P113 University of Rochester NAME ~~Soln key~~ Soln key – SLM
S. Manly Fall 2004

Final Exam (December 17, 2004)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

Problem 1 (7 pts, show your work):

put on vector symbols

Consider two vectors:

A, with components $A_x = 4$, $A_y = 9$, $A_z = -3$

B, with components $B_x = -3$, $B_y = 2$, $B_z = 8$

a) What is the magnitude of vector $\vec{A}$?

$$|\vec{A}| = \sqrt{4^2+9^2+3^2} = 10.2$$

b) What is the magnitude of vector $\vec{B}$?

$$|\vec{B}| = \sqrt{3^2+2^2+8^2} = 8.8$$

$\vec{A}+\vec{B}$

c) What is the resultant of the two vectors?

$\vec{A}+\vec{B}=\vec{R}$ $\quad R_x = A_x+B_x = 1 \quad R_z = 5$

$R_y = A_y + B_y = 11 \quad \vec{R} = (1, 11, 5)$

"–" hard to see

d) What is $\vec{A}$-$\vec{B}$?

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = -12+18-24 = -18$$

e) What is the dot product of the two vectors?

$(\vec{A}-\vec{B})_x = A_x - B_x = 7 \quad (\vec{A}-\vec{B})_z = -11$

$(\vec{A}-\vec{B})_y = A_y - B_y = 7$

f) What is the opening angle between the two vectors?

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \quad -18 = (7.2)(8.8)\cos\theta$

$\theta = 106.5°$

Magnitude

g) What is the cross product of the two vectors?

$$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta = (\cancel{7.2}\ 10.2)(8.8)\sin(106.5) = \cancel{60.7}\ 86$$

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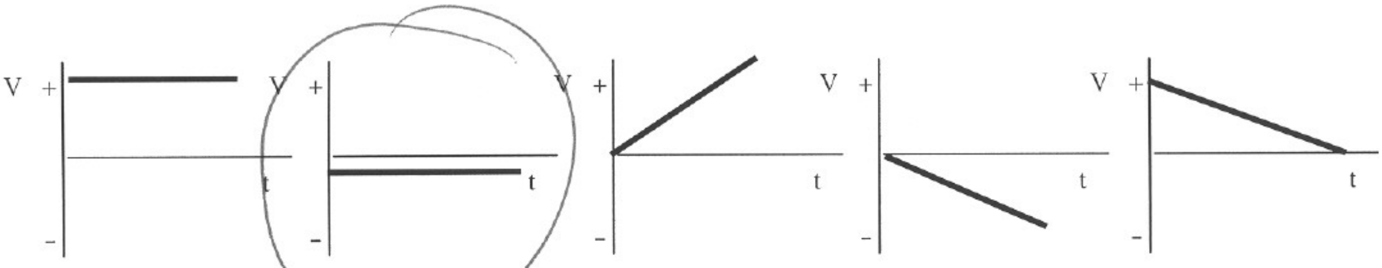
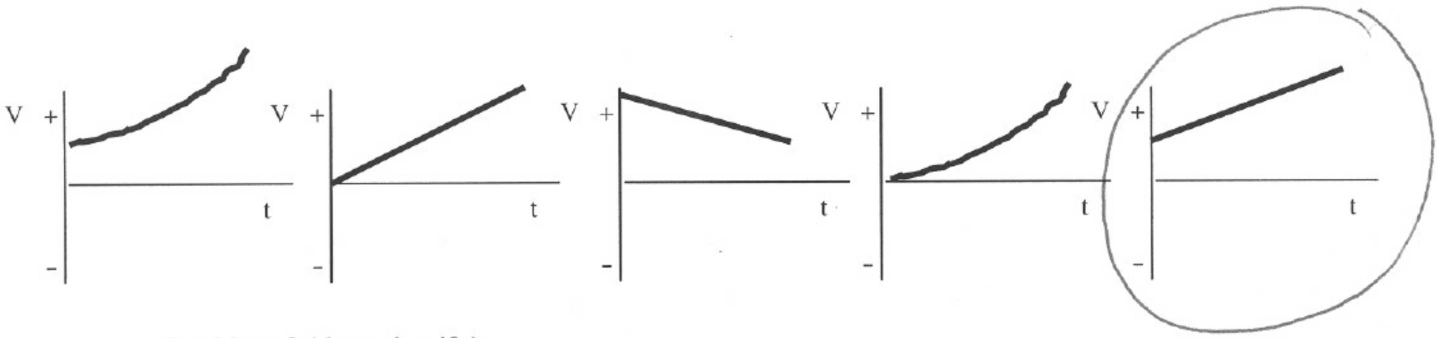
Problem 2 (6 pts, no justification necessary):

Circle the graph of v versus t does the particle end up closest to its starting point?



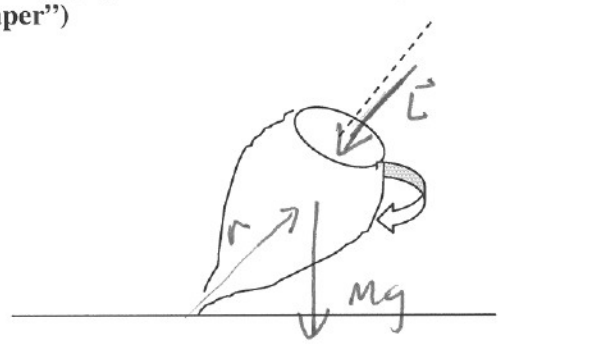
A car accelerates uniformly from a velocity of 10 km/hr to 30 km/hr in one minute. Circle the v versus t graph that best describes the motion of this car. (Ignore the jitter in the curves. That is a software artifact.)



Problem 3 (6 pts, justify):

In the sketch below the spinning top rotates about the axis shown. Think of the axis shown as being in the plane of the paper. The sense of rotation is such that points on the right edge of the top are coming out of the paper toward you and points on the left edge of the top are moving into the paper. State in what direction you think the top will precess and why. (Directions should be one of "into the paper", "out of the paper", "clockwise in the plane of the paper", "counterclockwise in the plane of the paper")

"out of paper"



$\vec{\tau}$ $\vec{r} \times M\vec{g}$ into paper

$\vec{\tau} = \frac{d\vec{L}}{dt}$

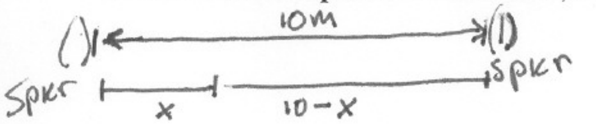
$\therefore$ $\vec{L}$ goes into paper

This happens if Top precesses out of page

Problem 4 (7 pts, show your work):

1)	/7
2)	/6
3)	/6
4)	/7
5)	/6
6)	/9
7)	/7
8)	/6
9)	/8
10)	/7
11)	/10
12)	/8
13)	/10
14)	/3
tot	/100

The famous rock star, Axle Ross, walks in a straight line between two speakers that are driven by the same amplifier. The speakers are separated by 10 meters. Each speaker emits a sound with a frequency of 300 Hz. The speed of sound in air is 340 m/s. At what positions, relative to the location of one of the speakers, will Axle find minima in the intensity of the sound (corresponding to destructive interference)? That is to say, as Axle walks from one speaker to the other, where will he encounter destructive interference?



$v = \nu\lambda$

$340 = 300\lambda \Rightarrow \lambda = 1.13\,m$

Get destructive interference when

$(10-x) - x = (n+\frac{1}{2})\lambda, \; n = 0,1,2\ldots$

$10 - 2x = (n+\frac{1}{2})\lambda$

$\frac{10-(n+\frac{1}{2})\lambda}{2} = x = \frac{10-(n+\frac{1}{2})1.13}{2}$

$n=0 \quad x = 4.7\,m$

$n=1 \quad x = 4.15\,m$

$n=2 \quad x = 3.6\,m$

$n=3 \quad x = 3.0\,m$

$n=4 \quad x = 2.45$

$n=5 \quad x = 1.9$

$n=6 \quad x = 1.3$

$n=7 \quad 0.76$

$n=8 = 0.2\,m$

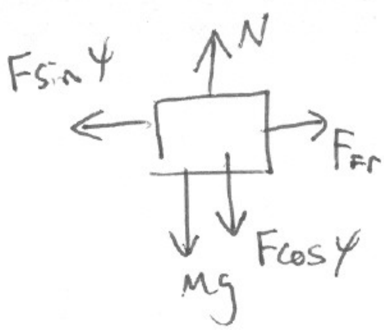
~~symmetric~~ symmetric about $x = 5\,m$

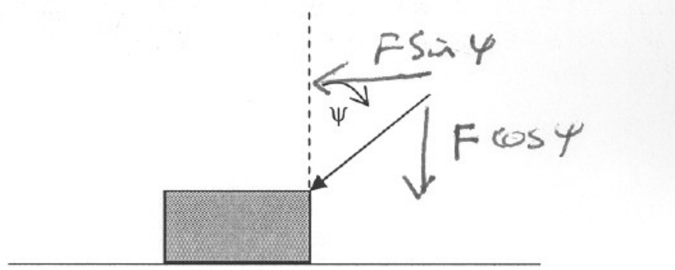
~ Every 0.55 - 0.6 m

Problem 5 (6 pts, show your work):

A block of mass m is pushed across a rough floor at a constant velocity by a force F. The coefficient of kinetic friction between the block and the floor is μ_k. The magnitude of the frictional force is

a) $\mu_k mg$
b) $\mu_k F\cos\psi$
c) $\mu_k F\sin\psi$
d) $\mu_k(mg - F\cos\psi)$
e) $\mu_k(mg - F\sin\psi)$
f) $\mu_k(mg + F\cos\psi)$
g) $\mu_k(mg + F\sin\psi)$





$|F_{fr}| = \mu_k N$

$\Sigma F_y = N - mg - F\cos\psi = 0$

$N = mg + F\cos\psi$

$|F_{fr}| = \mu_k(mg + F\cos\psi)$

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Problem 6 (9 pts, show your work):
On Christmas morning, young David was excited to see that Santa had left a bow and a set of arrows for him under the tree. In accordance with his position as president of the school science club, young David immediately set out to measure the parameters of his new bow. He found that at the moment of release each arrow moves at 24 m/s.

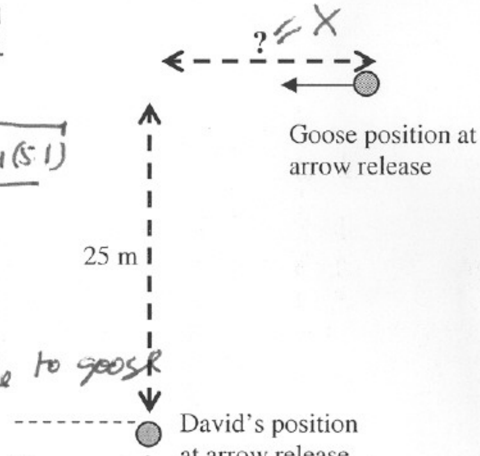
Part (a): (2 pts) How high can David's new bow shoot an arrow? ***(Assume David's height is negligible compared to the height of the arrow flight.)***

$$v^2 = v_0^2 + 2a(y - y_0)$$

$$0 = 24^2 - 2(9.8)(y) \qquad y = 29.4\text{ m}$$

Wanting to help out with Christmas dinner, young David goes out into his backyard to shoot a goose with his new bow. He spots a goose flying directly toward him at a height of 25 meters. David determines that the goose is flying at 7 m/s.

Part (b): (3 pts) If David's arrow is to fly straight up and hit the goose, at what point in the goose's flight should David release the arrow? That is to say, how far from being directly overhead should the goose be when the arrow is released? (You want to calculate the distance signified by "?" in the sketch below.)



$x = V_{goose} t$

$x = 7t$

$t \equiv$ Flight time to goose

$$y = y_0 + V_{0y} t + \tfrac{1}{2} a t^2$$

$$25 = 0 + 24t - \frac{9.8}{2} t^2$$

$$t^2 - 4.9t + 5.1 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{4.9 \pm \sqrt{4.9^2 - 4(5.1)}}{2}$$

$$t = \frac{4.9 \pm 1.9}{2}$$

$t = 1.5\text{ s}$ ← Time to goose
or
6.8 s ← Time it would take to be at 35 m on way down.

$x = (7)(1.5)$

$x = 10.5\text{ m}$

Unfortunately, David has not taken physics yet, so he missed the goose. But, he does not give up. You see, David's mom and dad are quite worried about homeland security and the such. So, they gave young David a model SA280 ground-to-air missile for Christmas. Saying something about "not only getting a goose, but cooking it too", David takes his missile out back to shoot down dinner. The missile specs state that the SA280 can accelerates straight upward with a(t)= Kt, where K is a constant with a value of 5 m/s^3. Again, David sees a goose flying directly toward him at 7 m/s at a height of 25 m.

a NOT CONSTANT

Part (c): (4 pts) As in part (b), determine for this case at what point in the goose's flight should David fire the missile?

$$v = \int_0^t a\,dt \qquad y(t) = \int_0^t v\,dt$$

$$v = \int_0^t Kt\,dt \qquad y(t) = \int_0^t \frac{Kt^2}{2}\,dt$$

$$v = \frac{Kt^2}{2} \qquad y(t) = \frac{Kt^3}{6}$$

$$25 = \frac{5t^3}{6} \qquad t = 3.1\text{ s}$$

$x = V_{goose} t$

$x = (7)(3.1)$

$x = 21.7\text{ m}$

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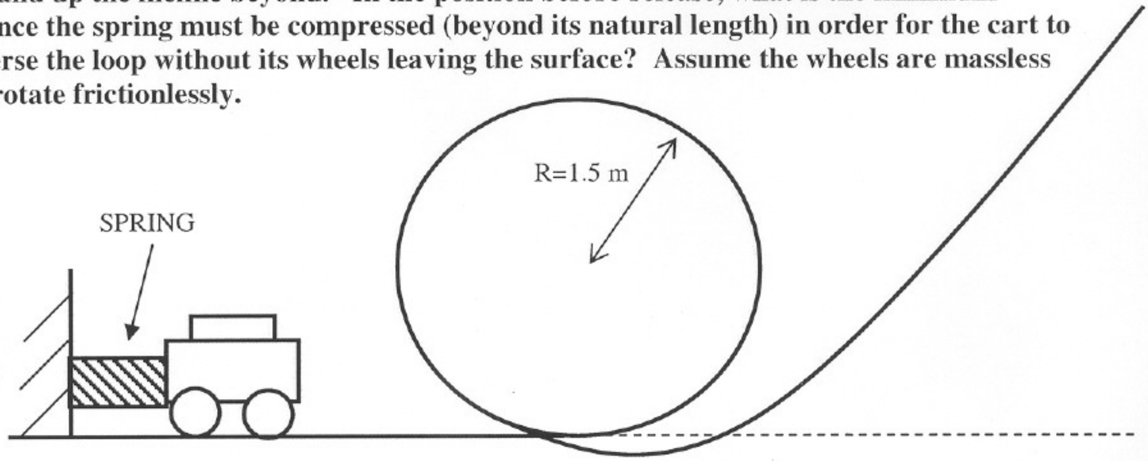
Problem 7 (7 pts, show your work):

As shown below schematically, a cart of mass 20 g is held against a compressed spring with spring constant k= 50 N/m. When the spring is released, the car is propelled around the loop and up the incline beyond. In the position before release, what is the minimum distance the spring must be compressed (beyond its natural length) in order for the cart to traverse the loop without its wheels leaving the surface? Assume the wheels are massless and rotate frictionlessly.



WANT minimum compression of Spring
$\Rightarrow$ Minimum velocity at top of loop

$$N + Mg = F_c = \frac{Mv^2}{R}$$

minimum velocity (compression) $\Rightarrow N = 0$

$$\cancel{M}g = \frac{\cancel{M}v^2}{R} \qquad v^2 = gR$$

Now use Energy Conservation

$$\frac{1}{2}Kx^2 = mgh + \frac{1}{2}mv^2$$

$$\frac{1}{2}Kx^2 = mg\underbrace{\cancel{h}}_{2R} + \frac{1}{2}MgR = \frac{5}{2}MgR$$

$$x^2 = \frac{5}{2}\frac{MgR}{K}(2) = 5\frac{MgR}{K} \qquad x = \sqrt{\frac{5MgR}{K}}$$

$$x = \sqrt{\frac{5(.02)(9.8)(3)}{50}} = 0.24\text{ m}$$

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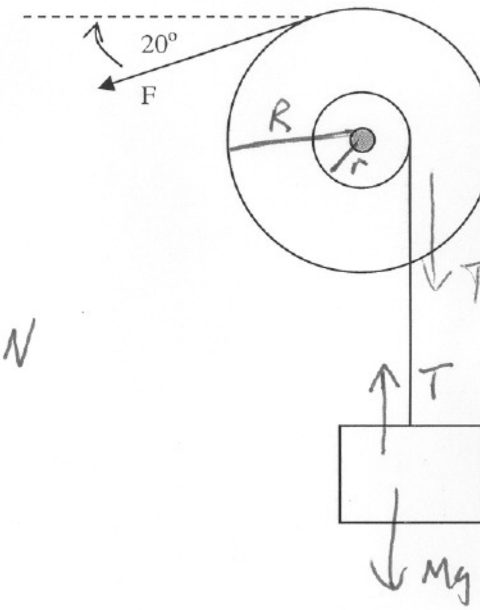
Problem 8 (6 pts, show your work):

Two pulleys are mounted rigidly on the same axis. The axis is free to rotate frictionlessly. A rope is attached to the large pulley of diameter 20 cm. A car engine of mass 300 kg is hung from the small pulley of diameter 8.0 cm. With what force F must a person pull the rope to hold up the engine?



$T = Mg$

$Tr - FR = 0$

$Mgr = FR$

$F = Mg\frac{r}{R} = (300)(9.8)\left(\frac{8}{20}\right) = 1176\ N$

Problem 9 (8 pts):

You graduate from college and enter graduate school where you are thrilled to work with the famous Professor Chip Rock, paleontologist extraordinaire. (In case you don't know, paleontology is the study of life that existed in past geologic times through fossil remains.) As part of your research, you discover evidence that during global warm periods in the past, the atmosphere expands (gases expand when warmed) and the length of the day changes. Professor Rock is dubious when presented with the data. Please briefly explain to us and Professor Rock why the length of the day changes and in what direction it changes.

When the atmosphere expands the mass distribution of the Earth is changed. Slightly more mass ~~axis~~ is pushed to a larger radius. This slightly increases the moment of Inertia of the earth.

Since Earth is not acted on by external torques, the Angular momentum = $I\omega$ is constant. Thus a larger I implies a smaller ω. Since ω is smaller it will take longer for the earth to rotate one time, i.e. the length of the day is increased.

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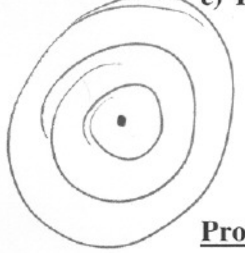
Problem 10 (7 pts, show your work):

At an outdoor bandstand, what would be the difference in the sound intensity level (in dB) received by a listener 5 m from the bandstand and a listener 40 m from the bandstand, assuming there is no sound reflection?

$I_5 \sim \frac{\text{Energy}}{4\pi 5^2}$

a) 8 dB
b) 9 dB
c) 10 dB
d) 18 dB
e) The answer depends on how loudly the band plays.

$I_{40} \sim \frac{\text{Energy}}{4\pi 40^2}$



$\text{Intensity} = \frac{\text{WATTS}}{M^2}$

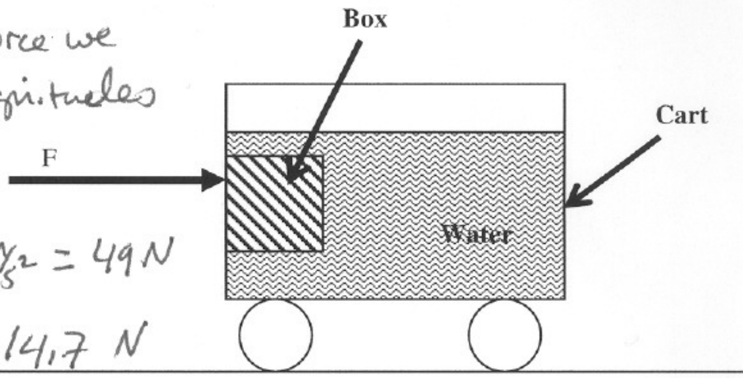
$I \sim \text{Energy}/\text{Area}$

$\beta = 10 \log \frac{I_5}{I_{40}} = 10 \log \left(\frac{40}{5}\right)^2 = 18$

Problem 11 (10 pts, show your work):

A cart is pushed by a force F. Initially a solid box with a mass of 1.5 kg and a volume of 0.005 m^3 is in the position shown in the sketch below. The cart is filled with water. The coefficient of kinetic friction between the surface of the box and the surface of the cart is μ_k=0.4. The coefficient of static friction between the surface of the box and the surface of the cart is μ_s=2.5. Assume the cart and water, without the box, has a mass of 1300 kg. What is the minimum force with which the cart must be pushed in order for the box to stay fixed in its initial position?
(Assume the wheels rotate on their axes frictionlessly and that they have no mass.)

To get direction of friction force we must determine relative magnitudes of F_B and Mg



$F_B = \left(1000 \frac{kg}{m^3}\right)(.005\ m^3)\, 9.8\ m/s^2 = 49\ N$

$Mg = (1.5\ kg)(9.8\ m/s^2) = 14.7\ N$

$\therefore F_B > Mg$ Box would float in absence of friction

$\Rightarrow F_{fr}$ is down

Solve for a

$\Sigma F_y = 0 = F_B - F_{fr} - Mg$

$a = \frac{F_B - Mg}{M\mu_s} = 9.1\ m/s^2$

$0 = F_B - N\mu_s - Mg$

$N = M_{box}\, a \qquad 0 = F_B - Ma\mu_s - Mg$

$F = (M_{box} + M_{\text{cart, water}})\, a = (1301.5)(9.1)$

$F = 11844\ N$

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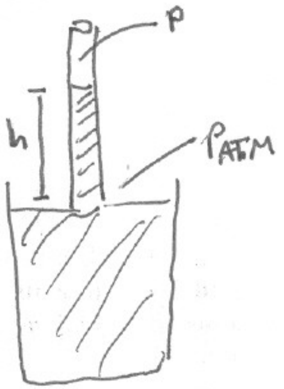
Problem 12 (8 pts):

Lucy Swanson is a legend on campus. She is the local champion in sucking down beers through a straw. In fact, Lucy is in training for the unofficial state championship in the event to take place soon at a large local party school near you. As part of her training regimen, she practices sucking water through straws of longer and longer length. She has noticed that no matter how hard she trains at sucking, she is unable to suck beer through a straw of height greater than a certain value. Because of your stellar physics expertise, the dejected Lucy sits by you one day at lunch and asks if you can explain to her why her training is fruitless. Below, briefly discuss how you would explain to Lucy, using the concepts we have studied recently, why there is a height beyond which she will be unable to suck a liquid, no matter how hard she trains. Use sketches, equations and text as you see fit. ***(Assume the straws Lucy uses of are very sturdy and do not collapse under suction.)***



From our studies of fluid statics we know the pressure at the bottom of a column of liquid is equal to the pressure at the top plus $\rho g h$. $P_{ATM} = P + \rho g h$

h is large when $P_{ATM} - P$ is large.

This is largest when P is equal to zero, that is to say, when Lucy pulls a true vacuum with her mouth. P cannot be negative or smaller than zero. Thus there is a limitation to how large h can be.

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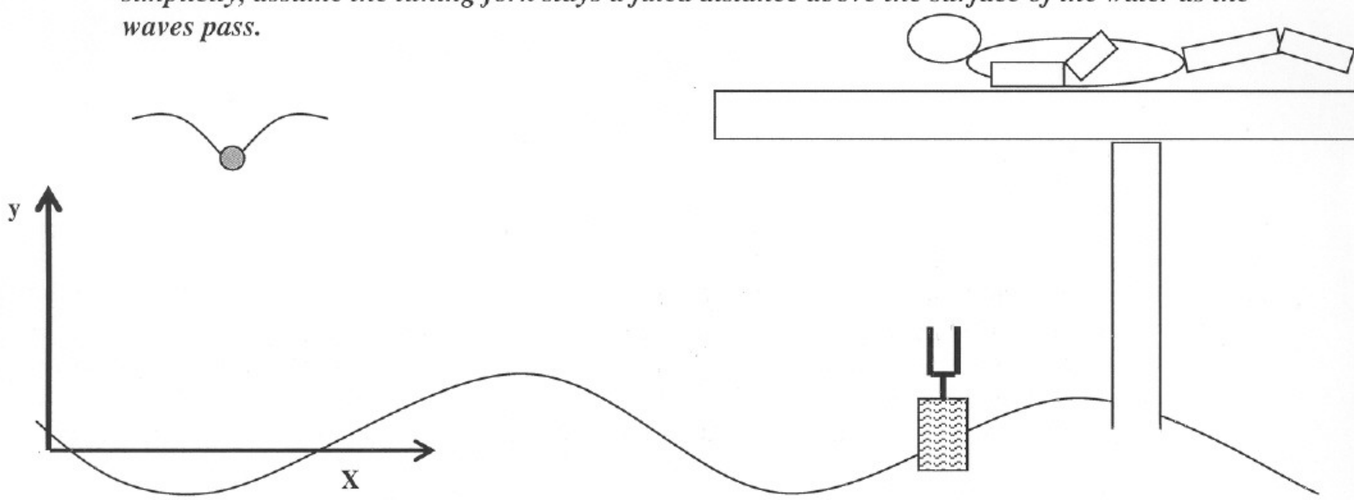
Problem 13 (10 pts, show your work):

After exams, Chaz Johnson travels to Jamaica to relax. He finds himself lying on a pier listening to the sound of the water below. For reasons that are best left obscure, one of Chaz's buddies has placed a tuning fork on a floating log directly beneath where Chaz's head lies on the pier. The tuning fork emits a frequency of 500 Hz. Chaz knows this and is surprised to find the frequency he hears varies with time. Being an analytical sort of guy, Chaz observes the water waves flowing toward the pier to be harmonic. He determines the water waves satisfy an equation describing the surface height (y) as a function of position (x) and time (t) as y(x,t)=Asin(kx-ωt)=2sin(1.5x-5t) in meters.

What is the range in frequencies Chaz hears coming from the tuning fork (not that which is emitted, but what Chaz hears)? Please calculate the range and show your work. ***For simplicity, assume the tuning fork stays a fixed distance above the surface of the water as the waves pass.***



$$y = A\sin(kx - \omega t)$$

$$\frac{dy}{dt} = -A\omega\cos(kx - \omega t)$$

$$V_{MAX\,y} = \pm A\omega = \pm(2)(5)$$

tuning fork bobs w/ this MAX V_y

Sound heard varies due to Doppler shift

$$f' = \frac{f}{1 \pm \frac{V_{source}}{V_{sound}}}$$

$$f' = \frac{500}{1 \pm \frac{(2)(5)}{340}}$$

Range of $f' = 486\ Hz - 515\ Hz$