m_{\rm com}}\frac{a_{\rm f}}{\alpha'\lambda' R'_{\rm p}}\frac{\alpha'}{\alpha}-1\right)m_{\rm com}\right]^{3/4} \qquad ;\ \text{if } a'_{\rm f} = r'_{\rm s},\\
&= (2m_{\rm wd})^{1/4}\left[\frac{2(m'_{\rm p}-m_{\rm wd})m'_{\rm p}}{m_{\rm wd}}\frac{R'_{\rm com}}{\alpha'\lambda' R'_{\rm p}} - \frac{R'_{\rm com}}{a_{\rm f}}\frac{\alpha}{\alpha'}m_{\rm com}\right]^{3/4} \qquad ;\ \text{if } a'_{\rm f} = r'_{\rm s},\\
&= (2m_{\rm wd})^{1/4}\left[\frac{2(m'_{\rm p}-m_{\rm wd})m'_{\rm p}}{m_{\rm wd}}\frac{1}{\alpha'\lambda'}\frac{R'_{\rm com}}{5.56\times10^4 f(m_{\rm wd})\,{\rm R}_\odot}\right.\\
&\qquad \left. - \left(\frac{4\pi^2}{G\,{\rm M}_\odot}\frac{1}{(m_{\rm wd}+m_{\rm com})P_{\rm f}^2}\right)^{1/3} R'_{\rm com}\frac{\alpha}{\alpha'}m_{\rm com}\right]^{3/4} \qquad ;\ \text{if } a'_{\rm f} = r'_{\rm s},
\end{aligned} \tag{13}$$

Given the approximations made already, one could also replace $m_{\rm wd} + m_{\rm com}$ with $m_{\rm wd}$ in the above equations, if desired.

9 Results

In the figures below, I vary the following parameters (see graph for annotations):

- $\alpha'\lambda = 0.1, 0.2, 0.3, 0.4,$
- $m'_{\rm p} = 1$ or 3,

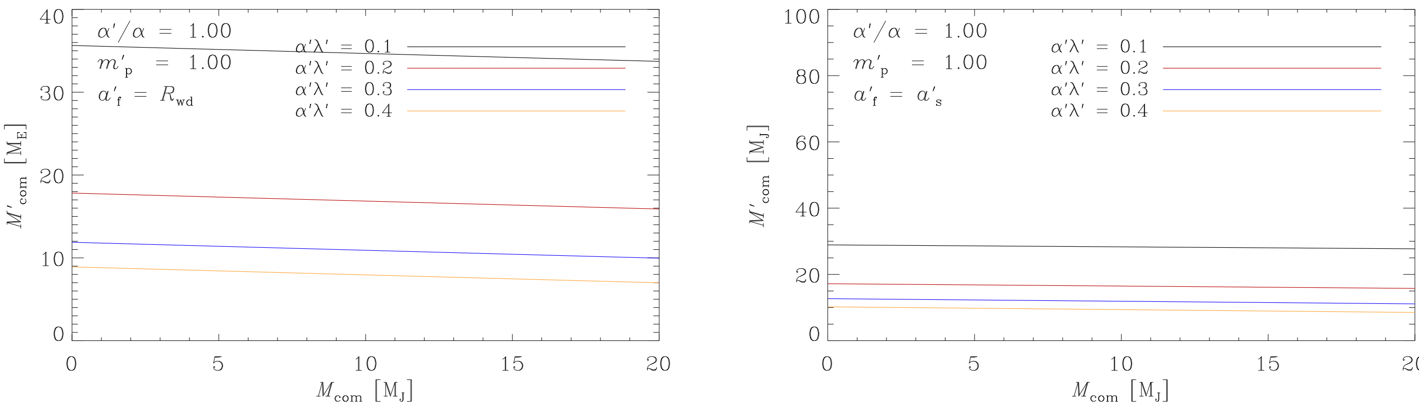


Figure 1: On the left is the case $a'_{\rm f} = R_{\rm wd}$, with $m'_{\rm com}$ in units of Earth masses, and on the right is the case $a'_{\rm f} = a'_{\rm s}$, with $m'_{\rm com}$ in units of Jupiter masses.

- $a'_{\rm f} = R_{\rm wd}$ or $a'_{\rm s}$,
- $\alpha'/\alpha = 1, 0.1, or 10$.

I assume the following parameters:

- $m_{\rm wd} = 0.52$ (observed),
- $R_{\rm wd} = 1.3 \times 10^{-3}\,{\rm R}_\odot$ (observed),
- $P_{\rm f} = 1.408\,{\rm d}$ (observed).

In addition, the observed planet radius is $R_{\rm com} = 10.4\,{\rm R_E}$.

Note that Vanderburg et al. (2020) estimate from Xu & Li (2010) that $\alpha\lambda \lesssim 0.4$, $\lesssim 2$ or $\lesssim 5$ for $m'_{\rm p} = 1$, 2 or 3, respectively. As discussed above, their estimated of the value of $\lambda\alpha$ to unbind the envelope is ~ 10 times higher in each case, so we expect β to be insensitive to the choice of $m'_{\rm p}$, if we increase $\alpha\lambda$ with $m'_{\rm p}$, as they suggest. However, if $\alpha\lambda$ is kept constant at a low value $\lesssim 0.5$, say, the higher $m'_{\rm p}$ cases give very low β.

References

Chen Z., Blackman E. G., Nordhaus J., Frank A., Carroll-Nellenback J., 2018, MNRAS, 473, 747

Vanderburg A., et al., 2020, Nature, 585, 363

Xu X.-J., Li X.-D., 2010, ApJ, 716, 114

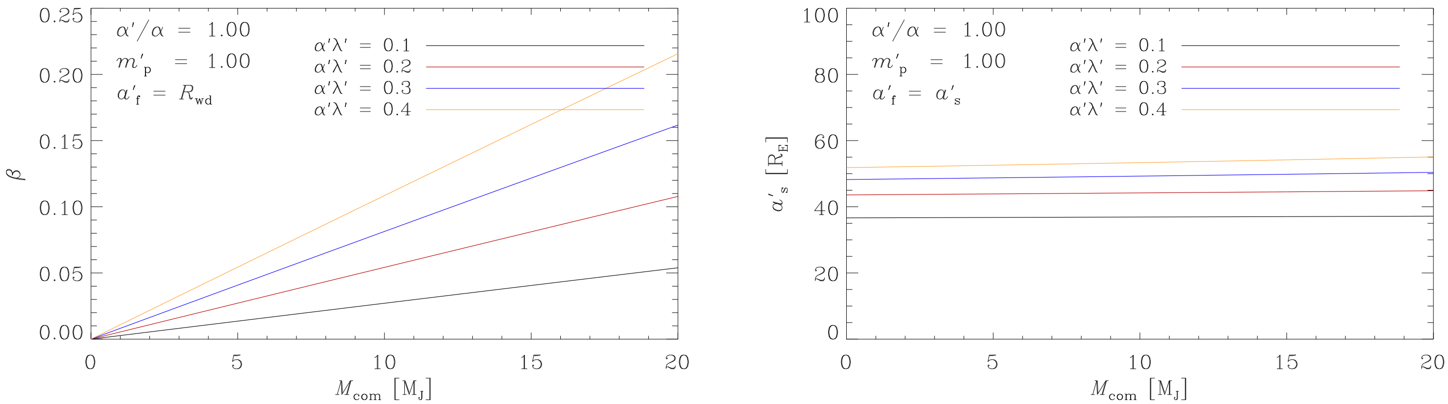


Figure 2: Note that β is identical for both cases ($a'_{\mathrm{f}} = R_{\mathrm{wd}}$ and $a'_{\mathrm{f}} = a'_{\mathrm{s}}$).

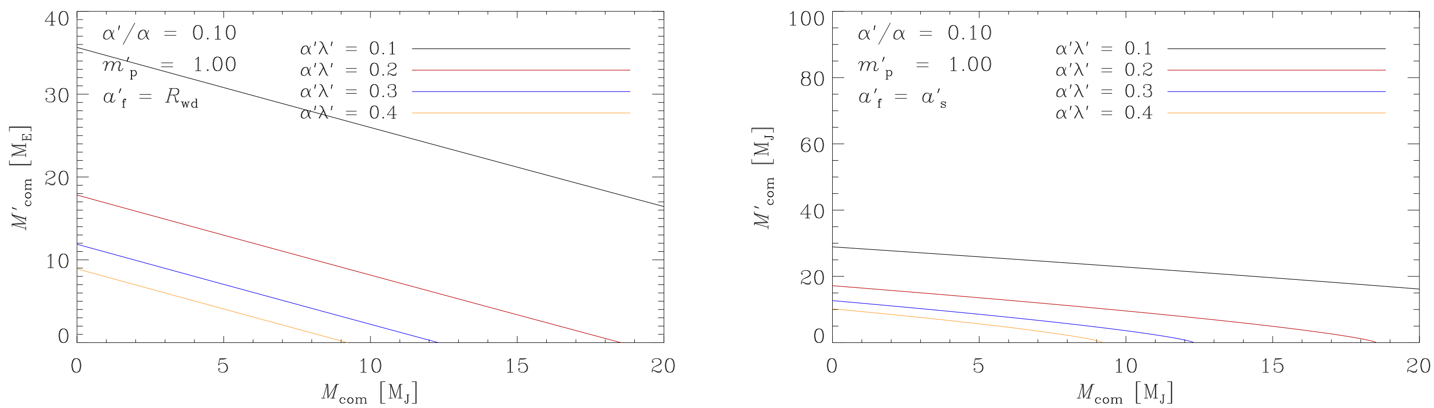


Figure 3: Now smaller value of α'/α.

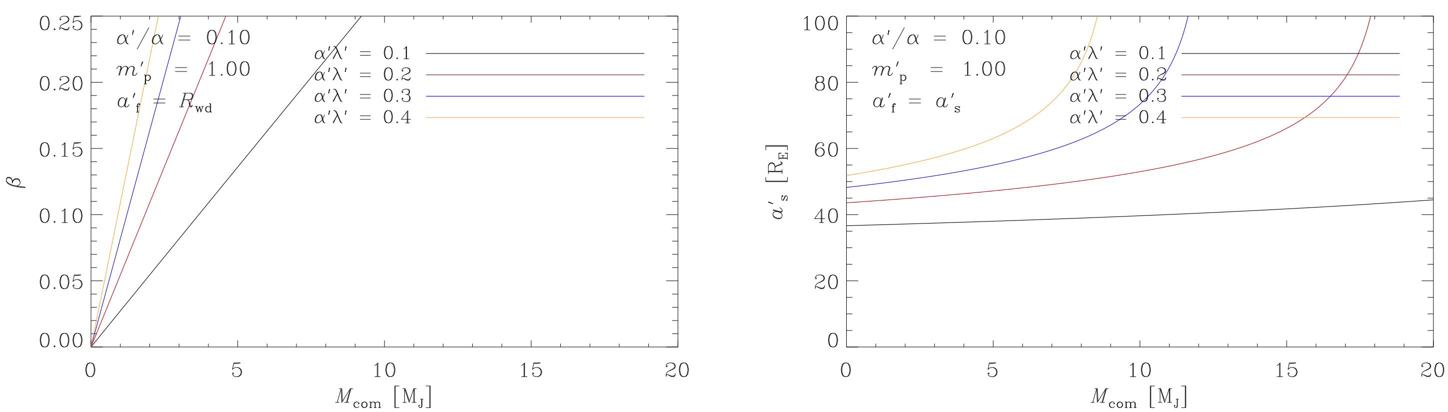


Figure 4:

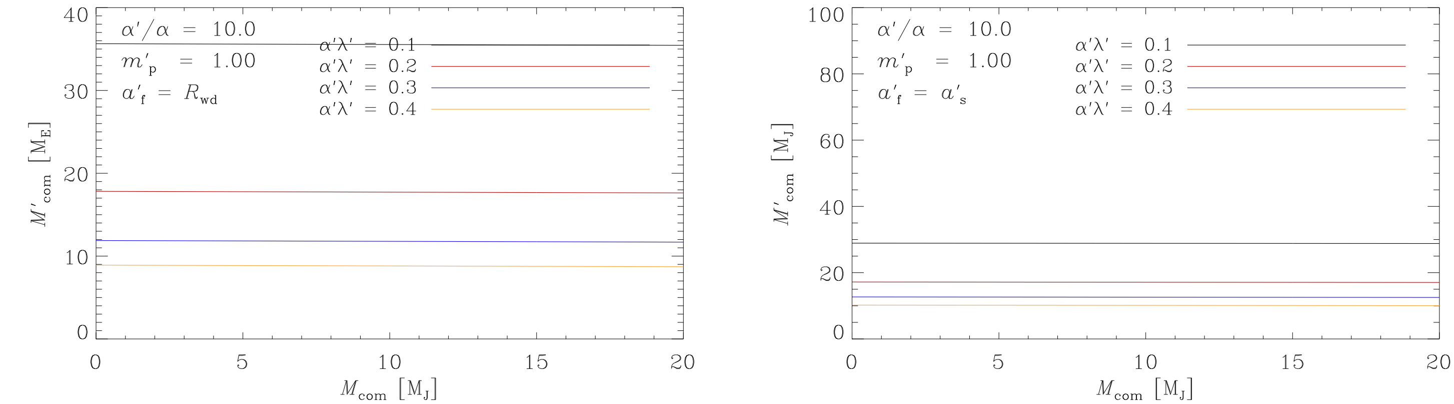


Figure 5: Now larger value of α'/α.

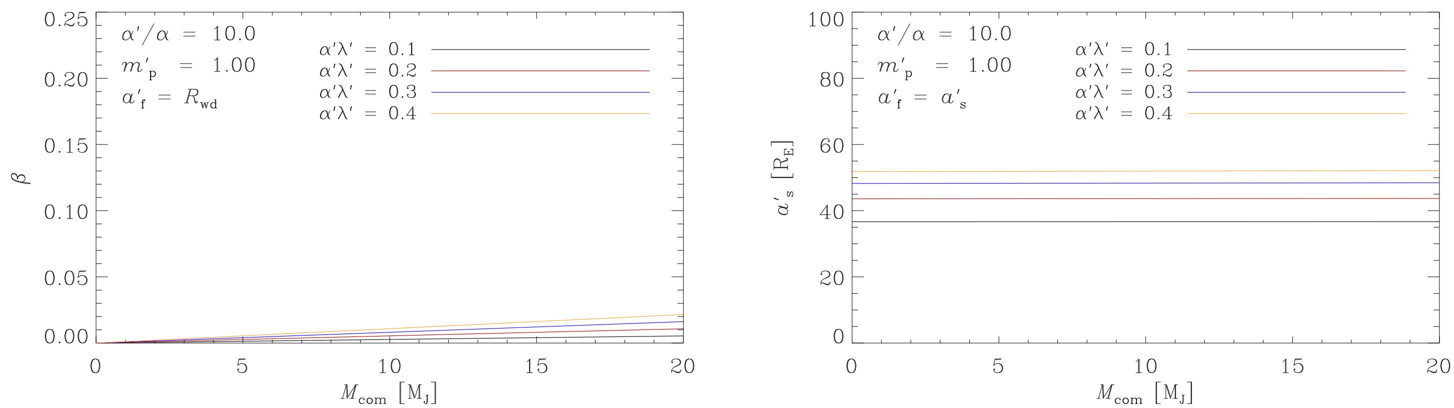


Figure 6:

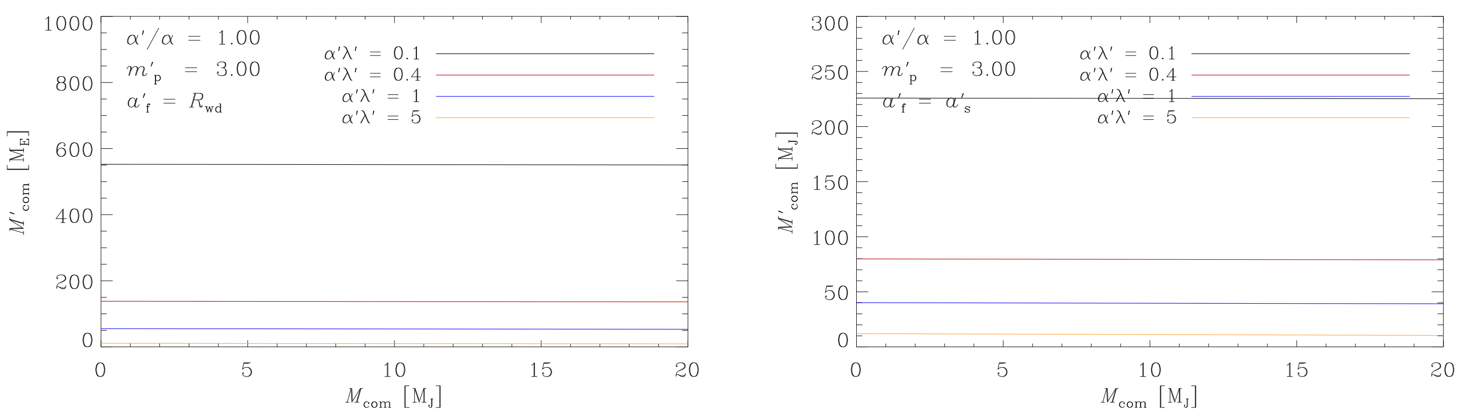
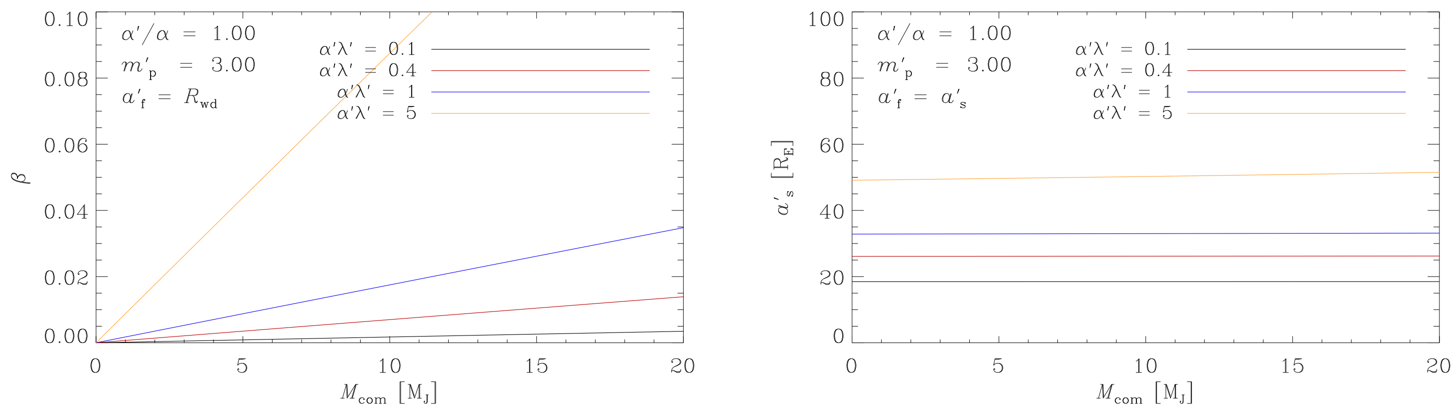


Figure 7: Now with larger value of m'_{p}.



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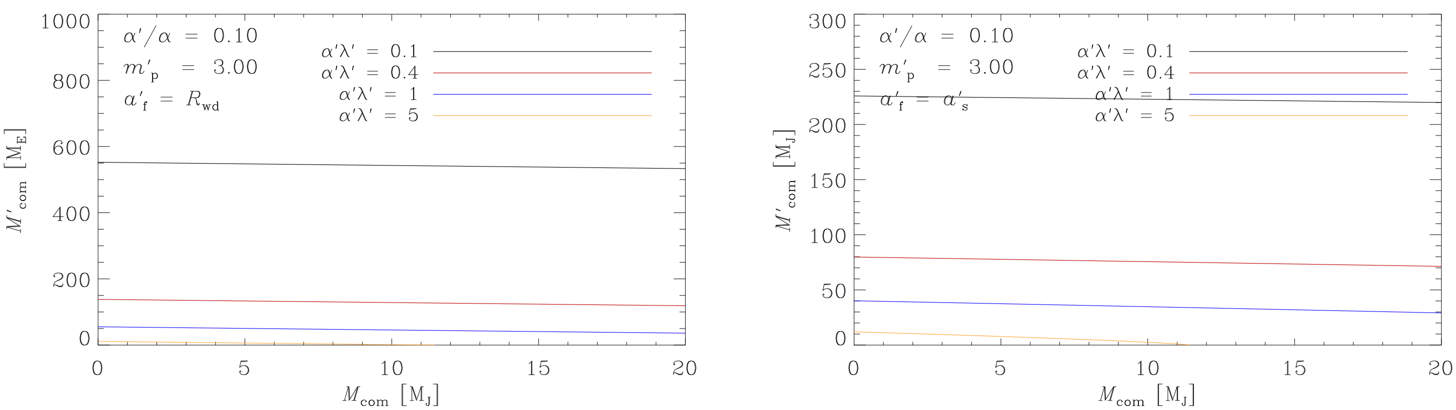

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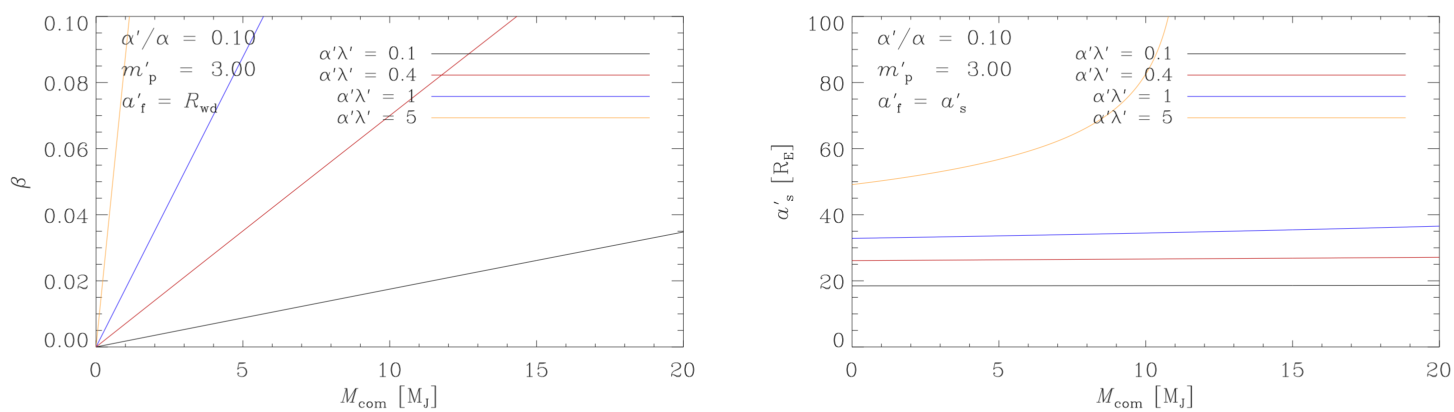

Figure 10:

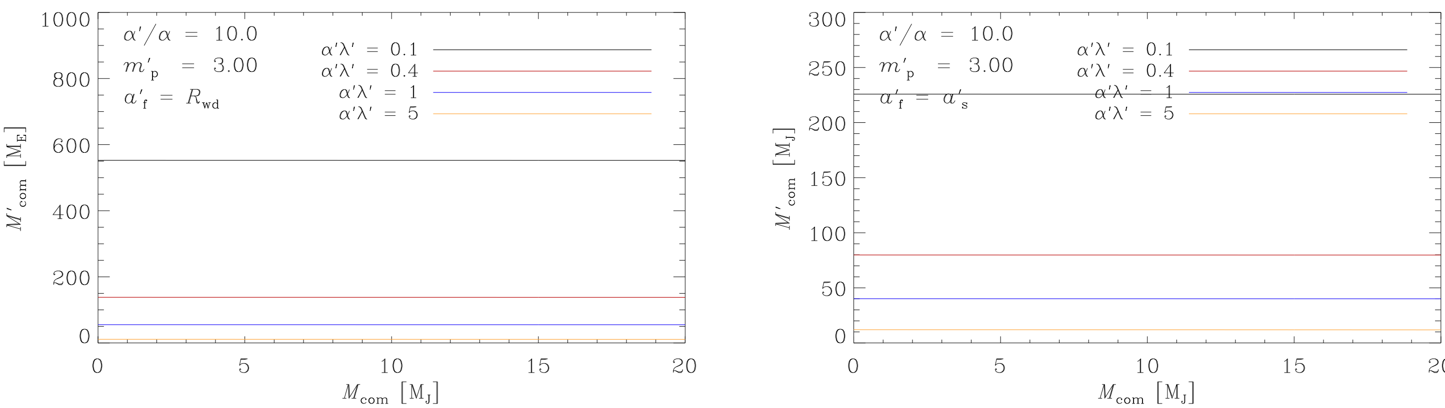

Figure 11:

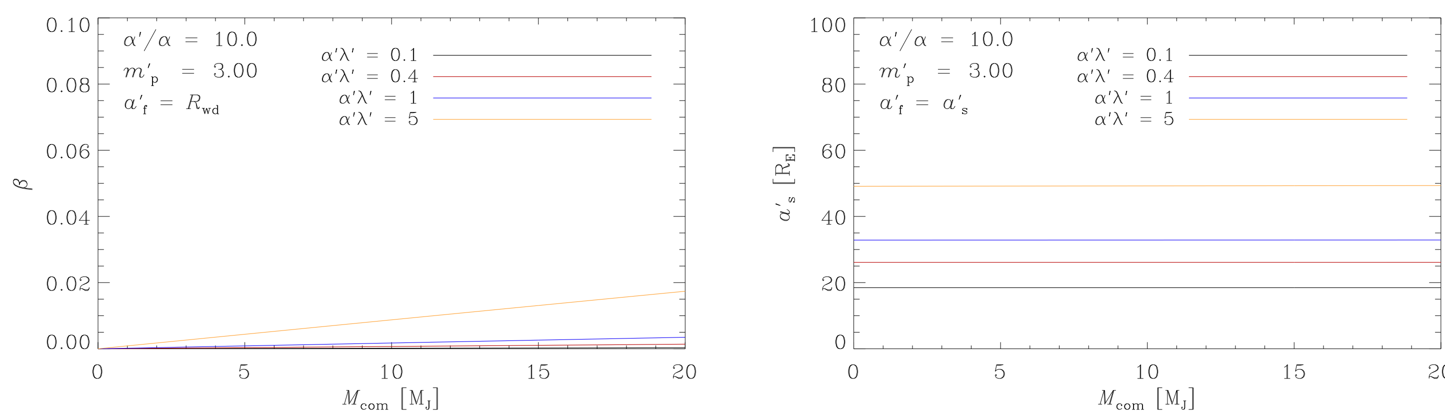

Figure 12: