# Conservative Forces (2 mechanical examples)

Gravitation (No friction or Air resistance)

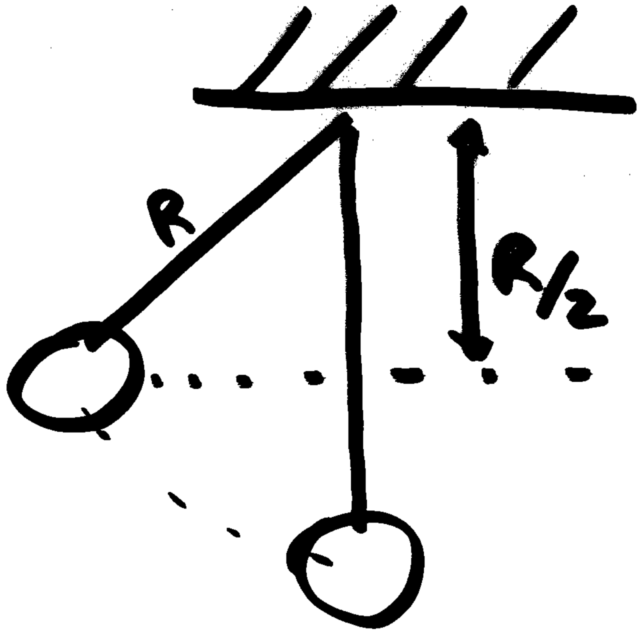
Springs (No friction)

Particularly useful to study

1) Energy flow "easy" to Analyze / understand

2) Concept of Potential Energy is useful AND depends only on position → NOT Path

Talked a little about gravity and gravitational PE
NOW let's look at Springs
+ come back to gravity




Release Mass m on string of length R.

As it passes lowest point, the tension in the string is

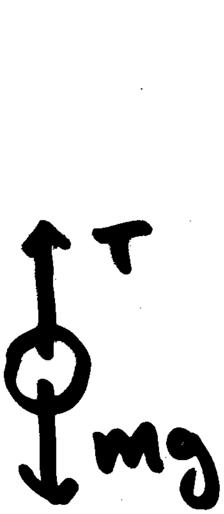
(1) impossible to determine

(2) mg

(3) 2 mg

(4) 3 mg

(5) none of these is correct




$$\underset{\substack{\text{lowest}\\ \text{pt}}}{\sum F_{up}} = \frac{mv^2}{R} = T - mg$$

$$T = Mg + \frac{mv^2}{R}$$

use energy cons. to get $v^2$

$$\underbrace{mgh = mg\frac{R}{2}}_{E_{init}} = \underbrace{\frac{1}{2}mv^2}_{E_{final}}$$

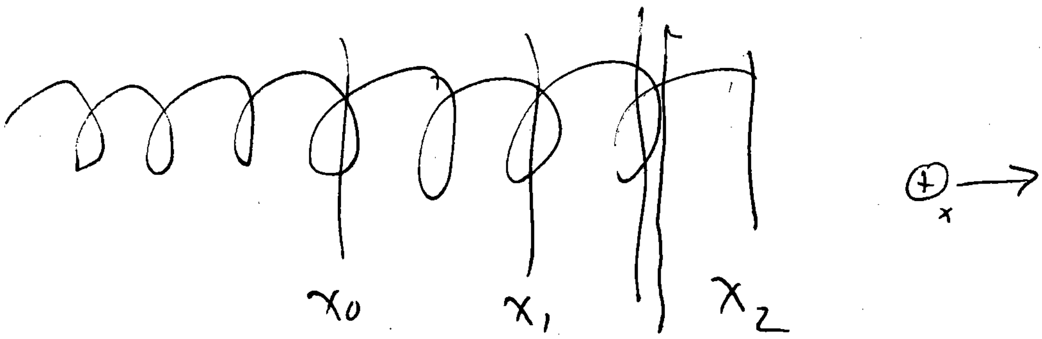
KE = 0

PE = 0

$$v^2 = Rg$$

$$T = Mg + \frac{M}{R}Rg$$

$$\boxed{T = 2Mg}$$

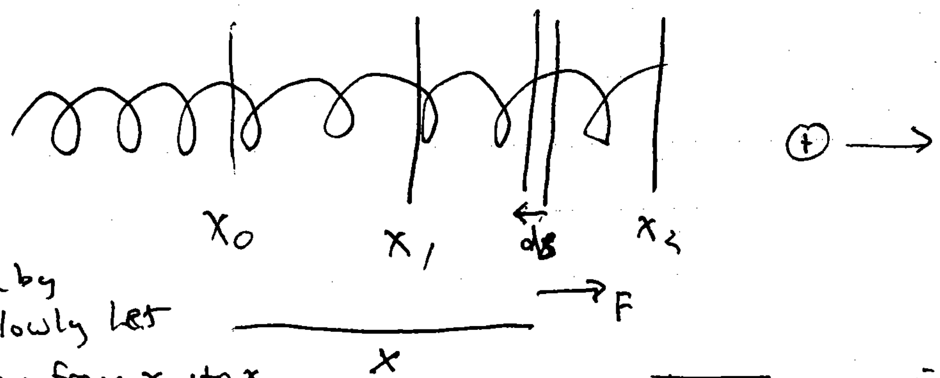



Work done by
Me as I Move Spring
From $x_1$ to $x_2$

$\left.\begin{matrix}\vec{ds} \\ \vec{dx}\end{matrix}\right)$ same sign

$\vec{F}\cdot\vec{ds}$ $\oplus$ - same direction

$$W = \int_{x_1}^{x_2} \vec{F}\cdot\vec{ds} = \int_{x_1}^{x_2} F\,dx = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$




Work done by
Me as I slowly let
Spring move from $x_2$ to $x_1$

x increasing to right
So $dx = -ds$

$$W = \int_{x_2}^{x_1} \vec{F} \cdot \vec{ds} = (-)(-) \int_{x_2}^{x_1} F\,dx$$

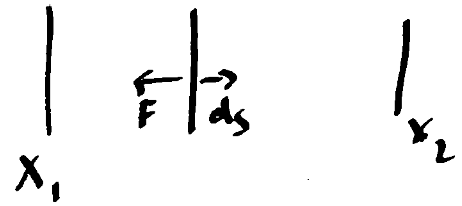
from dot Product $\vec{F}$ and $\vec{ds}$ in opposite directions

from fact that path s and x increase in opposite directions

$$W = \int_{x_2}^{x_1} F\,dx = \int_{x_2}^{x_1} kx\,dx = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

Work done on spring = − Work done by Spring

~~tot~~ consider work done <u>by</u> Spring from $x_1$ to $x_2$




$\vec{F}\cdot d\vec{s} = -kx\,dx$

$\Rightarrow W = \frac{k}{2}x_1^2 - \frac{k}{2}x_2^2$ $\ominus$

define elastic Potential Energy of a spring

$U \equiv \frac{1}{2}kx^2$ Ability to do WORK!

I do work on Spring $x_1 \longrightarrow x_2$ Stretching it

$\Delta$ Potential Energy $= \frac{k}{2}x_2^2 - \frac{k}{2}x_1^2$ is $\oplus$

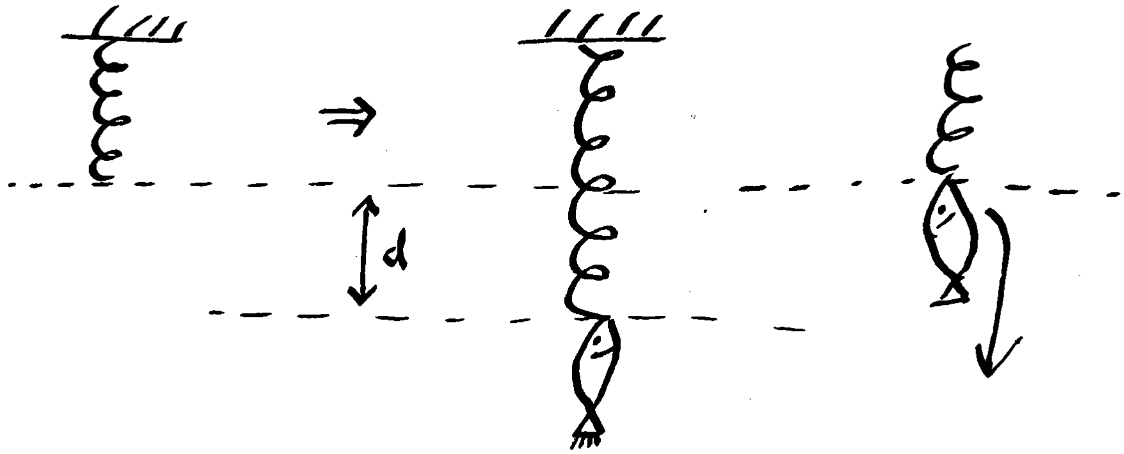
= Work I did on spring

I do work on spring letting it slowly relax from $x_1 \leftarrow x_2$

$\Delta$ Pot energy $= \frac{K}{2}x_1^2 - \frac{K}{2}x_2^2$ is $\ominus$

Also = to work I did in this case

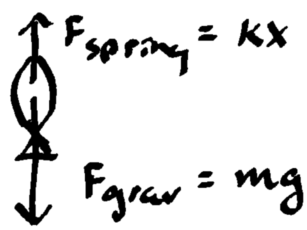
A fish is attached to a spring + slowly lowered to equilibrium position. Stretches spring by distance d.




If the same fish is attached to the unstretched spring and allowed to fall from rest, through what maximum distance does it stretch the spring?

one way to do this:

Know




$F_{NET} = mg - kx$

have a net force → can't use const accel eqns!

Easier to use Energy considerations

~~Find net work done by grav + spring~~

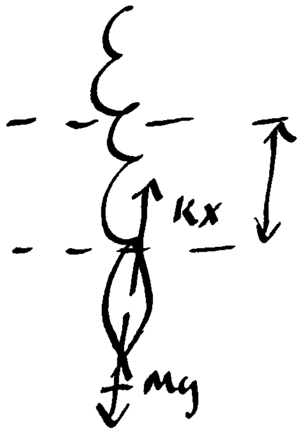
$\cancel{\int_0^d (mg - kx)\,dx = \text{increase in PE of spring}}$

Set Net work done by gravity = increase in PE of spring

$$mgh = \frac{1}{2}kh^2$$

but what is m? what is K?

We know from $1^{st}$ part of problem




$\leftarrow kx = kd$

have $a=0$ for this part and $x=d$

$\therefore$

$$mg = kd$$

$$k = \frac{mg}{d}$$

sub into

$$mgh = \tfrac{1}{2}kh^2$$

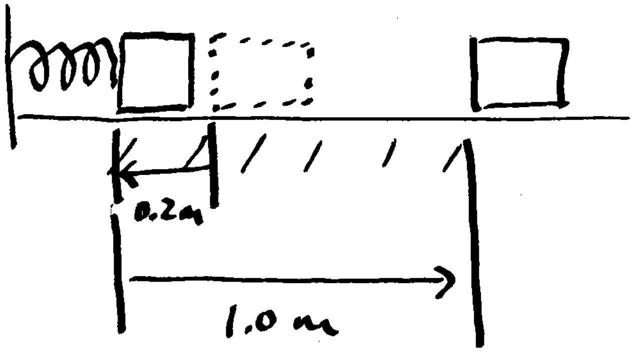
$$mgh = \frac{1}{2}\frac{mg}{d}h^2$$

$$\boxed{h = 2d}$$

Concept questions:

$\Rightarrow$ Explain how a person can bounce higher with each jump ~~from~~ on a trampoline from energy considerations

$\Rightarrow$ When the space shuttle lands most of the KE and grav. PE is gone .... Where did it go?




block w/ mass 0.5 kg

Spring compressed 0.2 m

block released and moves 1.0 m before coming to rest.

Spring constant, $k = 100$ N/m

What is $\mu_k$ between block and table ??

TOTAL PE of spring when compressed = TOTAL work done by friction on the block

($KE \rightarrow 0$)

$$\frac{1}{2}kx^2 = F_{fr}(\text{distance}) = mg\,\mu_k\,(1.0)$$

$$\frac{1}{2}\,100\,(0.2)^2 = (0.5)(9.8)\,\mu_k\,(1.0)$$

$$\boxed{\mu_k = 0.41}$$