P113 University of Rochester NAME Soln key - SM
S. Manly Fall 2004

Exam 3 (November 30, 2004)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

Problem 1 (10 pts, justify):

Identical bowling balls are placed at the top of two identical inclined planes. The surface of one inclined plane is covered in grease so that there is no friction between the ball and the surface. The surface of the other inclined plane is dry so the ball will roll without slipping. Both balls are released simultaneously from the same height.

[✓ a)] The ball on the greased inclined plane reaches the bottom first. ↙ diff version
b) The ball on the greased inclined plane reaches the bottom second.
c) Both balls reach the bottom at the same time.
d) From the information given, it cannot determined from the information provided which ball reaches the bottom first.

Ball on greased plane reaches bottom 1ST. Both balls start w/ the same amount of potential energy. All of this energy is converted into translational kinetic energy in the case of the greased plane because with no friction there is no torque to cause rotation. In the ~~cas~~ other case the energy is shared between the translational and rotational kinetic energy during descent.

larger translational KE $\Rightarrow$ larger $\frac{1}{2}Mv^2 \Rightarrow$ larger v ↙ Reaches bottom faster

Problem 2 (10 pts, justify):

Consider a cannon shell that explodes in midair at the top of its trajectory. After the explosion, the center of mass of the shell fragments has an acceleration, in the absence of friction,

a) of less than g downward.
[✓ b)] equal to g and downward. ↙ diff version
c) greater than g and downward.
d) of g downward plus a forward component.
e) of g downward plus a backward component.

~~In the absence of external forces, the CM~~

The CM moves in response to the only external force, which is gravitation. This is true whether or not there is an explosion. Force of gravity causes the CM to accelerate downward w/ magnitude g.

The irrelevance of the explosion in this is a manifestation of the power of the C.M. concept.

Problem 3 (10 pts, no justification necessary):

changed for diff version

Consider a car that moves quickly past you from left to right, accelerating as it goes. Please state the direction of the following vectors for the right front wheel or its center-of-mass as the car's motion brings the wheel directly in front of you. Please state your answer in terms of the direction from your perspective as the problem is laid out. For each vector, your answer should be either, "no direction, magnitude is zero", "left", "right", "up", "down", "toward me", "away from me", or "cannot be determined from given information".

1)	/10
2)	/10
3)	/10
4)	/15
5)	/15
6)	/20
7)	/20
tot	/100

a) angular velocity

Away from me

b) linear momentum

To the Right

c) angular acceleration

Away from me

d) angular momentum

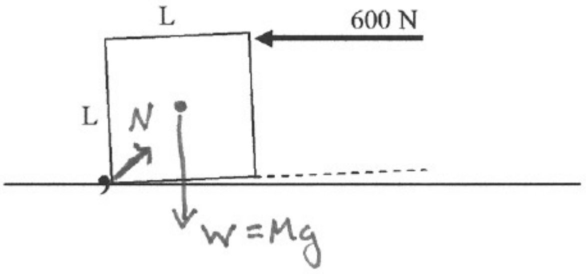
Away from me

e) torque

Away from me

Problem 4 (15 pts, show your work):

A box rests on a horizontal surface with one edge up against a small ridge in the floor as shown in the sketch below. If the horizontal force of 600 Newtons on the upper right corner of the box is just sufficient to lift the right edge of the box, what is the weight of the box? (*Assume the angle the box bottom makes with the floor is negligibly small. Also assume the box is a cube with a uniform mass distribution.*)



$\Sigma \tau$ abt lower left corner $= 0$

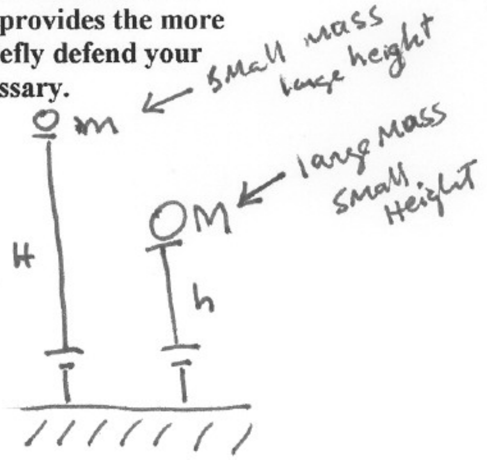
$$0 = 600\,L - W\,\tfrac{1}{2}L$$

$$600\,L = \frac{W}{2}L$$

$W = 1200$ Newtons

Problem 5 (15 pts):

You are a medical consultant to a construction company. You decide that it would be valuable to minimize the number of blows the workers do with hammers. So, you set out to determine the most efficient way to hammer a nail. You design an experiment whereby a hammer is hoisted in the air and dropped on a nail. You consider two cases, a small hammer raised very high and a large hammer raised to a smaller height. If the same amount of work is done in lifting each mass to its respective drop point and there is no rebounding when each hammer strikes its respective nail, which case provides the more efficient driver (in terms of number of strokes) of the nail? Please briefly defend your answer below using sketches, equations, and the written word as necessary.



What is the same in each case is the amount of Potential Energy to start with. Thus, after descent, the amount of KE is the same in each case (due to energy conversation).

Thus $mgH = Mgh$ AND $\frac{1}{2}mV_m^2 = \frac{1}{2}MV_M^2$

What causes the Force on the nail during the collision is the change in momentum of the nail which is equal and opposite to that of the hammer. The momentum of each hammer is ≈ zero after the collision. The hammer with the largest momentum at impact will be the one that delivers the largest force. The one that delivers the largest force will enable the driving of a nail with the fewest strokes.

$$\frac{1}{2}mV_m^2 = \frac{1}{2}MV_M^2$$

$$P_m V_m = P_M V_M$$

with $V_m > V_M$

m drops from H $\frac{1}{2}mV_m^2 = mgH$

$\therefore V_m = \sqrt{2gH}$

M drops from h

$\therefore V_M = \sqrt{2gh}$

$H > h \Rightarrow V_m > V_M$

$$\boxed{P_m < P_M}$$

∴ The large hammer has the largest momentum at impact. Thus the large hammer will impart the largest impulse / Force during the collision. A large hammer lifted a small distance is a more efficient driver of a nail.

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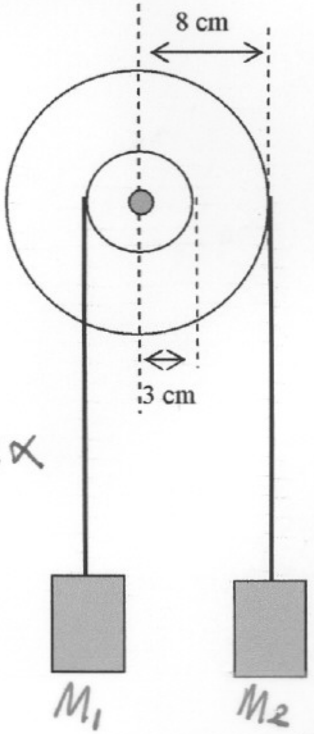
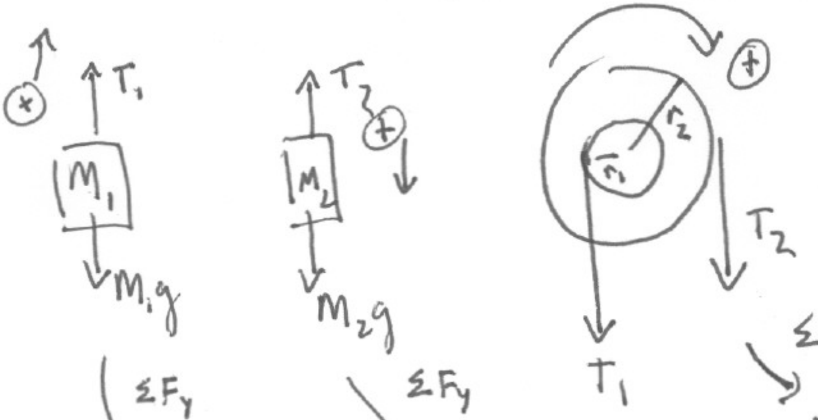
Problem 6 (20 pts, show your work):

Two blocks are attached to massless ropes which are wrapped around spools of different radii. The two spools share the same (frictionless) axis. The spools are rigidly attached to one another so that they rotate together. Each block has a mass of 5 kg. The combined spool has a moment of inertia of 12 kg-m^2. If this system is released from rest, determine the tension in each rope and the acceleration of each mass.





ΣF_y $\qquad$ ΣF_y $\qquad$ $\Sigma \tau$

$-r_1 T + r_2 T_2 = I\alpha$

$m_1 a_1 = T_1 - m_1 g \qquad m_2 a_2 = m_2 g - T_2$

Must relate a_1, a_2, α

α is same for both spools $\qquad s = r\theta, \; a = r\alpha$

$$\frac{a_1}{r_1} = \frac{a_2}{r_2} = \alpha$$

Put all eqns in terms of α, find α, T_1, T_2

$m_1 r_1 \alpha = T_1 - m_1 g$ ① $\qquad$ 3 eqns, 3 unknowns solve

$m_2 r_2 \alpha = m_2 g - T_2$ ②

$r_2 T_2 - r_1 T_1 = I\alpha$ ③

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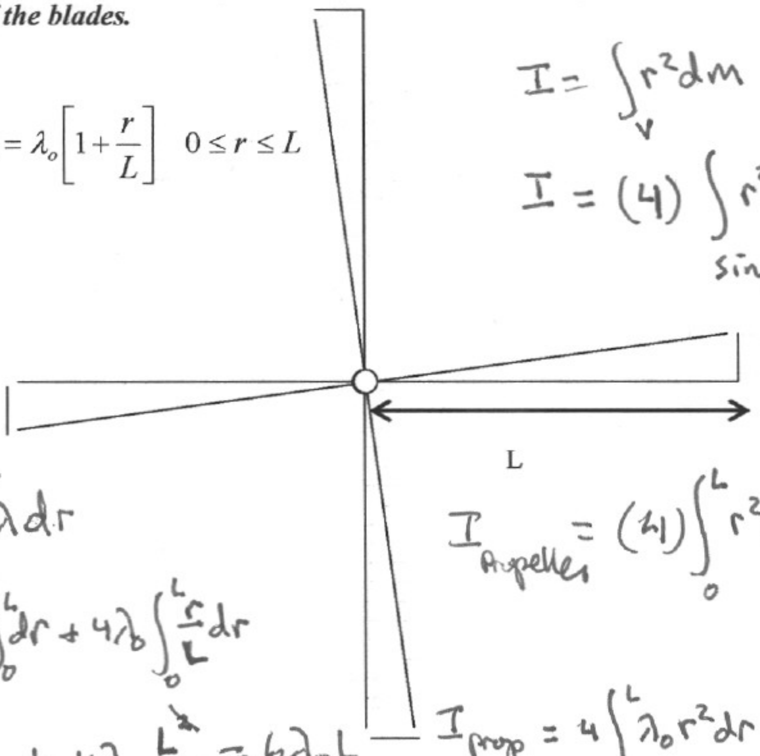
Problem 7 (20 pts, show your work):

Whoosh! Graduation comes and goes. After blowing your last dollar during your post-graduation celebratory trip to Webster, reality sinks in and you decide to get a job. Ce la vie. Given your AMAZING physics skills, you land a job in the airplane design division of Al's Aerospace Emporium. How cool is that?!

For your first assignment, Al asks you to calculate the moment of inertia of a new airplane propeller design. He supplies you with the sketch shown below of the propeller and an equation (also given below) for the linear mass density of each propeller blade as a function of distance, r, from the propeller axis. Determine an expression for the moment of inertia of the propeller in terms of the total mass M of the propeller and the length of each blade L. ***Ignore the moment of inertia of the small circular part in the center that holds the end of each of the blades.***

$$\lambda = \lambda_o\left[1+\frac{r}{L}\right] \quad 0 \le r \le L$$



$$I = \int_V r^2 dm$$

$$I = (4)\underbrace{\int r^2 dm}_{\text{single Blade}} = (4)\int_0^L r^2 \rho(r)\, dr$$

one for each Blade

$$M = (4)\int_0^L \lambda\, dr$$

$$I_{\text{Propeller}} = (4)\int_0^L r^2 \rho(r)\, dr = (4)\int_0^L r^2 \lambda_0\left[1+\frac{r}{L}\right] dr$$

$$M = 4\lambda_0\int_0^L dr + 4\lambda_0\int_0^L \frac{r}{L}\, dr$$

$$M = 4\lambda_0 L + 4\lambda_0 \frac{L^{\cancel{2}}}{2\cancel{L}} = 6\lambda_0 L$$

$$\therefore \quad \lambda_0 = \frac{M}{6L}$$

$$I_{prop} = 4\int_0^L \lambda_0 r^2 dr + (4)\int_0^L \lambda_0 \frac{r^3}{L} dr$$

$$= 4\lambda_0 \frac{r^3}{3}\Big|_0^L + 4\lambda_0 \frac{r^4}{4L}\Big|_0^L$$

Intermediate Answer in terms of λ_0

$$I_{prop} = 4\lambda_0\frac{L^3}{3} + 4\lambda_0\frac{L^3}{4} = \boxed{\lambda_0 L^3 \frac{7}{3} = I_{prop}}$$

$$I_{prop} = \frac{7}{3}\lambda_0 L^3 = \frac{7}{3}\frac{M}{6L}L^3$$

Asked for Answer in terms of M, L not λ_0, L

$$\boxed{I_{prop} = \frac{7}{18} M L^2}$$

Answer to problem