Workshop 9 – our last workshop :(

PHY142: Honors Introductory E&M

12-11-2013

Topics: Anything we have previously talked about in this course, however let's focus on Magnetostatics in the beginning of the workshop. After the questions we'll review for the final.

A steady current I flows down a long cylindrical wire of radius a (Figure 1). Find the magnetic field, both inside and outside the wife, if...

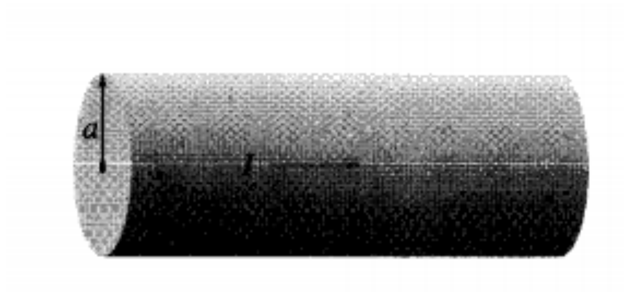

Figure 1: Figure 1. A long cylindrical wire with radius a. Figure taken with crappy cellphone from D.J. Griffith's Introductory E&M textbook.

1. The current is uniformly distributed over the outside surface of the wire.
   (a) Will we use the Biot-Savart law or Ampére's law? Explain why you'd use one instead of the other.
   (b) What is the $d\mathbf{l}$ for this geometrical object? Let it have a radius s. Recall in Gauss's law we have a "Gaussian surface." When using Ampére's law we have an "Ampérian loop."
   (c) Now it is really important that our Ampérian loop has a different radius than that of our physical object – hence, why it has radius s and not a, like our actual wire. What is the $\mathbf{B}$ for $s < a$ and $s > a$? Don't forget the direction.
   (d) What would have to be different about the statement in (a) to indicate to the reader that they wouldn't need to find the $\mathbf{B}$ in the two regimes stated above? Is this the most general case one could have to find the magnetic field for this geometrical construction?
2. The current is distributed in such a way that J is proportional to s, the distance from the axis.
   (a) I know, I am a terrible person making you guys work with something that is completely out of the scope of PHY142: $\mathbf{J}$. However I'll help you out and we'll simply use it for math

fun time. Recall that I previously told you that $\mathbf{J}$ was the current density, or the electric current per unit area of cross section - it is a vector, but for the sake of this problem, let's work with its magnitude. This implies that we can define $I = \int_0^a J da$. Construct an expression for J based on what I told you in (b), then solve for I.

- Now that you have J, what is da equal to? Solve this for some arbitrary radius s from 0 to a.

(b) Now that you've got I, solve for your multiplicative constant.

(c) Oh you've got I, but you *don't have* $I_{\text{encl.}}$. You can't use just I for Amp. law, unless $I_{\text{encl.}} = I$. How do we do this? Solve a similar expression to what is stated in the first bullet *for all* s – hence all the currents being enclosed. Hint: $I_{\text{encl.}} = \int_0^s J da = ???$

(d) Simplify this expression for $I_{\text{encl.}}$ the best you can in terms of I.

(e) What is $I_{encl.}$ for $s < a$ and $s > a$?

(f) What is $\mathbf{B}$ for $s < a$ and $s > a$?