Workshop 3

PHY142: Honors Introductory E&M

09-25-2013

"Within a few years a simple and inexpensive device, readily carried about, will enable one to receive on land or sea the principal news, to hear a speech, a lecture, a song or play of a musical instrument, conveyed from any other region of the globe."
- Nikola Tesla, A Means of Furthering Peace (1905)

Objective: Grad-div-curl in relation to electrostatics; Gauss's Law to find the electric field, electrostatic potential.

Writing Exercise: (This is important, physicists write all the time!) Take 10 minutes and write an e-mail to your father and tell him what you've learned thus far in PHY142 in a way that he'll have an idea of what you're talking about. You can use your notes, but do not use any math (say he studied photography in college... and hasn't seen math since HS algebra).

Problems:
1 Suppose the electric field in some region is found to be $\mathbf{E} = kr^3\hat{\mathbf{r}}$, in spherical coordinates (k a constant).

(a) Find the charge density ρ.

(b) Find the total charge contained in a sphere of radius R, centered at the origin (Do it two different ways).

2 A charge q sits at the back corner of a cube, as shown on the board (tell me to draw it please :)). What is the flux of $\mathbf{E}$ through the shaded side?

3 One of these is an impossible electrostatic field. Which one is it?
(a) $\mathbf{E} = k\left(xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3xz\hat{\mathbf{z}}\right)$, or

(b) $\mathbf{E} = k\left(y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}\right)$?

Here k is a constant with the appropriate units. For the *possible* one, find the potential, using the *origin* as your specific reference point. Check your answer by computing ∇V. [Hint: You must select a specific path to integrate along. It doesn't matter what path you choose, since the answer is path-independent, but you simply cannot integrate unless you have a particular path in mind.]

Equation Sandbox:

$$\text{For } V \text{ and } \mathbf{E}\text{: } |\mathbf{E}| = -\nabla V \quad \longrightarrow \quad V = -\int \mathbf{E} \cdot d\mathbf{l} \tag{1}$$

$$\text{For } \rho \text{ and } \mathbf{E}\text{: } \mathbf{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathbf{r}'}}{r'^2} \rho d\tau \quad \longrightarrow \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \quad \nabla \times \mathbf{E} = 0 \tag{2}$$

$$\longrightarrow \quad \oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{Q_{encl.}}{\epsilon_0}, \quad \oint_C \mathbf{E} \cdot d\mathbf{l} = 0 \tag{3}$$

$$\text{For } V \text{ and } \rho\text{: } V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho}{r'} d\tau \quad \longrightarrow \quad \nabla^2 V = -\frac{\rho}{\epsilon_0} \tag{4}$$

(1) Is simply how to get V from **E** and vice versa.

(2-3) Is how to get the **E** using some volume element (this is harder, you'd use this if there was no clear symmetry.) Then you have Gauss's law in differential (2) and integral form (3), then the curl of **E** for electrostatics in differential (2) and integral form (3). Both of these last equations are the first two Maxwell's equations. Use Gauss's law if you have nice symmetry (cylindrical, spherical, pillbox).

(4) This last differential equation you don't need to worry about till PHY217, it is called Poisson's equation, if $\rho \to 0$, then it is Laplace's Equation. It does not hurt to see it now.