Work and Energy

→ Ability to do work
CAN TAKE Many different Forms

→ Force x distance
CAN change Energy of a system

Intimate relationship between Work and Energy

do work to incr. energy of system

Work $\rightleftarrows$ Energy

Expend energy
doing work

System where such "Energy flow" is repeatable
and easy to follow $\Rightarrow$ Conservative
System

We'll come back to this

Physics Work VS. Biological Work

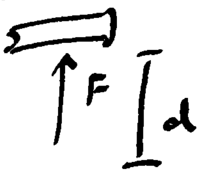
Demos
MV-1 Pile Driver
Mr-6 large pendulum

Mr-3 stopped pendulum

Consider book ... Pick it up

do work against gravity

Physical work vs. Biological work

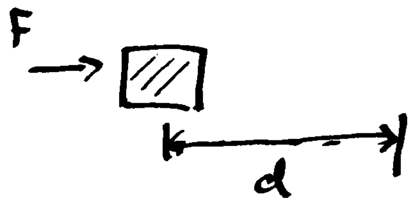


Suppose I carry it w/out
Δ'ing height
Is this work?

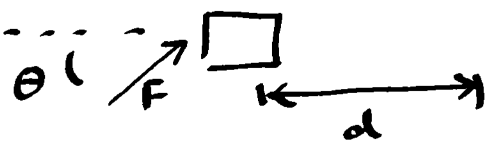
Pile Driver
Potential Energy

It may seem so ... but in physics we say NO.
You're NOT ~~[illegible]~~ changed ability of book to do work
i.e. changed its energy.
whereas you did when you picked it up

Work ≡ The work done by a force on a particle (or object) is equal to the force times the distance through which that particle (or object) is moved along the line of the force

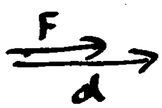


work = Fd

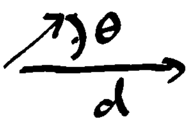


work = $F\cos\theta\ d$

$\vec{d}$ is a vector ... displacement



work = Fd



work = $F\cos\theta\ d$

or

$Fd\cos\theta$

Work in the World of Physics

The work done by a force on a Particle or object ...

= $|\vec{\text{Force}}|$ × distance through which particle or object moved along the line of the force

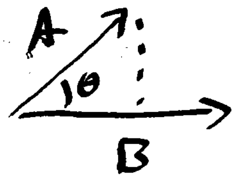
-OR-

= (distance moved) × Magnitude of component of force along direction of Movement

Work ≡ component of force along direction of movement times the distance moved

-or-

≡ Force times component of distance moved along the direction of force.



2 vectors w/ angle between

$$|\vec{A}|_{\text{along } B} = A\cos\theta$$

$$|\vec{B}|_{\text{along } A} = B\cos\theta$$

use this very often ... let's define a vector Dot product

$$\underbrace{\vec{A}\cdot\vec{B}} = |\vec{A}|\,|\vec{B}|\cos\theta$$

gives the (projection of $\vec{A}$ along $\vec{B}$) x ($|\vec{B}|$)

or ($|\vec{A}|$) x (projection of $\vec{B}$ along $\vec{A}$)

Also

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

NOTE these are scalars, NOT vectors

Dot product also called scalar product

~~Popular on p 19/20 of text~~

$$\vec{A}\cdot\vec{A} = ?$$

$$= |\vec{A}|\,|\vec{A}|\cos 0 = |\vec{A}|\,|\vec{A}|$$

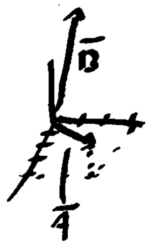
$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos 90 = 0$

No projection possible here!

<u>Example</u>

$\vec{A} = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$\vec{B} = 1\hat{i} + 0\hat{j} + 9\hat{k}$



Ask for Arbitrary (bet 0 + 10) components from class

$\vec{A} \cdot \vec{B} = (3)(1) + (4)(0) + (2)(9) = 4 + 18 = 22$

What is opening angle between $\vec{A}$ and $\vec{B}$?

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

$|\vec{A}| = \sqrt{3^2 + 4^2 + 2^2} = \sqrt{29}$

~~22 = ...~~

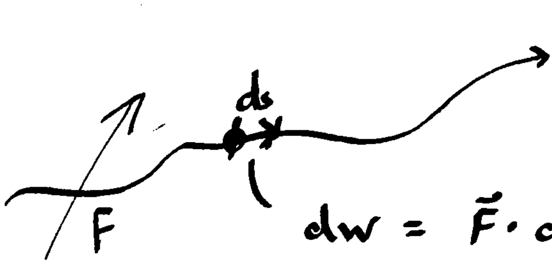
$|\vec{B}| = \sqrt{82}$

$22 = \sqrt{29}\sqrt{82} \cos\theta$

Solve for θ $\quad \theta \sim 63^\circ$

Since <u>Work</u> uses the projected component of Force along distance $\Rightarrow$ conveniently defined in terms of vector scalar or dot product

$W \equiv \vec{F} \cdot \vec{S}$ for constant vectors



$dW = \vec{F} \cdot d\vec{s}$

$W = \int \vec{F} \cdot d\vec{s}$ for varying ~~force~~ force / path

$W = \int (F_x dx + F_y dy + F_z dz)$

Show $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

EXPAND –

$$\vec{A}\cdot\vec{B} = A_x\hat{i}\cdot B_x\hat{i} + A_x\hat{i}\cdot B_y\hat{j} + A_x\hat{i}\cdot B_z\hat{k}$$
$$+ A_y\hat{j}\cdot B_x\hat{i} + A_y\hat{j}\cdot B_y\hat{j} + A_y\hat{j}\cdot B_z\hat{k}$$
$$+ A_z\hat{k}\cdot B_x\hat{i} + A_z\hat{k}\cdot B_y\hat{j} + A_z\hat{k}\cdot B_z\hat{k}$$

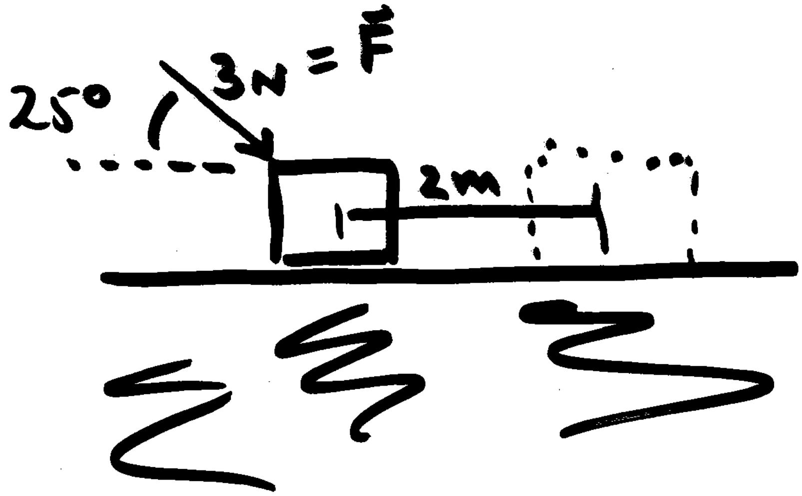
$$= A_x B_x\,\hat{i}\cdot\hat{i} + A_x B_y\,\hat{i}\cdot\hat{j} + \cdots + A_z B_z\,\hat{k}\cdot\hat{k}$$

NOTE

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = (1)(1)\cos 0 = 1$$

$$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = (1)(1)\cos 90 = 0$$

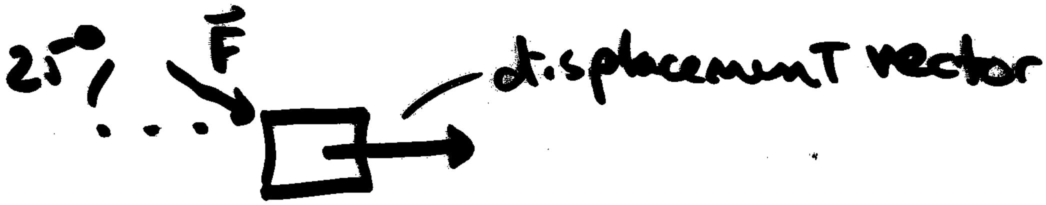
only 3 Terms survive $\Rightarrow$ $\hat{i}\cdot\hat{i}$, $\hat{j}\cdot\hat{j}$, $\hat{k}\cdot\hat{k}$

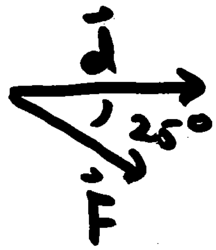


$\vec{F}$ is applied to Block of Mass M as it Slides 2 M to right.

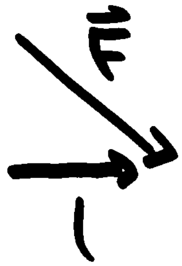
What is work done by $\vec{F}$?

1. 6 Joules
2. 6 Sin 25 Joules
3. 6 Cos 25 Joules
4. −6 Sin 25 Joules
5. −6 cos 25 Joules
6. −6 Joules





$$W = |\vec{d}||\vec{F}| \cos 25$$
$$W = 6 \cos 25$$



component of $\vec{F}$ along d

$$= \vec{F} \cos 25$$

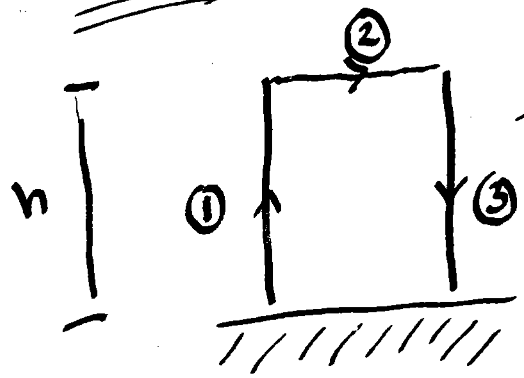
$$W = (|\vec{F}| \cos 25) \cdot d = 6 \cos 25$$

A particle moves halfway around a circle while acted on by a radial force F.

The work done by the radial force is

1. zero
2. $F \pi R$
3. $F 2\pi R$
4. FR
5. $2FR$

Example



~ Rock climber's path on a cliff

climber carries a ~~[illegible]~~ 35 mm camera. How much work does the climber perform to lug camera on trip.
Assume movement at constant velocity → $a = 0$
⇒ hell of a good rock climber!!

free body diagram

$a_y = 0 \therefore N = mg$

← force of gravity on camera

force rock climber exerts to support camera

① $\vec{S}$ ~ displacement $|\vec{S}| = h$, $\vec{N}$

$$\text{Work}_{①} = \vec{N}\cdot\vec{S} = |\vec{N}||\vec{S}|\overbrace{\cos\theta}^{+1} = mgh$$

$|\vec{N}| = mg$
$|\vec{S}| = h$

② $\vec{S}$, $\vec{N}$

$$\text{Work}_{②} = \vec{N}\cdot\vec{S} = |\vec{N}||\vec{S}|\overbrace{\cos\theta}^{0} = 0$$

③ $\vec{N}$, $\vec{S}$

$$\text{Work}_{③} = \vec{N}\cdot\vec{S} = |\vec{N}||\vec{S}|\overbrace{\cos\theta}^{-1} = -mgh$$

$$\text{Net TOTAL Work} = \text{Work}_{①} + \text{Work}_{②} + \text{Work}_{③} = 0 \text{ Joules}$$

Negative Work! ... don't you wish you could get a job doing that!

What work does gravity do in part ① ?

① $\uparrow$ on climb up ... there is NET work
where did it go?

Lift camera up a distance h ... can let it go ... it can fall and can do work! ... So by lifting camera up you've increased its energy ≡ ability to do work

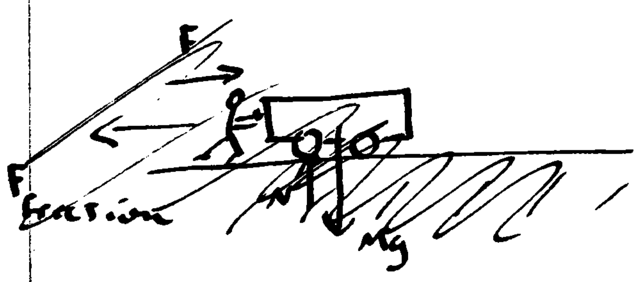
We say you have increased its "Potential Energy"

Converted chemical energy in body to perform work on camera to increase its potential energy.

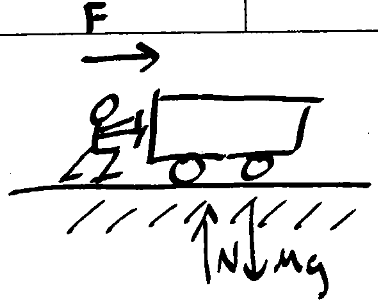
② $\longrightarrow$ NOT increasing or decreasing Potential energy ... NO work

③ $\downarrow$ decreasing potential energy
Negative Work

NeT work = Net potential Energy change = 0



You push cart with
~~Hard to~~
F



Push a frictionless cart on a frictionless horizontal surface.

Net horizontal force $F = m_{cart} a_{cart}$

Say you push w/ const force F over Distance D

$$W = FD$$

You put work into the system ... where has energy gone

You've NOT changed the Potential Energy of the system.

However $v^2 = v_0^2 + 2a(x - x_0)$

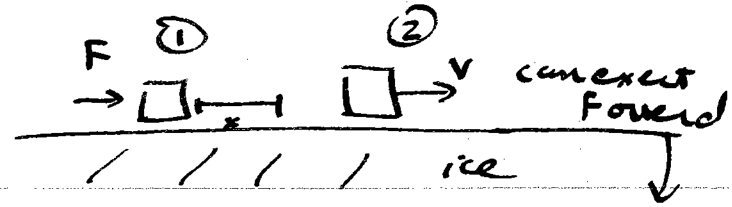
$$v^2 = 2 \frac{F}{m} D$$

$$\Rightarrow \frac{1}{2} m v^2 = FD \leftarrow \text{work input}$$

Kinetic Energy ≡

The work is converted into "kinetic energy" — Energy of Motion.

Kinetic Energy of an object mass m, moving w/ velocity of magnitude $|\vec{v}|$ $\equiv \frac{1}{2} m v^2$ / Important + General

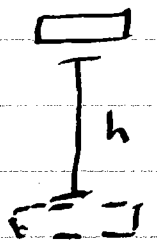


So,

Work ⟶ KE

Also

lift Book up a distance h



$F \cdot d = mgh$ amount of work

Now let it go – falls a distance h w/ $a = g$

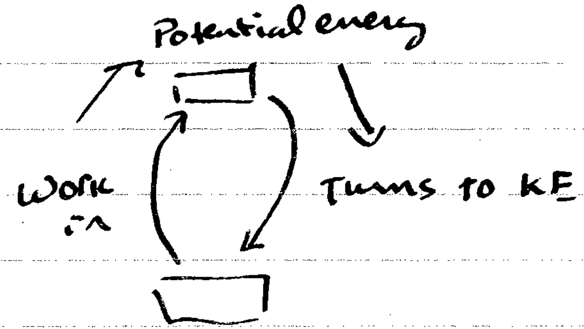
$\cancel{h = \frac{1}{2} g t^2}$ $\quad v^2 = v_0^2 + 2ah$

$v^2 = 2gh$

F/M

$\frac{1}{2} m v^2 = \cancel{\frac{1}{2}} m \, 2gh$

$\frac{1}{2} m v^2 = Fh$ = work done by gravity



In general

$$\boxed{E_{TOTAL} \text{ is Conserved}}$$

what do I mean by "is conserved"?

$$\left(\sum_i E_i\right)_{initial} = \left(\sum_j E_j\right)_{Final}$$

The form of the energy may change

Sometimes is NOT so obvious – consider pushing a car

$$W_{\substack{push\\car}} = \Delta PE_{\substack{car\\height}} + \Delta KE_{\substack{car\\motion}}$$

$$+ W_{friction} + \text{heat energy}$$

$$+ \text{sound energy}$$

$$+ \ldots$$

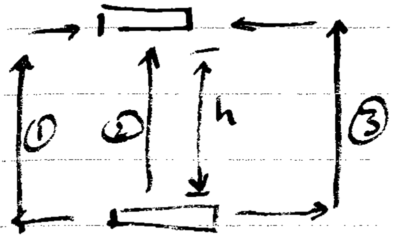
In the real world

$$W_{\substack{Push\\Car}} > \Delta PE + \Delta KE$$

Most of the time

useful to look at situations where other losses are NOT present.

Consider slide w/ NO friction or



lift book by 3 separate paths

Work done in each case = Mgh

ΔPE in each case = Mgh

$W = \Delta PE + \Delta KE$

The change in potential energy is Path Independent

Due to a Conservative force

With a conservative force

Work = $\Delta PE + \Delta KE$

one can study energy flow

Conser. systems that are commonly studied

* Springs ← Force is NOT CONSTANT

Gravity ← force const over small dist. at earth's surface

Very Impt. (chemical bonds + other things)

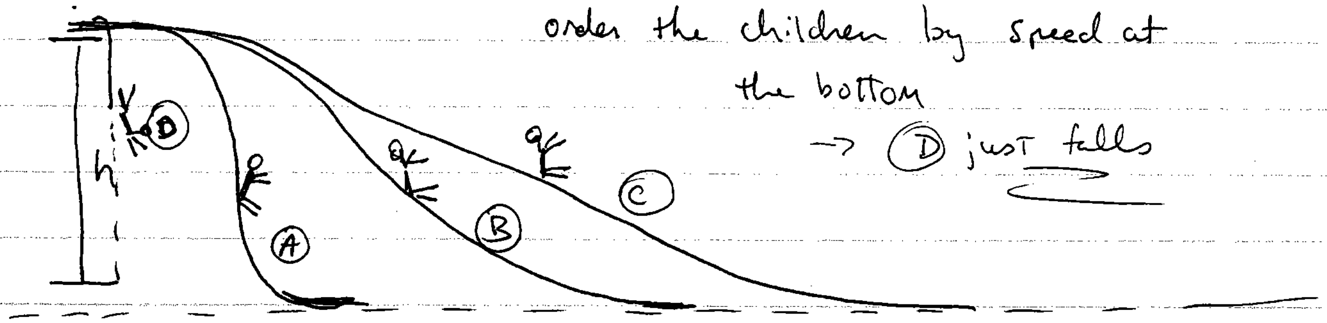
$\frac{1}{r^2}$ force in general

EXAMPLE Consider 3 Slides with No friction

(or Air resistance)

order the children by speed at the bottom

→ D just falls



$A = B = C = D$ because

$$E_{initial} = E_{final}$$

$$mgh_i + \tfrac{1}{2}m\underbrace{\cancel{v_i}}_{0}{}^2 = mg\underbrace{\cancel{h_f}}_{0} + \tfrac{1}{2}mv_f^2$$

$V_f = \sqrt{2gh}$ independent of Path !!