Sept 9, 2004

Study Group ANNouncement

Prob set due at end of class
PRS # Sheet
Will Try to Post Solns ASAP

P.S. 2 will appear shortly

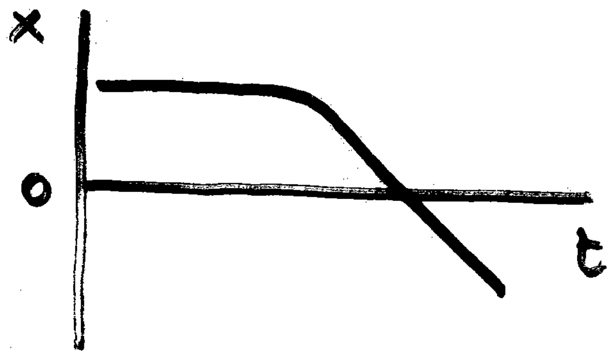
1-d kinematics

$$\bar{v} = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt}$$

$$\bar{a} = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

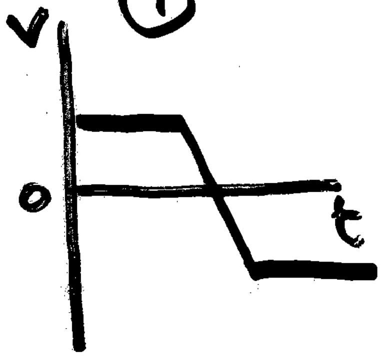
	MKS	cgs	English
x ~ Position	m	cm	foot
v ~ velocity	m/s	cm/s	ft/s
a ~ acceleration	m/s²	cm/s²	ft/s²
t ~ time	s	s	s

Consider the x-t graph



which graph best represents the corresponding v-t graph

1



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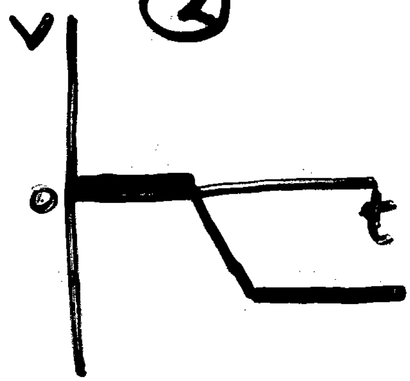

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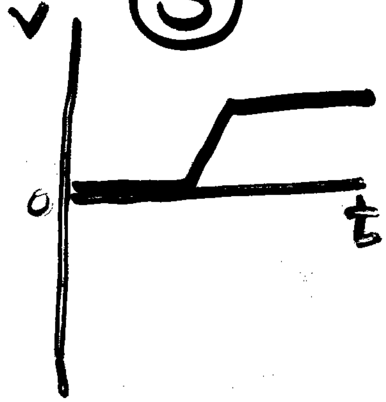
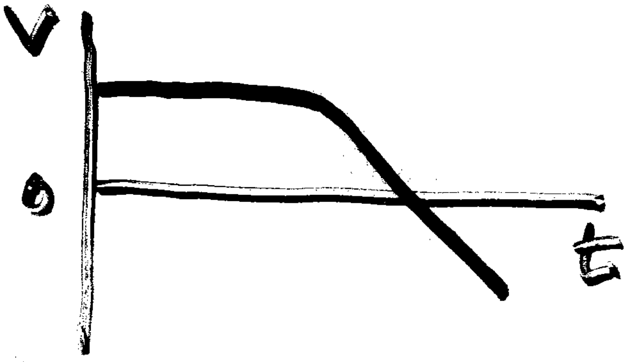


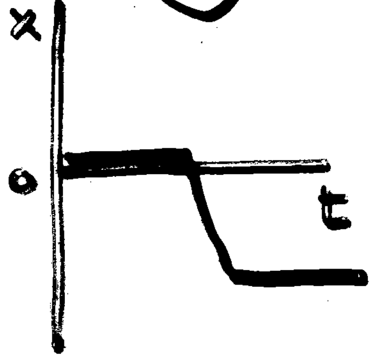
Consider The V-t graph



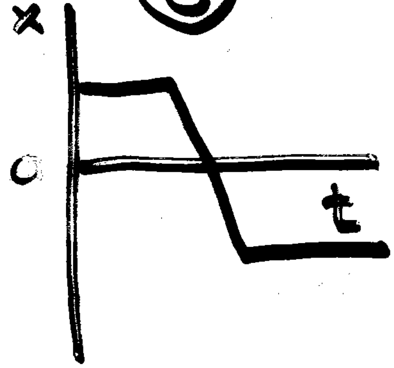
Which graph best Represents the corresponding x-t graph

①

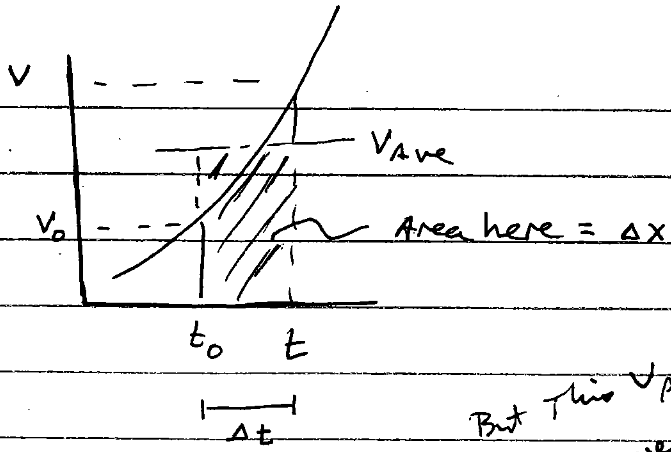
②



③



$$\frac{\Delta x}{\Delta t} = V_{Ave}$$



$$\Delta x = V_{Ave}\,\Delta t$$

But this V_{Ave} not nec. good one

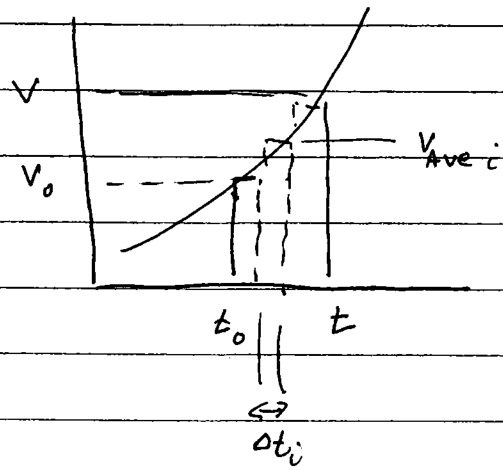
$t_0 + \delta t$

where

$\delta t < \Delta t$

Could try to be more accurate

$$\Delta x = \sum_i V_{Ave_i}\,\Delta t_i$$



In the limit $\Delta t_i \to 0$

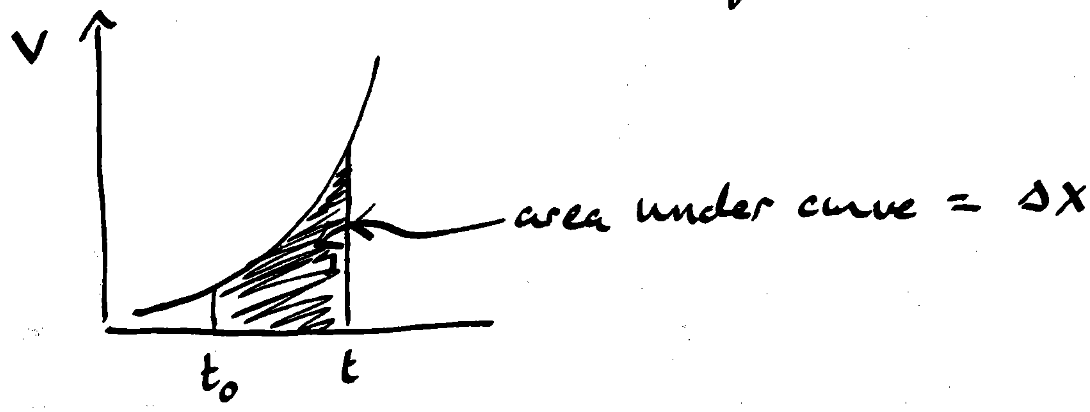
$$\Delta x = \lim_{\Delta t_i \to 0} \sum_i (V_{Ave})_i\,\Delta t_i$$

$$\left.\begin{array}{l} \Delta t \to dt \\ V_{Ave} \to V_{instantaneous} \end{array}\right\}\ \text{in this limit}$$

$$\Delta x = \int_{t_0}^{t} v\,dt$$

In this limit

Δx = exact area under v-t curve between limits of $t=t_0$ and $t=t$



Should this surprise you?

recall from calculus

$$\text{If } g = \int f\,dt, \text{ then } f = \frac{dg}{dt}$$

Integrals + derivatives are $\approx$ inverses of each other

↳ Think of as a slope

↳ Think of as a sum

$$\left.\begin{matrix} dx \\ dt \\ dv \\ \vdots \end{matrix}\right\} \equiv \text{differentials} \ldots$$

Think of as a little tiny Δx ....

* give table of integration formulas

$$\Delta x = \int_{t_0}^{t} v\, dt$$

example

$V = 3t + 4$

$x - x_0 = \int_{t_0}^{t} (3t+4)\, dt$

$= \left[\frac{3t^2}{2} + 4t\right]_{t_0}^{t}$

$x - x_0 = \frac{3t^2}{2} + 4t - \frac{3t_0^2}{2} - 4t_0$

$$\boxed{x - x_0 = \int_{t_0}^{t} v\, dt}$$

Very general eqn

V can vary w/ t

Can evaluate if $V(t)$ is known

$$\frac{\Delta v}{\Delta t} = a_{ave}$$

→ do same steps we just did ...

$$\boxed{v - v_0 = \int_{t_0}^{t} a\, dt}$$

Very general eqn

a can vary w/ t

can evaluate if

$a(t)$ is known

NOW Consider special case of a CONSTANT w/ Time

→ CONSTANT Acceleration problem ⇒ Very IMPORTANT

$$v - v_0 = \int_{t_0}^{t} a\, dt = a \int_{t_0}^{t} dt = a(t - t_0)$$

- gravity on surf of earth
- Any constant "Force" situation

Usually define $t_0 = 0$

$$\boxed{v = v_0 + at}$$

uses v, a, t NOT x

Substitute this into general eqn for x

$$x - x_0 = \int_{t_0}^{t} (v_0 + at)\,dt$$

NOTE:

$$\int (A + B)\,dt = \int A\,dt + \int B\,dt$$

$$x - x_0 = \int_{t_0}^{t} v_0\,dt + \int_{t_0}^{t} at\,dt$$

$$x - x_0 = v_0 \int_{t_0}^{t} dt + a \int_{t_0}^{t} t\,dt$$

$$x - x_0 = v_0\, t \Big|_{t_0}^{t} + a \frac{t^2}{2} \Big|_{t_0}^{t}$$

$$x = x_0 + v_0 (t - t_0) + \tfrac{1}{2} a (t^2 - t_0^2)$$

If $t_0 = 0$

$$\boxed{x = x_0 + v_0 t + \tfrac{1}{2} a t^2}$$ uses x, t, a NOT v

End lect Sept 9

The game is To find eqns that use variables you have or need to find ... and NOT Variables you don't have

So far have ① eqn That does NOT ~~have~~ use x

① eqn that does NOT use v

Need ① eqn that does NOT use t

① eqn that does NOT use a

Consider $\quad \Delta x = v_{Ave} \Delta t$

for case w/ CONSTANT Acceleration

$\Rightarrow$ V changes at a CONSTANT rate

$\therefore \quad v_{Ave} = \dfrac{v + v_0}{2}$ v_{final} .. at end of interval

$\therefore \quad x - x_0 = \left(\dfrac{v + v_0}{2}\right) t$

$$\boxed{x = x_0 + \left(\frac{v + v_0}{2}\right) t}$$ x, v, t No a

Finally find eqn w/ NO t

START w/ $\quad v = v_0 + at$

Solve for t $\rightarrow$ $t = \dfrac{v - v_0}{a}$

Subst. in above eqn

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right)$$

$$2a(x - x_0) = v^2 - v_0^2$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$ v, a, x NO t

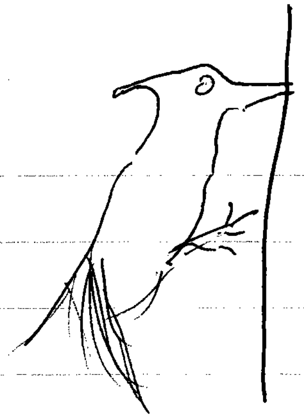
Hang time

Basketball

Conceptual Example

Why do basketball players appear to hang in the air at the top of their motion during a jumpshot

Woodpecker's head



$V_0 = 0.600$ m/s

Comes to stop in 2.00 mm

(a) What is acceleration of head in units of g ?

What is final v ? $\longrightarrow 0$

$$\underset{0}{v^2} = \underset{0.6}{v_0^2} + 2a\underbrace{(x - x_0)}_{0.002}$$

$$\frac{-(0.6)^2}{2(.002)} = a \qquad a = -90 \text{ m/s}^2$$

deceleration at ~ 9 g's
wow!

(b) how much time to Stop?

$$v = v_0 + at \qquad 0 = .6 - 90t$$

$$t = \frac{-.6}{-90} = 6.7 \text{ ms}$$

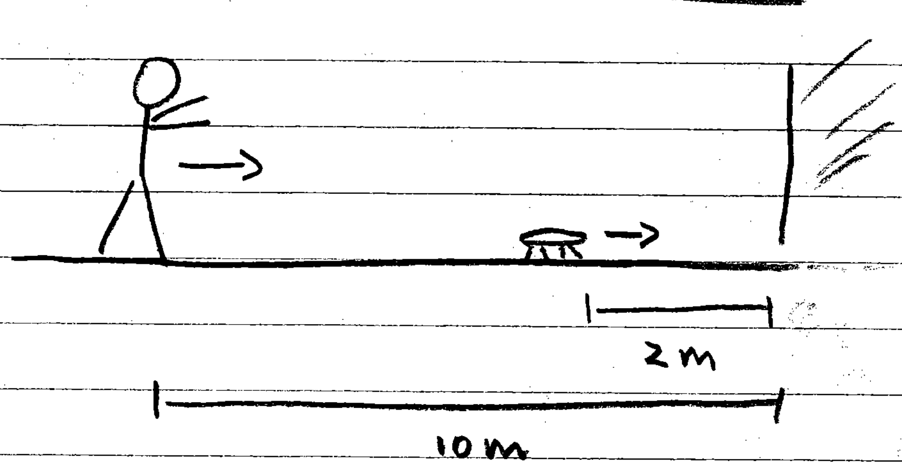
Tendons cradle bird's brain
They stretch - allowing brain to stop in 4.5 mm
What is brain's Acceleration?

$$v^2 = v_0^2 + 2a(x - x_0) \qquad \frac{-(0.6)^2}{2(.0045)} = a = -40 \text{ m/s}^2$$

or ~ 4 g

CAN Tie 2 1d problems into one problem

example



you see cockroach at $t=0$

you start from rest and accelerate (CONSTANT)

toward cockroach at $8\ m/s^2$

cockroach moves at const v of $1\ m/s$

does cockroach make it to safety?

Time for cockroach to make it safely

$a=0$ const

$$x_c - x_{o_c} = \left(\frac{v_{o_c}+v_{f_c}}{2}\right)t \qquad 2m = 1\ m/s\ t(s)$$

$$t = 2s$$

If person can reach wall in $t \leq 2s$

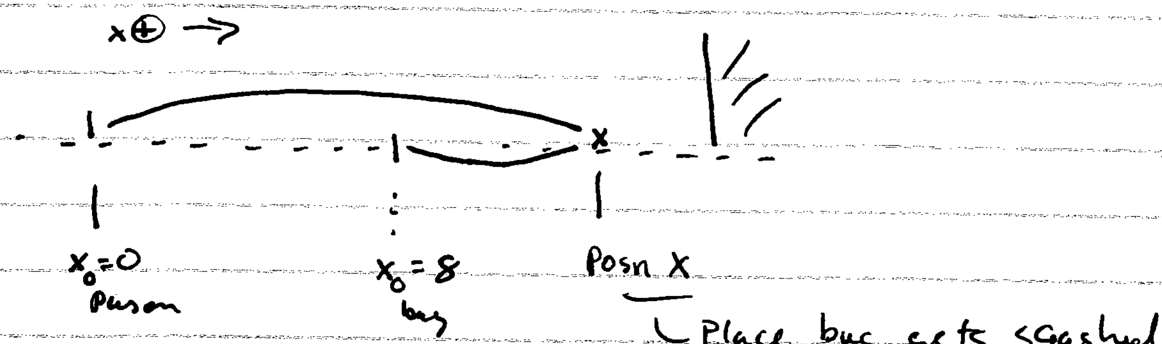
cockroach dies!

$$x_p = x_{o_p} + v_{ox_p}t + \frac{1}{2}a_{x_p}t^2$$

$$10\ m = (0)t + \frac{1}{2}8t^2 \qquad 10 = 4t^2 \qquad t = 1.6\ s < 2s$$

Want to put a little wreath :)

Where and When does bug get Squashed?



Person

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$x = \frac{8}{2} t^2$$

2 eqns
2 unknowns t, x
Solve Simultaneously

Bug $x = x_0 + v_0 t + \tfrac{1}{2} a t^2$

$$x = 8 + 1t + 0 \qquad x = 8 + t$$

$$8 + t = \frac{8}{2} t^2$$

$$0 = t^2 - \frac{2}{8} t - 2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \frac{\frac{2}{8} \pm \sqrt{\left(\frac{2}{8}\right)^2 + 4(2)}}{2}$$

.25 2.84

$t = 1.54\ s$ or $t = -1.3\ s$ Which is Correct?

$x = 8 + t = 8 + 1.54 = 9.54\ m$