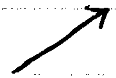
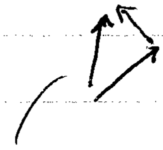
Last Time

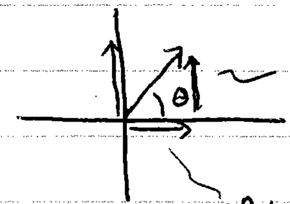
Vector - magnitude <u>and</u> direction



A  + B  = 

Resultant

graphical Addition of vectors



$|A|\cos\theta$ in Mag.

$|A|\sin\theta$ in Mag

Analytical - Break each vector up into components along x, y axes

Algebraically sum components along each axis using Proper Sign

Then ~~recombining~~ combine summed components into resultant vector

3 vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ have the following x and y components

	$\vec{A}$	$\vec{B}$	$\vec{C}$
x comp.	+6	−3	+2
y comp.	−3	+4	+5

What is the magnitude of the resultant of $\vec{A}$, $\vec{B}$, and $\vec{C}$?
That is to say, what is $|\vec{A}+\vec{B}+\vec{C}|$?

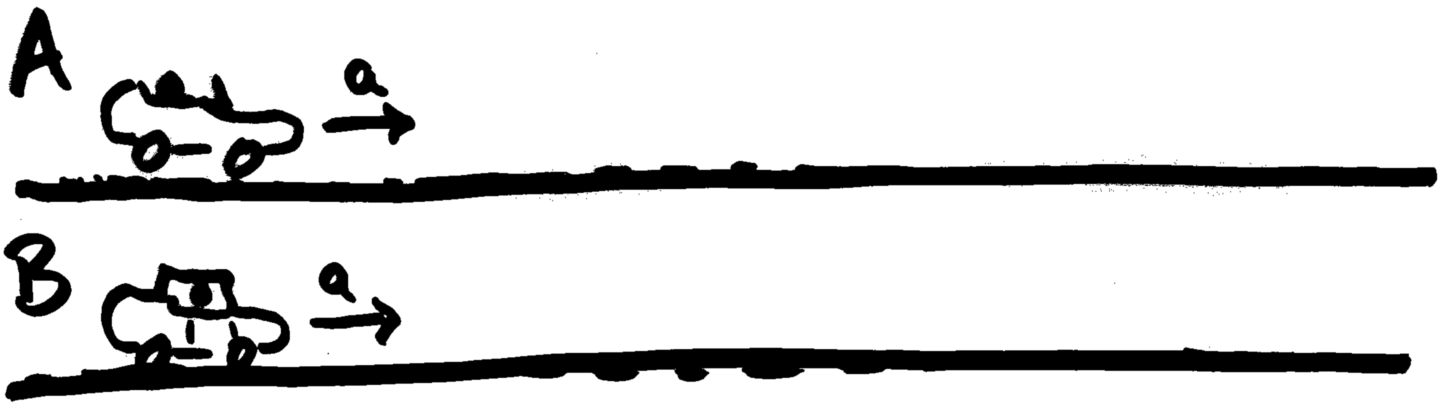
1. 3.3
2. 5.0
3. 11
4. 7.8
5. 14

$$R_x = +6-3+2 = +5$$

$$R_y = -3+4+5 = +6$$

$$|R| = \sqrt{R_x^2+R_y^2} = \sqrt{61} = 7.8$$

x component of resultant is the algebraic sum the x components of the 3 vectors



Car A and Car B start from rest and accelerate with the same CONSTANT acceleration, a.

Car A Travels for 1 minute
(Then stops ... instantaneously)

Car B Travels for 2 ~~minut~~ minutes before stopping (instantaneously)

Compared w/ Car A, Car B Travels

1. twice as far
2. 3 times as far
3. 1.41 Times as far
4. 4 times as far
5. 1/2 as far

Relevant relation between x and a and t

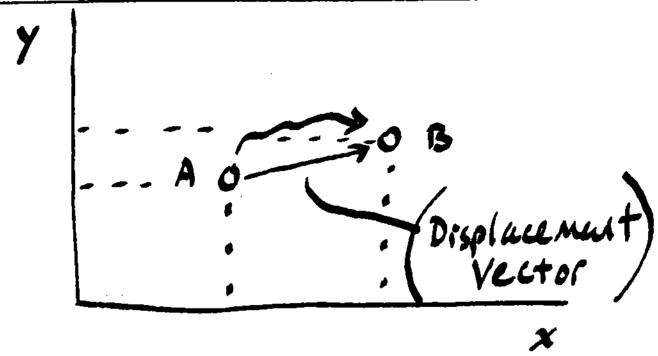
$$x = \cancel{x_0} + \cancel{v_0} t + \frac{1}{2} a t^2$$

0 in both case A and B

A $\rightarrow$ $d = \frac{1}{2} a t^2$

B $\rightarrow$ $d_B = \frac{1}{2} a (2t)^2 = 4 d_B$

Digress on vectors

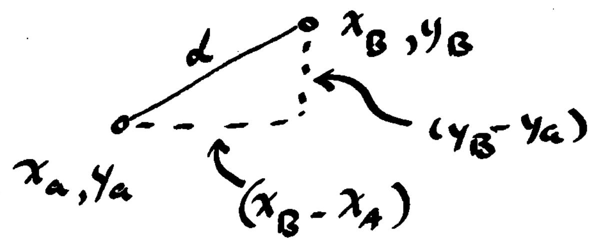


consider ball moving from A to B in the x-y plane

displacement from A to B has both a <u>Magnitude</u> and a <u>direction</u>

represent w/ a vector

length of vector = distance from A to B



Pythagorian Theorem

$$d = \sqrt{\underbrace{(x_B - x_A)^2}_{(\Delta x)^2} + \underbrace{(y_B - y_A)^2}_{(\Delta y)^2}}$$

direction $\equiv$ along the line joining A and B
Pointed from A toward B
components of vector $(\Delta x, \Delta y, 0)$

In 3 dimensions — see next page

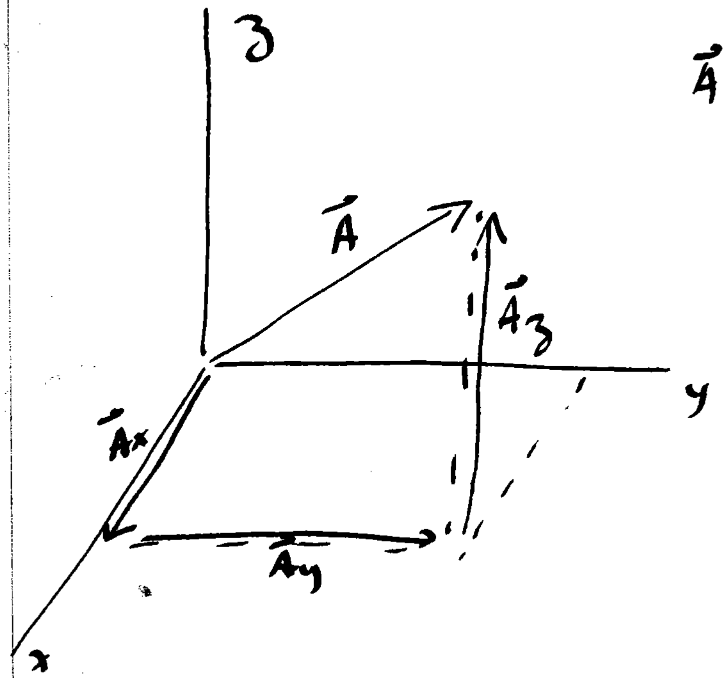
Suppose you want to use a vector to define a direction only

Can do this by dividing out the magnitude To create a "unit vector"

The unit vector in the $\vec{r}$ direction is

$$\frac{\vec{r}}{|\vec{r}|} \equiv \hat{r} \qquad |\hat{r}| = 1$$

note that $|\vec{r}|$ is Not Necessarily 1

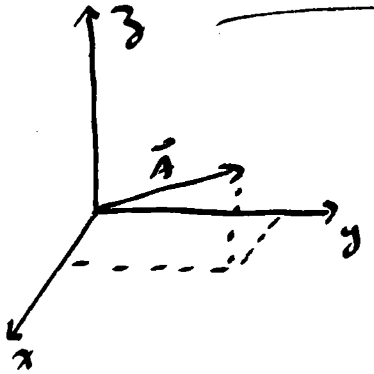


$\vec{A}$ is vector sum of $\vec{A}_x + \vec{A}_y + \vec{A}_z$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Complexity of all this may not seem necessary to you now ... fine ... hang in there and you will see why we do it the way we do.

Position



$$\vec{A} = (A_x, A_y, A_z)$$

- or -

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

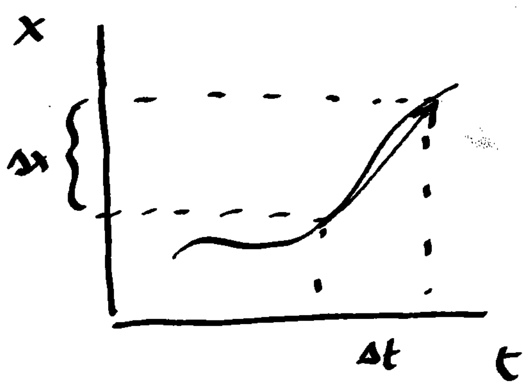
- or -

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\hat{x} \equiv \hat{i}, \quad \hat{y} \equiv \hat{j}, \quad \hat{z} \equiv \hat{k}$$

Acceleration + Velocity as vectors

$$(V_{Ave})_x = (\Delta v)_x = \frac{\Delta x}{\Delta t}$$



generalize to 3-d

$$\vec{V}_{Ave} = \vec{\Delta v} = (\Delta v)_x \hat{i} + (\Delta v)_y \hat{j} + (\Delta v)_z \hat{k}$$

$$\vec{V}_{Ave} = \vec{\Delta v} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k}$$

for instantaneous $\vec{v}$

$$\vec{V} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k}$$

- or -

$$\vec{V} = \vec{V}_x + \vec{V}_y + \vec{V}_z$$

Similarly

$$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

-or-

$$\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

Projectile Motion

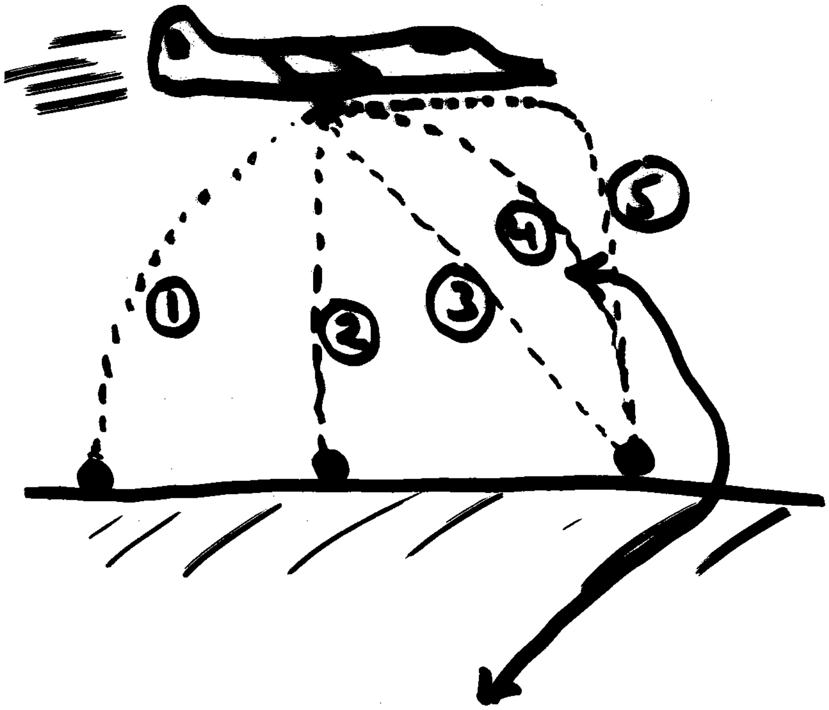
given an initial velocity, position
projectile follows path determined by effects
of gravity + air resistance

The ideal motion of bullets, basketballs, baseballs,
kids jumping out of swings, etc ... is projectile motion

Method of Solution:

Break 3-dimensional projectile motion up into
(3) 1-dimensional problems and "solve them
simultaneously" ... i.e., each constrains the others

Let's do an example or two ....



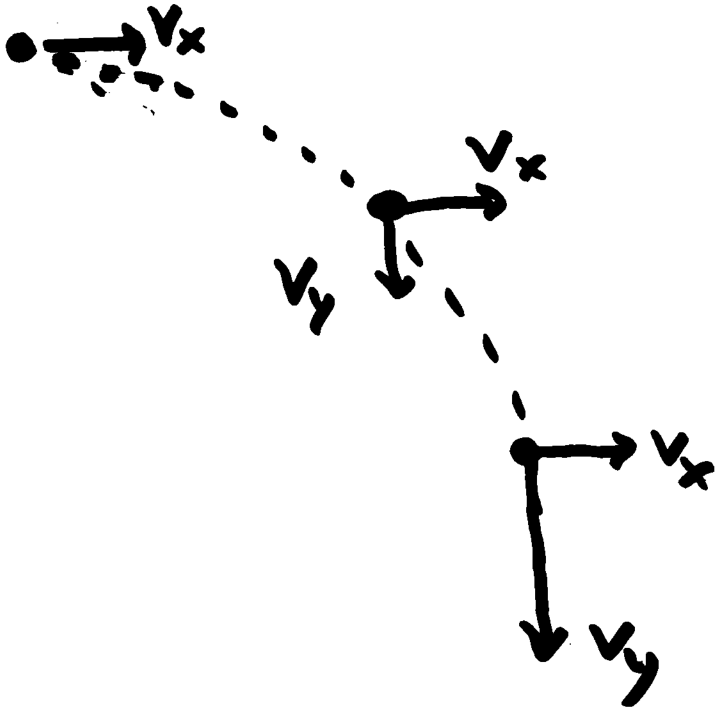
Plane drops bowling ball

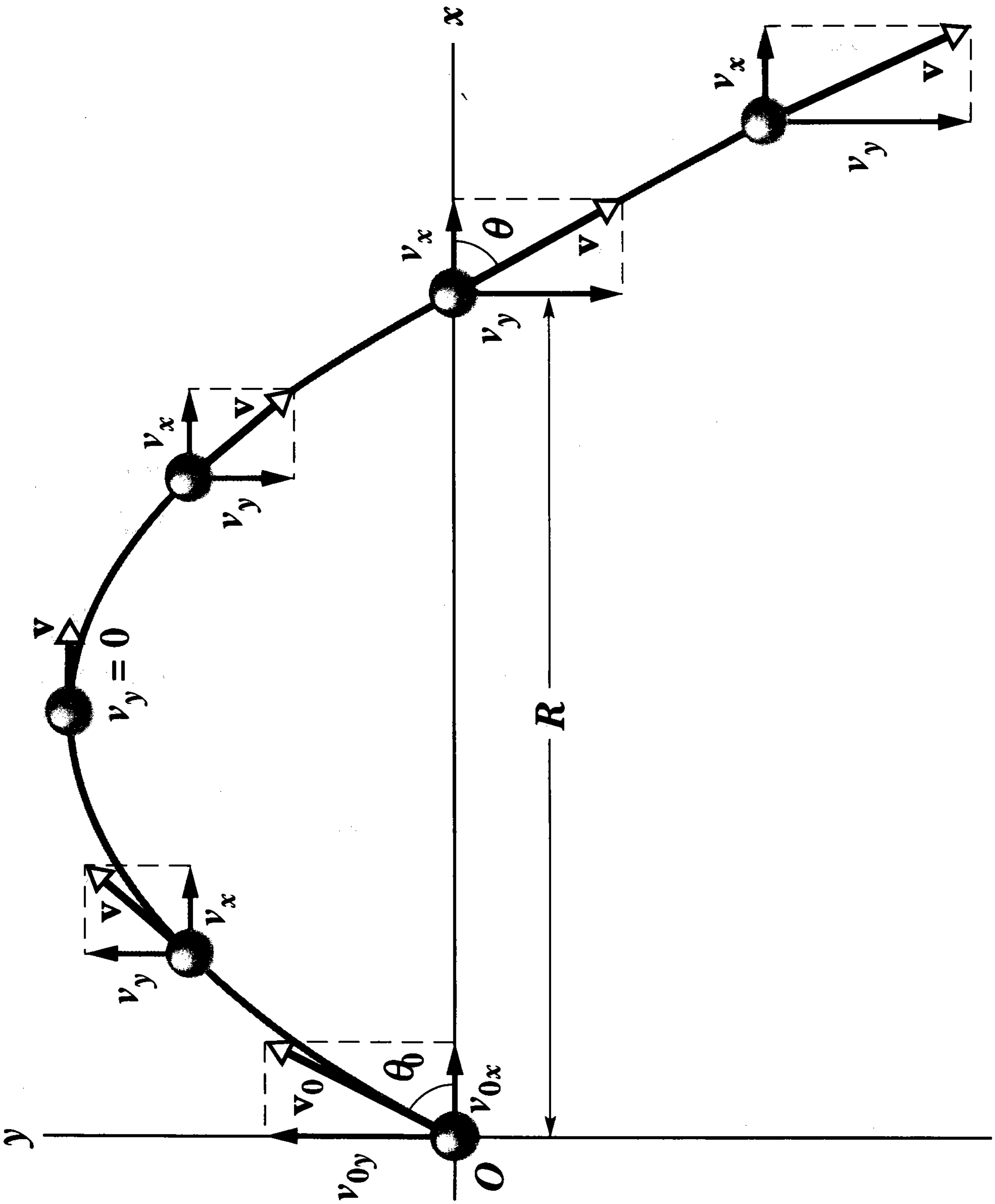
Which path will bowling ball follow most closely? Ignore Air Friction.

Correct answer is 4

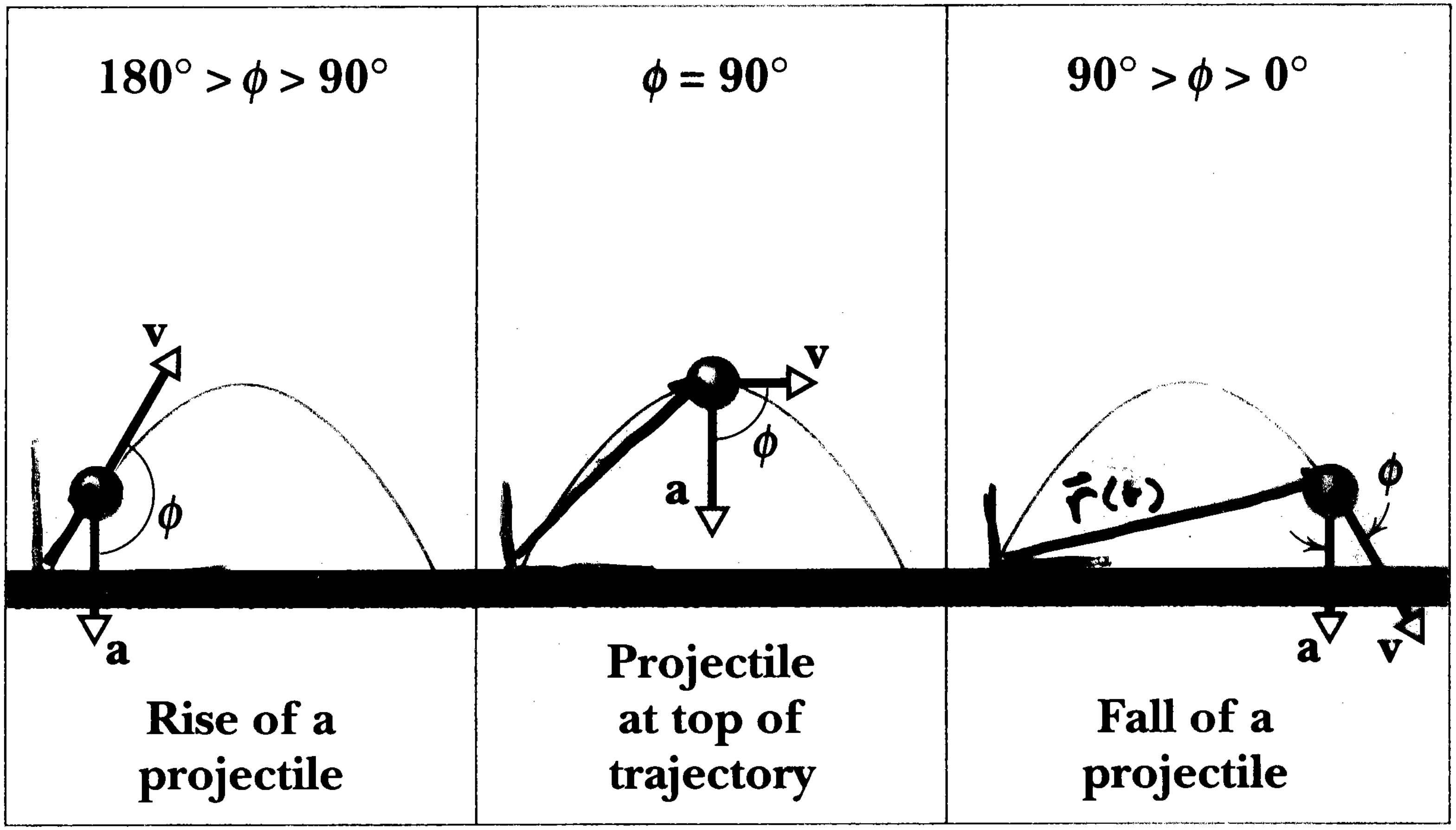
$a_x = 0 \Rightarrow V_x = \text{const}$

$a_y = -9.8\ \text{m/s}^2 \Rightarrow V_{y_0} = 0 \rightarrow$ grows more "−" linearly



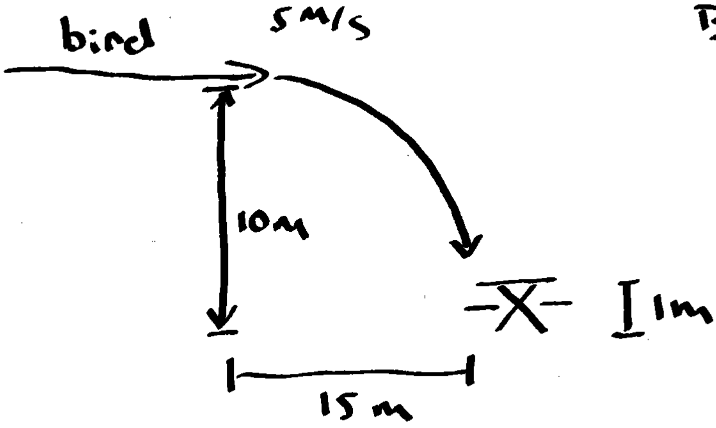


x
y
O
v_{0y}
v_{0x}
$\mathbf{v}_0$
θ_0
v_x
v_y
$\mathbf{v}$
$v_y = 0$
R
θ

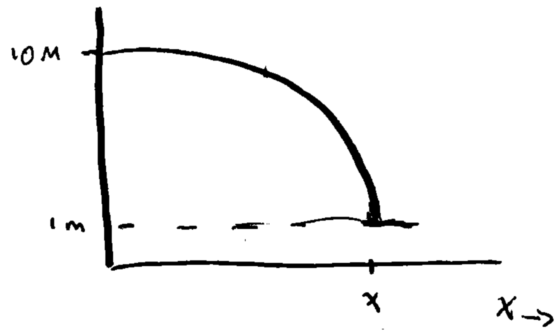


180° > φ > 90°
φ = 90°
90° > φ > 0°
v
a
φ
r(t)
Rise of a projectile
Projectile at top of trajectory
Fall of a projectile

Bird + Picnic Example



Does Bird poop reach ~~bit~~ table ?



$a_y = -9.8\ m/s^2$ $\quad a_x = 0$ $\quad \Big\rangle$ Both constant Accel. problems

$V_{oy} = 0$ $\quad V_{ox} = 5\ m/s$

$y_0 = 10\ m$ $\quad x_0 = 0$

$y = 1\ m$

what is x when y = 1 m ?

$\cancel{x = V_{ox} t}$

$$x = \cancel{x_0}_{\,0} + V_{ox} t + \tfrac{1}{2} \cancel{a_x}_{\,0} t^2$$

$$x = V_{ox} t = 5\ m/s\ t$$

t is the time it takes for poop to fall from y=10 to y=1

$$y = y_0 + V_{oy} t + \tfrac{1}{2} a_y t^2$$

$$1m = 10m + 0 - \tfrac{1}{2}(9.8\ m/s^2) t^2$$

$$9 = \tfrac{1}{2}(9.8) t^2$$

$$t = \pm\sqrt{\frac{18}{9.8}}\ s$$

"−" soln is not physical for this problem

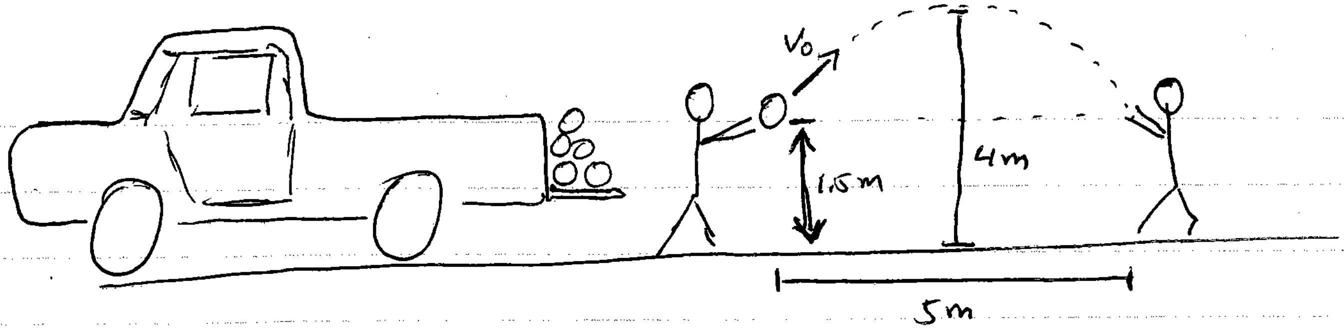
$$t \approx \overset{1.9}{\cancel{1.36}}\ s$$

$$x = (5\ m/s)(\overset{1.9}{\cancel{1.36}})s$$

$$x = 7 \text{ meters}$$

The picnic is safe !!

~~end of Material for Exam I~~



Fraternity brothers Party
ice for iced Tea.
What is $\vec{V}_0$?

2 1d problems

<u>Y Motion</u>

$a = -9.8\ m/s$

$Y_0 = 1.5\ m$ $\qquad t_y = ?$

$Y_f = 1.5\ m$

$V_{0y} = ?$

* What is ~~V_{0f}~~ V_{Fy} ?

<u>x motion</u>

$a = 0$

$x_0 = 0$ $\qquad t_x = ?$

$x_f = 5$

$V_{0x} = ? = V_{fx}$

* How are t_x and t_y related ?

Look at y motion

$$V_y^2 = V_{oy}^2 + 2a_y(y - y_o)$$

look at ice at top of motion

$$0^2 = V_{oy}^2 - 2(9.8)(4 - 1.5) \qquad V_{oy} = 7\ \text{m/s up}$$

Total Time of flight = 2 × time to top → symmetry

Time to top

$$V_o = V_{oy} + a\,t_{top}$$

$$0 = 7\,\text{m/s} - (9.8\,\text{m/s}^2)\,t_{top} \qquad t_{top} = \cancel{1.4\,s}\ 0.7\,s$$

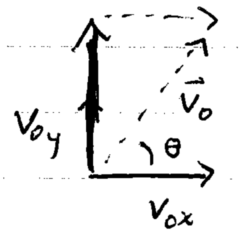
Total time of flight $t = (2)(\cancel{1.4}\ 0.7) = \cancel{2.8s}$ 1.4

X motion

$$x = x_o + V_{ox}t$$

$$5\,m = 0 + V_{ox}(\cancel{2.8}\ 1.4) \qquad V_{ox} = \cancel{1.8}\ 3.6\ \text{m/s to right}$$

Have V_{ox}, V_{oy} WANT $\vec{V}_o$



$$|\vec{V}_o|^2 = |V_{oy}|^2 + |V_{ox}|^2$$

$$|\vec{V}_o| = 7.8\ \text{m/s}$$

Need Direction

$$\tan\theta = \frac{7}{\cancel{1.8}\ 3.6} \qquad \theta = \cancel{75.6}\ 62.8^\circ$$