Solutions to EXAM 1 P114 Univ. of Rochester

Spring 1999

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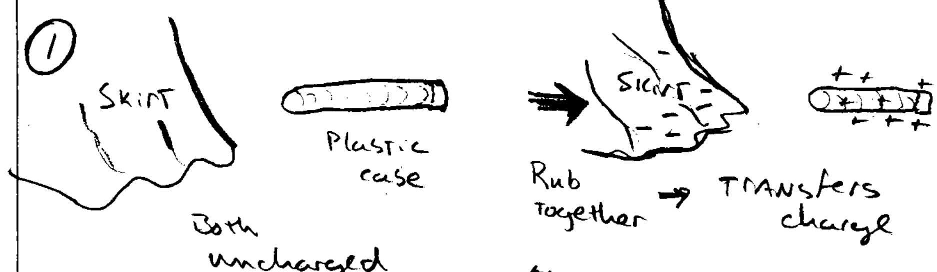
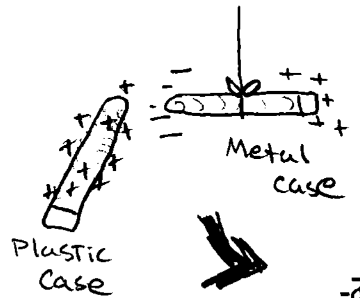
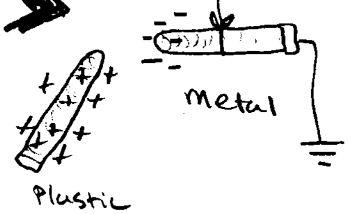
(1)

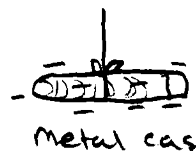




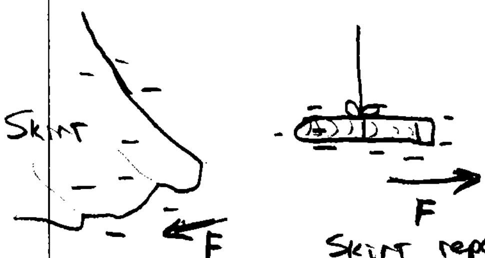
Positive charge on Plastic case induces a seperation of charge on the Metal case



Bill's Finger provides a "Path to ground" for Positive charge



Plastic case is removed, Metal case left with net negative charge



Skirt repels Metal case

(2)

use $U = \frac{1}{2}CV^2$

- Find equivalent capacitance for config I

$$C_I = C_1 + C_2 + C_3 + C_4 = 12\,\mu F$$

$$U_I = \frac{1}{2}(12\,\mu F)(2\text{ volts})^2 = 24\,\mu\text{ Joules} = 24 \times 10^{-6}\text{ J}$$

- Find equivalent Capacitance for Config II

Top, Bottom Paths each $\frac{1}{C_{T,B}} = \frac{1}{C_{1,4}} + \frac{1}{C_{3,2}} = \frac{C_3 + C_1}{C_1 C_3}$ $\quad C_{T,B} = \frac{5}{6}\,\mu F$

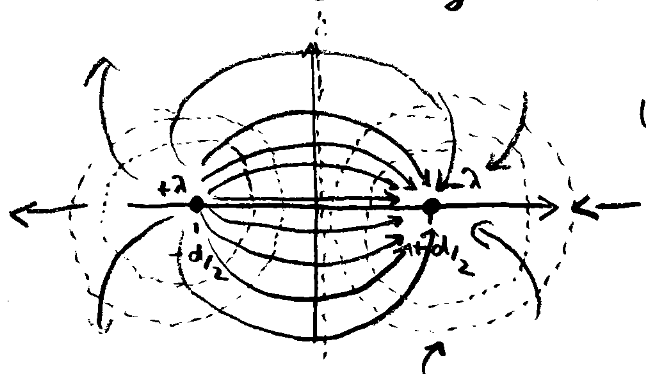
$$C_{II} = C_T + C_B = \frac{5}{6}\,\mu F + \frac{5}{6}\,\mu F = \frac{5}{3}\,\mu F$$

$$U_{II} = \frac{1}{2}\left(\frac{5}{3}\,\mu F\right)(2\text{ volts})^2 = \frac{10}{3}\,\mu\text{ Joules} = (3.33) \times 10^{-6}\text{ J}$$

→ Use Configuration (I), The stored energy is ~7 × that of Config (II)

(3) Due to Symmetry $E_z = 0$, i.e. $\bar{E}$ confined to planes parallel To xy plane

(a)

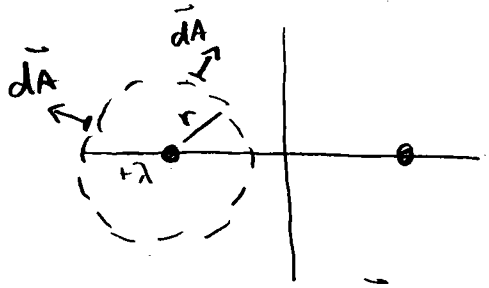


lines of force in plane parallel To xy plane look like Those of a dipole formed from point charges

equipotential lines } always $\perp$ to Lines of force

Everything projects in z

(b) Calculate force of one line on the other
Calculate E field of one line (say $+\lambda$) at other (say $-\lambda$)



Draw cylindrical Gaussian surface around line of charge

$$\int E \cdot dA = \frac{Q_{encl}}{\epsilon_0}$$

By Symmetry $\vec{E}$ is radial, $\perp$ to round surface of cylinder
$\vec{E}$ and $d\bar{A}$ are $\perp$ on endcaps of cylinder
Assume cylindrical Gaussian surface is L in length

Gauss's Law gives $\quad |E|\, 2\pi r L = \dfrac{\lambda L}{\epsilon_0} \implies |E| = \dfrac{\lambda}{2\pi\epsilon_0 r}$

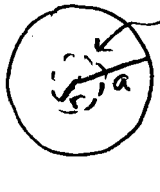
direction of $\vec{E}$ at $-\lambda$ line ($x = +d/_2$) due to $+\lambda$ line ($x = -d/_2$) is in the $+x$ direction

Force per unit Length $= \dfrac{F \text{ on } -\lambda \text{ line due to This } \vec{E}}{L} = \dfrac{q}{L}\vec{E} = (-\lambda)\left(\dfrac{\lambda}{2\pi\epsilon_0 r}\hat{x}\right)$

$\therefore \left|\dfrac{\vec{F}}{L}\right|$ of one line on other $= \dfrac{\lambda^2}{2\pi\epsilon_0 r}$

The force is attractive, i.e. both lines are pulled Toward the middle ($x=0$)

(4) $r < a$ $\qquad \oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{encl}}{\epsilon_0}$



Gaussian surface

$$|E|\, 4\pi r^2 = \int_0^r \frac{\rho\, dv}{\epsilon_0} = \frac{1}{\epsilon_0}\int_0^r \frac{Q}{r^2}\, 4\pi r^2 dr = \frac{4\pi Q}{\epsilon_0}\int_0^r dr$$

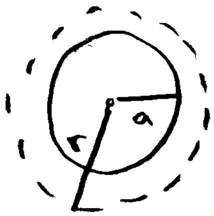
$dV = 4\pi r^2 dr$

$$= \frac{Q}{\epsilon_0} 4\pi r$$

use spherical symmetry

$$\boxed{E_{r<a} = \frac{Q}{r\epsilon_0}\hat{r}}$$

$r > a$ $\qquad \oint E \cdot dA = \dfrac{Q_{encl}}{\epsilon_0}$



$$|E|\, 4\pi r^2 = \frac{1}{\epsilon_0}\int_0^a \frac{Q}{r^2} 4\pi r^2 dr = \frac{4\pi Q}{\epsilon_0}\int_0^a dr = \frac{4\pi Q a}{\epsilon_0}$$

$$\boxed{E_{r>a} = \frac{Qa}{\epsilon_0 r^2}\hat{r}}$$

note: goes as $1/r^2$ ✓

(5)

He^- Through 20 Million Volts $\rightarrow KE = 20$ MeV

He^{++} through 20 Million Volts $\rightarrow KE = (2)\,20$ MeV $= 40$ MeV

Charge of ion is +2

$\therefore$ Total KE = 60 MeV = 60 Million electron Volts