Workshop 7

PHY142: Honors Introductory E&M

11-06-2013

Two coaxial cylinders with an inner radius a and outer radius b, illustrated in Figure 1.

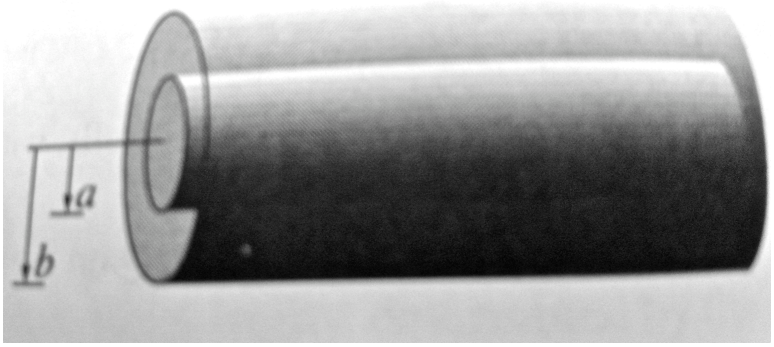

Figure 1: Cartoon depiction of two coaxial metal cylindrical tubes, of radii a and b. Picture with a crappy cellphone from D.J. Griffiths Problem §2.39, 3rd Ed.

(a) Say there is a charge on the inner cylinder, Q, for some length L. Using Gauss's law, find the electrostatic field between the two cylinders.

(b) What is the potential difference, $V = V(b) - V(a)$ between the two cylinders?

(c) As set up by the problem, which of the two cylinders (the one with radius a or radius b) has the higher potential? How does would considering this fact impact how we define V?

(d) Since $\mathbf{E}$ is proportional to Q, so is V. What is the proportionality constant between Q and V called? What are the units? Can this quantity be positive or negative, or both?

(e) For convenience, let's call this constant, C, define it for our coaxial cylinders.

(f) Okay, so I guess I will just tell you that C is capacitance. To "charge up" a capacitor, such as the one in Figure 1, we have to remove electrons from the positive plate and carry them to the negative plate. By doing this, we FIGHT against the electric field. The field wants them to go back to the positive conductor, and pushes them away from the negative one. Now: how much work does it take to charge the capacitor up to a final amount Q?

- We know that, letting $\mathbf{a}$ and $\mathbf{b}$ be arbitrary, $V(\mathbf{b}) - V(\mathbf{a}) = \frac{W}{Q}$, where W is the work done. Using this equation and the definition of capacitance, (i.e. suppose the charge on the positive plate is q, so the potential difference is q/C, how would we generally define the work you must do to transport the next piece of charge, dq?
- Integrate this quantity, and find the total work necessary when considering $q = 0$ to $q = Q$.
- Or, since $Q = CV$, how can you define the work done by a capacitor in terms of potential, V?

(g) Now, using your equations that you've just derived. What would be the work done by our cylindrical capacitors?