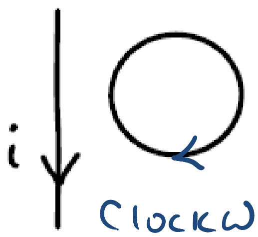
P142 University of Rochester NAME Soln key - SLM
S. Manly Fall 2010

Final Exam (December 20, 2010)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

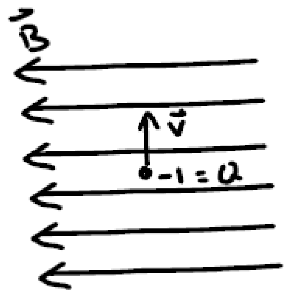
<u>Problem 1 (~~9~~ pts):</u> 12 PTS

In the following, four physical situations are sketched for you. In each case determine the direction of the force or current as requested. Your choices in each case are: zero, left, right, up, down, into paper, out of paper, clockwise, counter-clockwise.



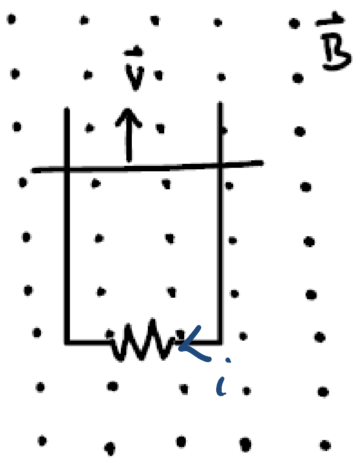
Determine the direction of the current in the loop (if any). The current i is increasing with time.

Clockwise



Determine the direction of the force on the electron moving in the magnetic field as shown.

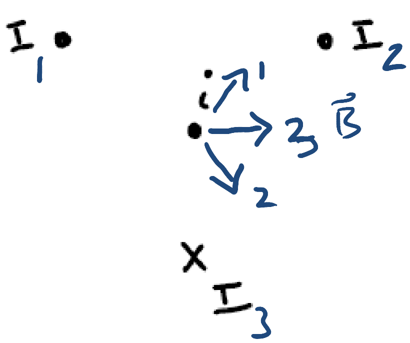
$\vec{F}$ is into paper



A rod slides on conducting rails at speed v in a magnetic field as shown. Determine the direction of the current induced in the resistor as shown (if any).

current is induced to left through resistor.

Also Accept clockwise (current around loop)



Three currents of magnitude I are situated at the points of an equilateral triangle in the plane of the paper. The currents I are perpindicular to the plane of the paper and are in the directions shown. Determine the direction of the force (if any) on a fourth current i coming out of the paper at the center of the triangle.

B at current in middle is to Right

So $\vec{F}$ on i is up (on paper) ↑

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1)	~~/9~~ 12
2)	~~/4~~ 3
3)	~~/4~~ 3
4)	~~/8~~ 7
5)	/7
6)	/8
7)	/8
8)	/8
9)	/8
10)	/8
11)	/8
12)	/10
13)	/10
tot	/100

Problem 2 (~~4~~ 3 pts):

If the resistance for a simple series RLC circuit is increased, the resonant frequency of that circuit (provided it still readily oscillates)

a) Is increased.
b) Is decreased
c) Remains unchanged
d) Insufficient information is give for a response.

Resonant frequency determined as $\frac{1}{\sqrt{LC}}$ . . .

No R dependence

Problem 3 (~~4~~ 3 pts):

A spherical balloon contains a positively charged particle at its center. As the balloon is inflated to a greater volume while maintaining the charged object at the center,

a) the electric potential at the surface of the balloon increases while the electric flux through the balloon's surface decreases.
b) the electric potential at the surface of the balloon increases while the electric flux through the balloon's surface increases.
c) the electric potential at the surface of the balloon decreases while the electric flux through the balloon's surface decreases.
d) the electric potential at the surface of the balloon decreases while the electric flux through the balloon's surface increases.
e) None of the above is correct.

Flux is unchanged

$V \sim \frac{kQ}{r}$ decreases

flux same

Problem 4 (~~8~~ 7 pts, show work):

Biff the spacefarer travels to the Centauri star system 4.5 light years away at a speed of 0.9c. How long does it take Biff to travel to the Centauri star system in Biff's frame of reference?

$$\gamma = \frac{1}{\sqrt{1-(0.9)^2}} = 2.3$$

4.5 ly is distance in Earth's frame

it is proper length for this problem

$$\underset{4.5\,ly}{d_{Earth}} = d_{Biff}\,\gamma$$

$$d_{Biff} = \frac{4.5}{2.3} = \underbrace{1.9}_{1.96}\ ly$$

Biff perceives star system to be 1.9 ly distance due to relativistic length contraction

$d = vt$

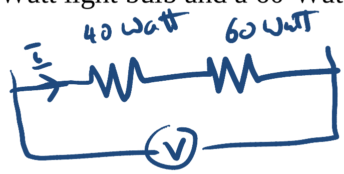
$t = d/v$

$$\frac{1.9\,ly}{0.9\,c} = \frac{1.96}{0.9} = \underline{\underline{2.17\ years}}$$

Time Biff perceives trip to take

Problem 5 (7 pts, show work):

A difference in potential is applied across the combination of a 40-Watt light bulb and a 60-Watt light bulb connected in series. The 40-Watt light bulb



a) glows more brightly than the 60-Watt light bulb.
b) glows less brightly than the 60-Watt light bulb
c) glows with the same intensity as the 60-Watt light bulb.
d) and the 60-Watt light bulb do not glow at all.

$V = IR$

Same I

$P = VI = I^2R = V^2/R$

$\frac{120\,V^2}{40\,watt} = R_{40} = 360\,\Omega$

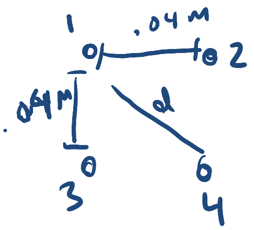
$\frac{120\,Volts^2}{60\,watt} = R_{60} = 240\,\Omega$

$P = I^2R$

More power in 40 watt bulb

Problem 6 (8 pts, show work):

How much work is required to assemble four -3.0 nC charges at the corners of a square of side 4.0 cm?



$.04^2 + .04^2 = d^2$

$d = .057\,m$

Bring in 1 $W = 0$

Bring in 2 $W = qV = (-3\,nC)(-3\,nC)\frac{k}{.04}$ 9×10^9 $= 2\,\mu J$

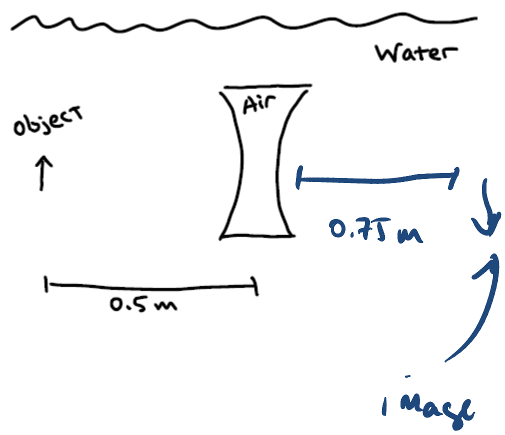
Bring in 3 $W = 2\,\mu J + \frac{(3\,nC)(3\,nC)\,9\times10^9}{.057} = 3.4\,\mu J$

Bring in 4 $W = 2 + 2 + 1.4 = 5.4\,\mu J$

Work to form full distribution $= 2 + 3.4 + 5.4 = 10.8\,\mu J$

Problem 7 (8 pts):

An object in water is 0.5 meters from a concave "air lens" with a focal length 0.3 meters. Assume the index of refraction of air is 1 and the index of refraction of water is 1.33. The air lens is a shaped cavity of air immersed in the water.



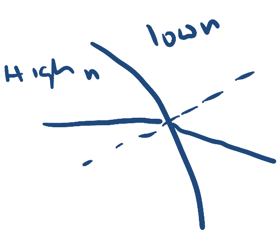
a) Is the image of the object real or virtual?

Acts as converging lens. So image will be real in this case

b) Where is the image located?

$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$

$\frac{1}{i} + \frac{1}{.5\,m} = \frac{1}{.3\,m}$



$i = 0.75\,m$ to right of Air lens

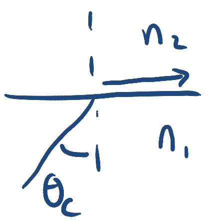
lens is converging

Problem 8 (8 pts):

a) Pure water is colorless and transparent, yet you can easily see water drops if you spill several onto a glass table. Briefly explain why this is so.

The water droplet presents a curved refractive surface. So rays of light passing through a drop are refracted causing images to seem distorted as you look at the drop.

b) Would you expect a diamond to have more or less sparkle when immersed in water relative to air? Briefly explain your answer. *Assume the thickness of the water is not a factor. That is to say assume the same amount of light is incident on the diamond in air or in water.*

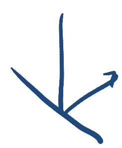


$n_1 \sin\theta_c = n_2$

diamond ↗ ↑ air or water

$\sin\theta_c = \frac{n_2}{n_1}$

as n_1 gets larger than n_2 θ_c is smaller



The smaller θ_c leads to more internal reflection inside the diamond leading to more sparkle (less light passes out the back). Relative to air, n_{water} is bigger and θ_c is larger — less sparkle.

Problem 9 (8 pts, justify your answer):

A capacitor of area A and plate separation D is fully charged across a potential difference of V and placed in series with and inductor of inductance L, causing LC oscillations to occur. While the LC oscillations are continuing in this resistance-free circuit, the distance between the capacitor plates is increased to 2D.

$C = \frac{\epsilon_0 A}{d}$

a) How will the frequency of the LC oscillations in this circuit change when the plate separation is increased to 2D?

$\omega_0 = \frac{1}{\sqrt{LC}}$

$D' \to 2D$ means $C' \to \frac{1}{2}C$

$\omega_{new} = \frac{1}{\sqrt{LC/2}} = \omega_0\sqrt{2}$

$C = \frac{\epsilon_0 A}{d}$

b) Relative to the initial situation, while the LC oscillations are continuing, how will the frequency of the LC oscillations in this circuit change if the space between the capacitor's plates are filled with a dielectric with dielectric constant K=4.

$C' \to KC = 4C$ $\omega_{New} = \frac{1}{\sqrt{L4C}} = \frac{\omega_0}{2}$

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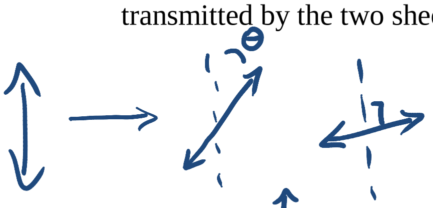
Problem 10 (8 pts, justify your answer):

A beam of polarized light is sent through a system of two polarizing sheets. Relative to the polarization direction of that incident light, the polarizing directions to the sheets are at angles θ for the first sheet and 90 degrees for the second sheet. If 10% of the incident intensity is transmitted by the two sheets, what is θ?



I_0 $\qquad I = I_0 \cos^2\theta$ $\qquad I = I_0 \cos^2\theta \underbrace{\cos^2(90-\theta)}_{\sin^2\theta}$

$I = I_0 \cos^2\theta \sin^2\theta$

From Formula sheet

$\sin 2x = 2 \sin x \cos x$

$I = I_0 \dfrac{\sin^2 2\theta}{4}$

$0.1 = \dfrac{\sin^2 2\theta}{4}$

$\boxed{\theta = 19.6^\circ}$

Problem 11 (8 pts, justify your answer):

The large radio telescope in Arecibo, Puerto Rico has been used to search for extra-terrestrial intelligence. The radio telescope has a diameter of 1000 feet = 304.8 meters. According to one of the researchers in Arecibo, the telescope can detect a signal that lays down over the surface of the Earth a power of 1 picowatt (1×10^{-12} Watts). If a signal emanating from the center of our galaxy (2.2×10^4 light years distant) were detected, what is the minimum power of the source of the signal (assuming the source radiates equally in all directions)? *The radius of the Earth is* 6.4×10^6 *m and the speed of light is* 3×10^8 *m/s.*

$$\frac{1\times10^{-12}\text{ Watts}}{\pi(6.4\times10^6)^2} = 7.8\times10^{-27}\text{ Watts/m}^2$$

Area of Disk of Earth

Area of large sphere

$$\text{Total power} = (7.8\times10^{-27}\text{ watts/m}^2)\left[4\pi(2.08\times10^{20})^2\right] = \boxed{4.2\times10^{15}\text{ Watts}}$$

$$2.2\times10^4\text{ ly} \times 365\frac{\text{day}}{\text{yr}} \times 24\frac{\text{hr}}{\text{day}} \times 60\frac{\text{Min}}{\text{hr}} \times 60\frac{\text{sec}}{\text{Min}} \times 3\times10^8\text{ m/s} = 2.08\times10^{20}\text{ m}$$

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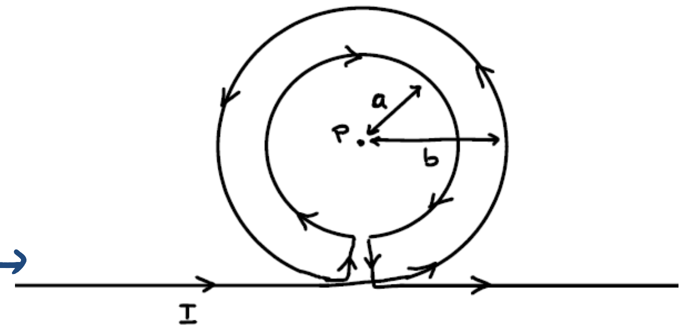
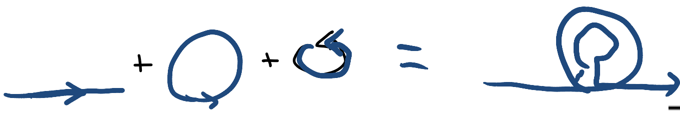
Problem 12 (10 pts, justify your answer):

An infinitely long, current-carrying wire is bent into the shape shown in the sketch below. The straight part of the wire is infinite in both directions. The circular parts of the geometry are centered on the point P. The radius of the smaller circle is "a". The radius of the larger circle is "b". Show that the magnetic field is zero at point P if a/b = π/(π+1).



Decompose problem into
$\vec{B}$ from parts

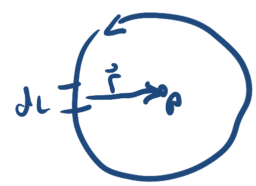


B_p from line current
use Ampere's law ↗ $2\pi b B = \mu_0 I$

$B = \frac{\mu_0 I}{2\pi b}\ \widehat{\text{out}}$

B_p from circle of radius b
use Biot-Savart

$dB = \frac{\mu_0}{4\pi} \frac{i\,d\vec{l} \times \hat{r}}{r^2}$



$B = \frac{\mu_0}{4\pi} \frac{I\,2\pi b}{b^2} = \frac{I\mu_0}{2b}\ \widehat{\text{out}}$

B_p from circle of radius a
use Biot-Savart

$B = \frac{I\mu_0}{2a}\ \widehat{\text{in}}$

B at P = 0

$$\frac{\mu_0 I}{2\pi b} + \frac{\mu_0 I}{2b} - \frac{\mu_0 I}{2a} = 0$$

$$\frac{1}{2\pi b} + \frac{1}{2b} - \frac{1}{2a} = 0$$

$$\boxed{\frac{a}{b} = \frac{\pi}{\pi + 1}}$$

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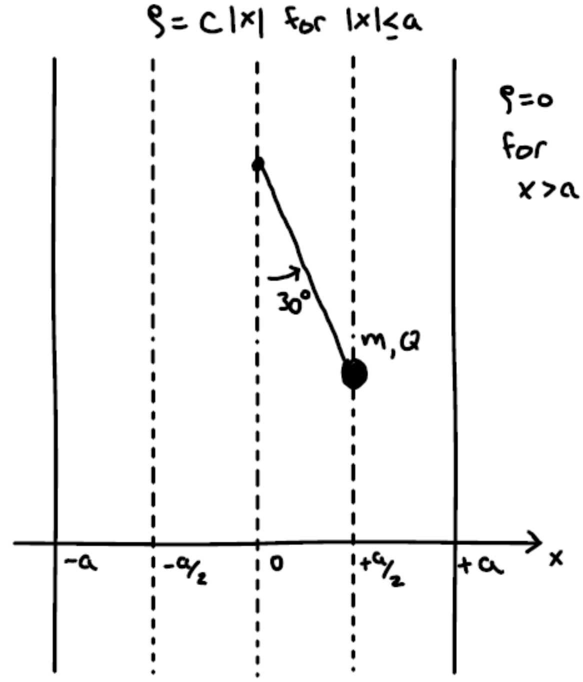
Problem 13 (10 pts, justify your answer):

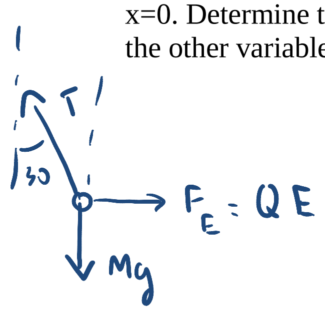
Near the surface of the Earth, a planar charge distribution is infinite in the y and z directions. It had a width of 2a in the x direction (which is horizontal) and is centered at x=0. Along the x direction, the charge density varies as

$\rho(x) = C|x|$, for $|x|\leq a$ and **$\rho(x) = 0$, for $|x|>a$,**

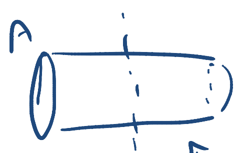
where C is a constant with units of coulombs/m^4.

A mass m with charge +Q is attached to a massless, uncharged, insulating string and is held at equilibrium at in the charge distribution at x=a/2 such that the string forms an angle of 30 degrees with the vertical axis at x=0. Determine the value of the constant C in terms of the other variables in the problem.





What is E at location of charge?



use Gauss's law

pillbox gaussian surface ends at $\pm a/2$

by symmetry $\vec{E}$ along x

$$\oint E \cdot dA = \frac{Q_{encl}}{\epsilon_0}$$

$$2|\vec{E}|A = \frac{2}{\epsilon_0} A \int_0^{a/2} \rho(x)\,dx$$

$$= \frac{2A}{\epsilon_0} C \int_0^{a/2} x\,dx$$

$$2|\vec{E}|A = \frac{2A}{\epsilon_0} C \frac{(a/2)^2}{2}$$

$$\rightarrow |\vec{E}| = \frac{C}{\epsilon_0}\frac{a^2}{8}$$

$$T_y = T\cos 30 = Mg$$

$$T_x = T\sin 30 = QE = Q\frac{Ca^2}{\epsilon_0 8}$$

$$\tan 30 = \frac{QCa^2}{\epsilon_0 8 Mg}$$

$$\boxed{C = \frac{\epsilon_0 8 Mg \tan 30}{Qa^2}}$$

Final Exam Formulas

$\vec{F} = q\vec{E}$

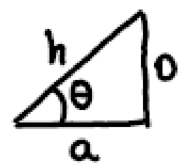
$|e| = 1.6\times10^{-19}$ coulombs

$k = 8.99\times10^{9}\ \frac{Nm^2}{C^2}$

$\vec{F} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12}$

$\epsilon_0 = 8.85\times10^{-12}\ C^2/Nm^2$

$\phi_E = \oint \vec{E}\cdot d\vec{A}$



$\sin\theta = \frac{o}{h}$ $\cos\theta = \frac{a}{h}$

$\tan\theta = o/a$

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$

Sphere: $A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

$\vec{E} = \int_{vol} \frac{k\,dQ}{r^2}\hat{r}$

cylinder: $A = 2\pi rL + 2\pi r^2$

$V = \pi r^2 L$

V = work/charge

$V_{\substack{point\\charge}} = \frac{kQ}{r}$

$V = \int_{vol}\frac{k\,dQ}{r}$

$E_s = -dV/ds$

$x' = \gamma(x - vt)$

$y' = y$

$z' = z$

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$U'_x = \frac{U_x - V}{1 - \frac{V}{c^2}U_x}$

$U'_{y,z} = \frac{U_{y,z}}{\gamma\left(1 - U_x\frac{V}{c^2}\right)}$

$E = \gamma mc^2$

$P = m\eta = m\frac{dx}{d\tau} = m\gamma v$

$$U_{capacitor} = \frac{1}{2}CV^2$$

$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

$$Q = CV$$

$$Q = Q_0 e^{-t/RC}$$

$$E_{\parallel plate} = \sigma/\epsilon_0$$

$$E = E_0/K$$

$$u_E = \frac{\epsilon_0}{2}E^2$$

$$\mathcal{E} = -d\Phi_M/dt$$

$$P = iV = i^2R = V^2/R$$

$$\Phi_M = \oint \vec{B}\cdot d\vec{a}$$

$$V = iR$$

$$\mathcal{E} = -L\,di/dt$$

$$F = q\vec{v}\times\vec{B}$$

$$\Phi = Li$$

$$|\vec{\mu}| = |niA|$$

$$u_B = B^2/2\mu_0 \qquad \mu = \mu_0(1+\chi)$$

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

$$B_{matter} = \chi B_{free}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$

$$B_{solenoid} = \mu_0 n i$$

$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$$

$$\int u^n du = \frac{u^{n+1}}{n+1}$$

$$\int \frac{du}{u} = \ln|u|$$

$$\int e^u du = e^u$$

$$\int \frac{x\,dx}{(x^2+a^2)^{1/2}} = \sqrt{x^2+a^2}$$

$$X_C = 1/\omega C$$

$$X_L = \omega L$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\bar{S} = \frac{E_0^2}{2\mu_0 c} = \frac{c B_0^2}{2\mu_0}$$

$$P = U/c$$

$$\text{Pressure} = S/c$$

$$n = c/v$$

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

$$m = -i/o$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

quadratic eqn

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin(x) = x - \frac{x^3}{3!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \cdots$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$