Topics: Anything we have previously talked about in this course, however let’s focus on Magneto-statics in the beginning of the workshop. After the questions we’ll review for the final.

A steady current $I$ flows down a long cylindrical wire of radius $a$ (Figure 1). Find the magnetic field, both inside and outside the wire, if...

![Figure 1](https://via.placeholder.com/150)

Figure 1: Figure 1. A long cylindrical wire with radius $a$. Figure taken with crappy cellphone from D.J. Griffith’s Introductory E&M textbook.

1. The current is uniformly distributed over the outside surface of the wire.
   
   (a) Will we use the Biot-Savart law or Ampère’s law? Explain why you’d use one instead of the other.

   (b) What is the $dl$ for this geometrical object? Let it have a radius $s$. Recall in Gauss’s law we have a “Gaussian surface.” When using Ampère’s law we have an “Ampérien loop.”

   (c) Now it is really important that our Ampérien loop has a different radius than that of our physical object – hence, why it has radius $s$ and not $a$, like our actual wire. What is the $B$ for $s < a$ and $s > a$? Don’t forget the direction.

   (d) What would have to be different about the statement in (a) to indicate to the reader that they wouldn’t need to find the $B$ in the two regimes stated above? Is this the most general case one could have to find the magnetic field for this geometrical construction?

2. The current is distributed in such a way that $J$ is proportional to $s$, the distance from the axis.

   (a) I know, I am a terrible person making you guys work with something that is completely out of the scope of PHY142: $J$. However I’ll help you out and we’ll simply use it for math
fun time. Recall that I previously told you that \( \mathbf{J} \) was the current density, or the electric current per unit area of cross section - it is a vector, but for the sake of this problem, let’s work with its magnitude. This implies that we can define \( I = \int_0^a J da \). Construct an expression for \( J \) based on what I told you in (b), then solve for \( I \).

- Now that you have \( J \), what is \( da \) equal to? Solve this for some arbitrary radius \( s \) from 0 to \( a \).

(b) Now that you’ve got \( I \), solve for your multiplicative constant.

(c) Oh you’ve got \( I \), but you don’t have \( I_{\text{encl.}} \). You can’t use just \( I \) for Amp. law, unless \( I_{\text{encl.}} = I \). How do we do this? Solve a similar expression to what is stated in the first bullet for all \( s \) – hence all the currents being enclosed. Hint: \( I_{\text{encl.}} = \int_0^a J da = ??? \)

(d) Simplify this expression for \( I_{\text{encl.}} \) the best you can in terms of \( I \).

(e) What is \( I_{\text{encl.}} \) for \( s < a \) and \( s > a \)?

(f) What is \( \mathbf{B} \) for \( s < a \) and \( s > a \)?