Why is it easier to climb a mountain via a zigzag route rather than climb straight up?

Do as group Exercise

Power

We've discussed work + different forms of energy
Often the time distribution of the input/output
of work/energy is important

Ave. Power $\equiv \frac{\Delta W}{\Delta t}$

instantaneous Power $= \frac{dW}{dt}$

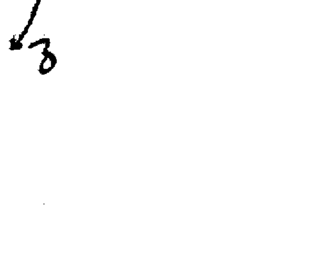
unit of Power is the Watt $\equiv$ J/s

Ex: A 100 Watt light bulb is one that
converts 100 Joules of electrical energy
into heat and light energy every second.

<u>A closer look at Gravity and Grav. Pot. energy</u>

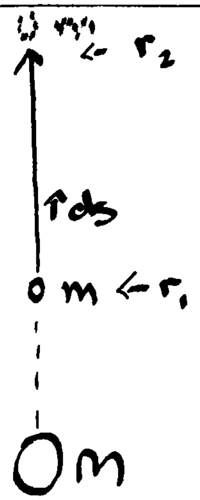
$$F_g = \frac{G m_1 m_2}{r^2}$$ Attractive along line joining "center of Masses"

more formally written



$$\vec{F}_{1m2} = -\frac{G m_1 m_2}{|\vec r_2 - \vec r_1|^2} \underbrace{\frac{(\vec r_2 - \vec r_1)}{|\vec r_2 - \vec r_1|}}_{\hat r \text{ from 1 to 2}}$$

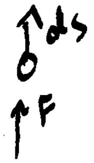
Now... do same for gravity as we did for Spring



move mass m slowly away from mass M along radial direction

What is the work done by me to move m ~~gravity on~~ m?

$$W = \int \vec{F} \cdot d\vec{s} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$



$$|d\vec{s}| = |d\vec{r}|$$

Force I exert and $d\vec{s}$ are in same direction

$$W = \int_{r_1}^{r_2} \frac{GMm}{r^2}\, dr$$

~~dot product gives "−" due to $\vec{F}_{grav}$ and $d\vec{s}$ pointing in opposite directions~~

$$W = GMm \int_{r_1}^{r_2} \frac{1}{r^2}\, dr$$

$$W = -\frac{GMm}{r}\Big|_{r_1}^{r_2} = -\frac{GMm}{r_2} + \frac{GMm}{r_1}$$

(+)

Recall

$$F = \frac{GM_1 m_2}{r^2} \Rightarrow$$

on earth

$$F = \left(\frac{GM_E}{R_E^2}\right) m$$

$$g = 9.8\ m/s^2$$

AMPAD 22-141 50 SHEETS 22-142 100 SHEETS 22-144 200 SHEETS

define grav Potential energy of system

$$PE = -\frac{GMm}{r}$$

when I move particle out from r_1 to r_2

sign chosen to make this $+$

$$\Delta PE = -\frac{GMm}{r_2} + \frac{GMm}{r_1} \quad +$$

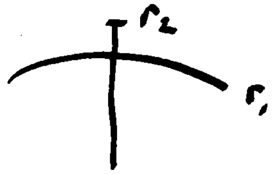
Particle ~~has~~ near a Mass

Particle has 0 potential Energy at ∞

" $-$ potential energy at finite r

but ΔPE going from small r to larger r is $+$

$$\Delta PE = \frac{GMm}{r_1} - \frac{GMm}{r_2}$$



$$\Delta PE = \frac{GMm(r_2 - r_1)}{r_1 r_2}$$

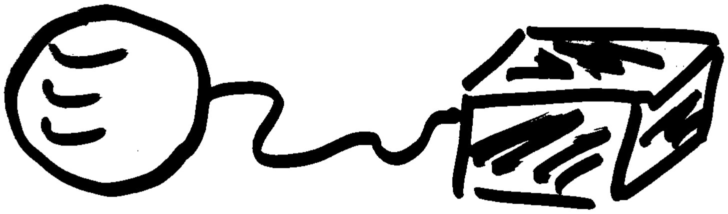
let r_1 = surface of Earth (radius) $= R_E$ $\quad M = M_E$

r_2 = small distance above surface of Earth

$\therefore r_2 = r_1 + \Delta r$ where $\Delta r \ll r_1$

$$\Delta PE = \frac{GM_E m(R_E + \Delta r - R_E)}{R_E(R_E + \Delta r)} \cong \underbrace{\frac{GM_E}{R_E^2}}_{\equiv g} m \underbrace{(\Delta r)}_{h} = mgh \; !$$

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
AMPAD



Bowling Ball

M

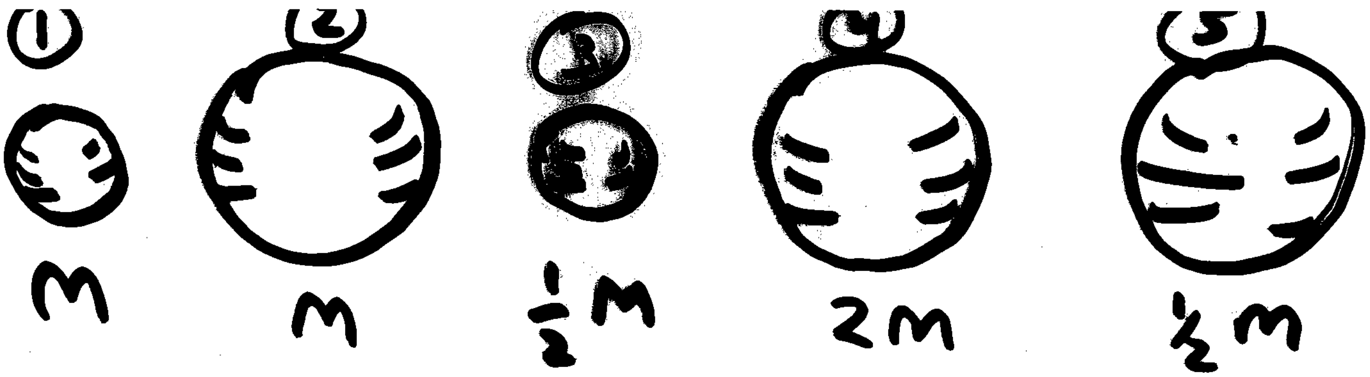
wood

m

$\boxed{M > m}$

Dropped in a Vacuum on Earth

1. Bowling Ball hits 1st
2. Wood hits 1st
3. Both hit at the same time



5 homogeneous planets have relative sizes and masses as shown.

A body of mass m would weigh least on which planet

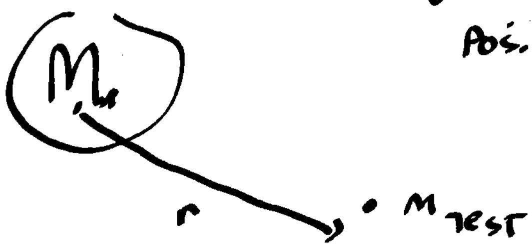
Our concept of gravity comes about from thinking about the Force of one object on another

what is the condition in space created by M_1 that causes a force on M_2 ??

Useful to think of the space around M_1 independent of any other mass. We say M_1 creates a condition in the space such that if a "TEST Mass" were there it would feel a gravitational force due to M_1.

We say M_1 ~~has~~ sets up a "gravitational Field" in the space surrounding it ... out to $r = \infty$.

Suppose we place a little "test mass" of small ~~arbitrary~~ mass in some position



$$\vec{F}_{\text{of } M \text{ on } M_{TEST}} = \frac{G M M_{TEST}}{r^2}\hat{r}$$

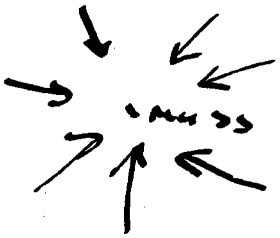
$$\frac{\vec{F}}{M_{TEST}} = \frac{\overrightarrow{\text{Force of } M \text{ on a mass}}}{\text{unit mass}} = -\frac{GM}{r^2}\hat{r} \equiv \text{Gravitational Field}$$

iT is a vector Field

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
AMPAD

$$\frac{\vec{\text{Force}}}{\text{unit test mass}} = \vec{g}$$ = gravitational field

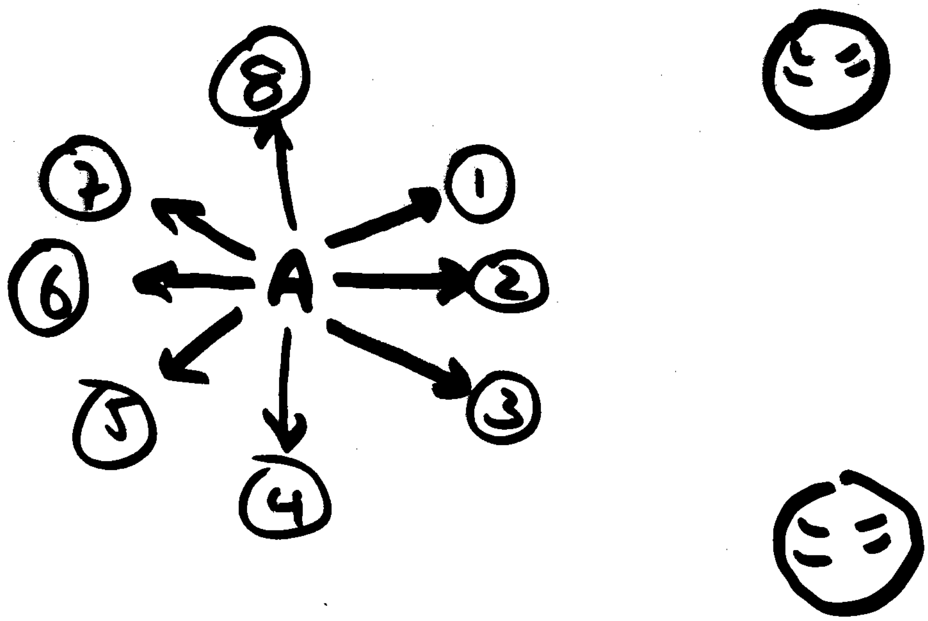
vector field at each point



Can visualize w/ "lines of force" that represent how a test mass would move at each point

$$\vec{F} = m\vec{g}$$

AMPAD 22-141 50 SHEETS 22-142 100 SHEETS 22-144 200 SHEETS

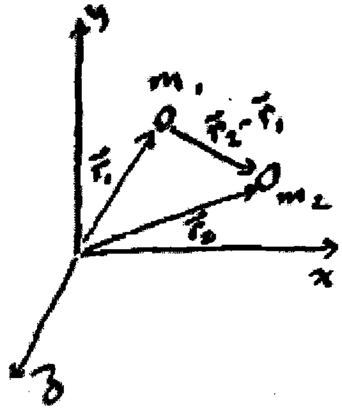


Point A exists near two planets as shown. Which vector most closely gives the direction of the gravitational field at point A?

A closer look at Gravity and Grav. Pot. energy

$$F_g = \frac{G m_1 m_2}{r^2}$$ attractive along line joining "center of Masses"

More formally written



$$\vec{F}_{1m2} = -\frac{G m_1 m_2}{|\vec{r}_2-\vec{r}_1|^2}\underbrace{\frac{(\vec{r}_2-\vec{r}_1)}{|\vec{r}_2-\vec{r}_1|}}_{\hat{r} \text{ from 1 to 2}}$$

Finally, I discussed at some length several different views of the essence of the gravitation force:

1) Newton – action at a distance
2) Einstein – curvature of spacetime
3) Quantum – exchange of virtual gravitons

We have functioning theories of gravitation for 1 and 2. Working on 3. Who is right? Discussed truth versus scientific models. Discussed regions of applicability of models.