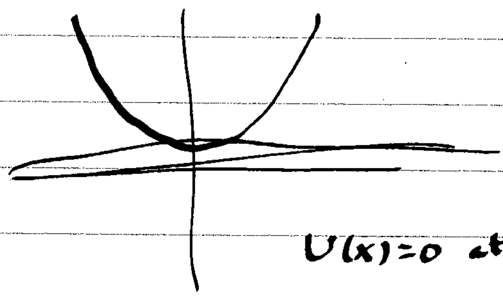
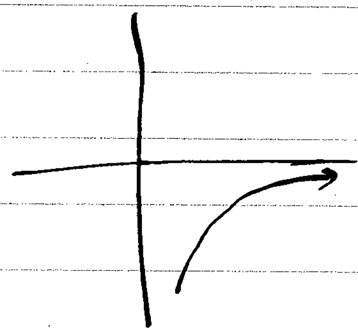
can discover functional dependence of PE: U

gravity $U(r) = -\frac{GMm}{r}$

Spring $U(x) = \frac{1}{2} K x^2$



$U(x)=0$ at $x=0$



$U(r)=0$ at $r=\infty$

Force vs. Potential

$$\boxed{-\frac{dU}{dx} = F_x}$$

$$-\frac{d(\frac{1}{2}kx^2)}{dx} = -kx = F_x$$

$$\frac{-d\left(-\frac{GMm}{r}\right)}{dr} = -\frac{GMm}{r^2} = F_r$$

Consider system w/ | Movement along x

$$F_x \Delta x = -\Delta PE = -\Delta U$$

$$F_x = -\frac{\Delta U}{\Delta x} \xrightarrow[\text{in limit of small } \Delta s]{} F_x = -\frac{du}{dx}$$

do for spring

do for gravity

Generalize to 3d

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$\vec{F} = -\frac{du}{dx}\hat{i} + \frac{du}{dy}\hat{j} + \frac{du}{dz}\hat{k}$$

but F is fn of x, y, z ∴ can't use normal derivatives
Must use partial derivatives

Let $B(x,y,z)$ $\quad \frac{\partial B}{\partial x} = \left.\frac{dB}{dx}\right|_{y,z}$ treated as constants

$$\vec{F} = -\frac{\partial u}{\partial x}\hat{i} - \frac{\partial u}{\partial y}\hat{j} - \frac{\partial u}{\partial z}\hat{k}$$

Why do you care?

⇒ In the end physics often boils down to determining $\vec{F}$ at a point or in a region

⇒ If we know $\vec{F}$, we know motions of particles

$\vec{F}$ is a vector "field" — Sometimes hard to determine for gravity $\frac{\vec{F}}{m} = \vec{g}$ ≡ gravitational field

for Electromagnetism

$$\frac{\vec{F}}{q} = \vec{E} \equiv \text{electric field}$$

Knowing $\vec{F}$ is to know $\vec{g}$ or $\vec{E}$

⇒ These are <u>vector</u> fields : often hard to calculate

<u>But</u> we just learned if we can understands the Potential Energy function in space we can derive the Force (or $\vec{g}$ or $\vec{E}$)

Potential Energy function is a <u>Scalar</u> field

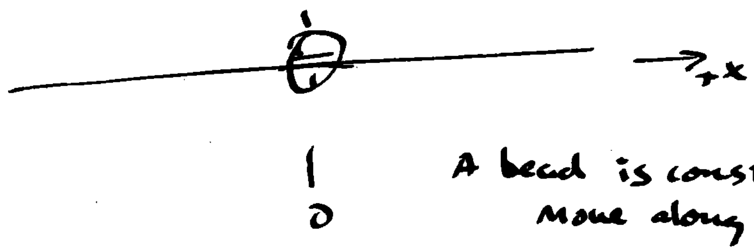
often easier to understand + get than $\vec{E}$ or $\vec{g}$ directly!

$$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

define $\vec{\nabla} \equiv \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

vector operator "del" ≡ gradient

$\vec{F} = -\vec{\nabla} U$ ... We say the Force is the negative gradient of the Potential Energy function!



A bead is constrained to move along the x-axis

Some combination of bubblegum, springs, + string exert force on the bead such that

$\cancel{\vec{F}_{NET} = F_x = Ax^4}$

The potential Energy, $U = Ax^4$

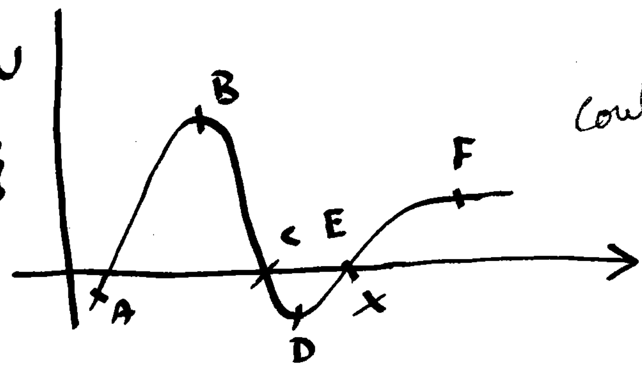
What is $F_x(x)$?

$$F_x = -\frac{dU}{dx} = -A\frac{x^3}{3}$$

Where is force $= 0$? $x = 0$

$\neq$

Some system has Potential Energy Described by this curve



Could ASK TO do for NexT class

AT each point is F_x $+$, $-$, or 0 ?

AT which point does F_x have the greatest Magnitude ?

AT which points can system be in equilibrium ?

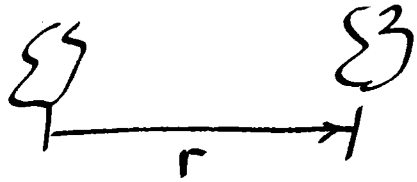
$\neq$

do spring | and gravity 1st

Example

Two atoms interact via the Van der ~~Walls~~ Waals interaction

$$U(r) = \frac{a}{r^{12}} - \frac{b}{r^6}$$



lets simplify + do as 1 d problem

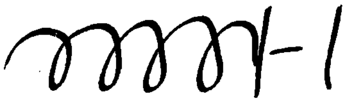
$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

$$\vec{F} = -\nabla u = -\frac{\partial u}{\partial x}\hat{i} - \underbrace{\cancel{\frac{\partial u}{\partial y}}}_{0}\hat{j} - \underbrace{\cancel{\frac{\partial u}{\partial z}}}_{0}\hat{z}$$

because u has no y or z dependence

$$\vec{F} = -\left(-\frac{12a}{x^{13}} + 6\frac{b}{x^7}\right)\hat{i} = \left(\frac{12a}{x^{13}} - \frac{6b}{x^7}\right)\hat{i}$$

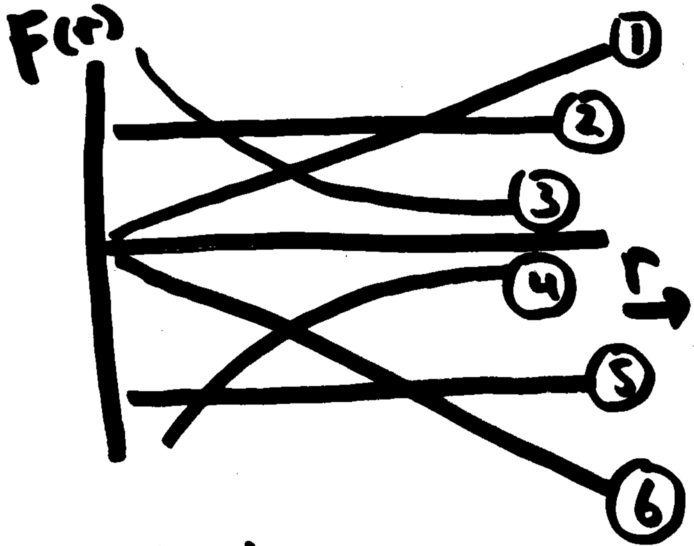
Example

Spring  $U(x) = \frac{1}{2}kx^2$

$$\vec{F}(x) = -\nabla u = -kx\,\hat{i}$$

You discover a new subatomic particle called a Squark. Book a ticket for Stockholm!

The potential energy between two Squarks as a fn of $r \equiv$ dist. between them looks like



The force between Squarks is best described by which curve?

Linear Momentum – Momentum Conservation

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$$

define $m\vec{v} \equiv \vec{P} \equiv$ momentum

vector quantity

$$P_x = mv_x$$

$$P_y = mv_y$$

$$P_z = mv_z$$

Typically $\frac{d(m\vec{v})}{dt} \Rightarrow$ m constant

$$\frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} + \cancelto{0}{\vec{v}\frac{dm}{dt}}$$

NOT <u>always</u> !

Consider a Rocket ... Burns fuel

$$\frac{d\vec{P}}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

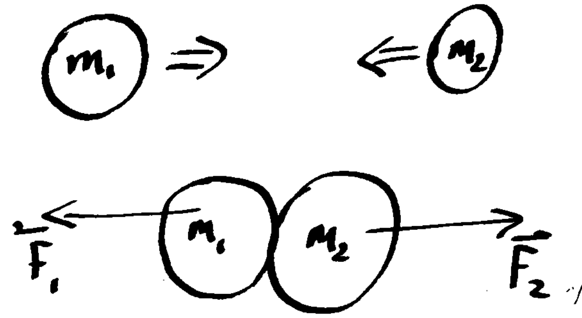
$$\vec{F} = \frac{d\vec{P}}{dt} \Rightarrow d\vec{P} = \vec{F}dt \Rightarrow \int_{P_1}^{P_2} d\vec{P} = \int_{t_1}^{t_2} \vec{F}dt$$

$$\Delta\vec{P} = \vec{P}_2 - \vec{P}_1 = \int_{t_1}^{t_2} \vec{F}dt$$

1 ≡ initial
2 ≡ Final } here

$\equiv J \equiv$ impulse

Consider two bodies colliding



not complicated

complicated

$$\Delta \vec{P}_1 = \int_{t_i}^{t_f} \vec{F}_1 \, dt \qquad \Delta \vec{P}_2 = \int_{t_i}^{t_f} \vec{F}_2 \, dt$$

but we know from Newton's 3rd Law that $\vec{F}_1 = -\vec{F}_2$ at all times

$$\therefore \underbrace{\int_{t_i}^{t_f} \vec{F}_1 \, dt}_{\Delta \vec{P}_1} = \int_{t_i}^{t_f} (-\vec{F}_2) \, dt = - \underbrace{\int_{t_i}^{t_f} \vec{F}_2 \, dt}_{\Delta \vec{P}_2}$$

$$\therefore \Delta \vec{P}_1 = - \Delta \vec{P}_2 \quad \text{or} \quad \Delta \vec{P}_1 + \Delta \vec{P}_2 = 0$$

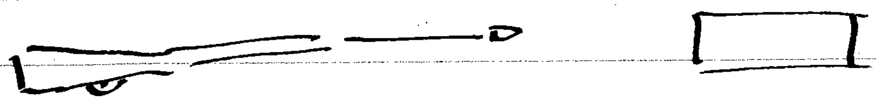
Momentum of individual Parts has changed

Σ changes in the Momentum $\rightarrow 0$!

Momentum Conservation

If There are No external Forces acting on a system, the total Momentum of that system is Conserved.

$$\text{-or-} \quad \Sigma \Delta \vec{P} = 0 \quad \text{-or-} \quad \Sigma \vec{P}_i = \Sigma \vec{P}_F$$



Ballistic pendulum

Don't know V_{Bullet} Know $V_{Bullet} + V_{Block}$

is energy conserved ?

always

Write it for me

$$\frac{1}{2} m V_{Bullet}^2 = \frac{1}{2}(m_{Bul} + M_{Block}) V^2$$

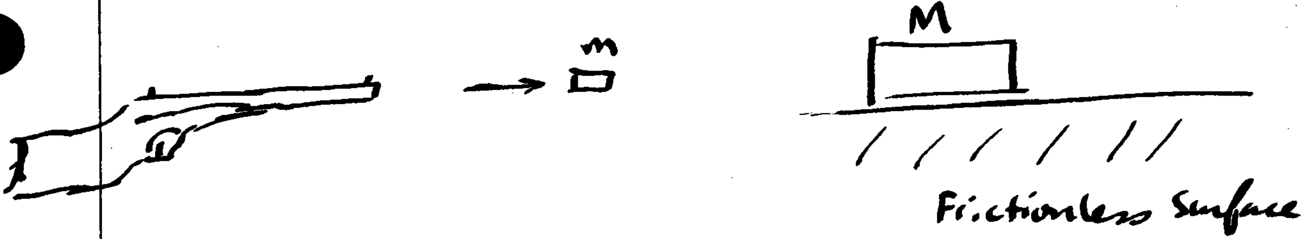
$\vec{P}$ Always conserved

KE NOT " "

if it is → elastic collision

if Not → inelastic collision

Example



bullet fired w/

unknown velocity $\vec{V}_{bullet}$ ← Find This

embeds in block "instantaneously"

block + bullet move, $\vec{V}_f$ Measured (Block + Bullet combo)

momentum cons ↙

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$m_{bullet} \vec{V}^{\,i}_{bullet} + M_{block} \underset{\uparrow \atop 0}{V^{\,i}_{block}} = (m+M)\vec{V}^{\,f}_{block/bullet}$$

$$\vec{V}^{\,i}_{bullet} = \left(\frac{m+M}{m}\right)\vec{V}^{\,f}_{block/bullet}$$

Is energy Conserved? → yes

$KE_{bullet} = KE_{bullet + Block}$? ⟹ <u>No</u>

(No change in Pot energy here)

Frictional energy loss to Stop bullet

⟹ Known as an <u>inelastic</u> <u>Collision</u>

in other words

$$\tfrac{1}{2} m v_i^2 \neq \tfrac{1}{2}(m+M) v_f^2$$

You drive around a curve on a narrow, one-way street at 25 mph when you notice a car identical to yours coming straight toward you at 25 mph.

You have 2 choices:

- hit other car head on
- Swerve into a massive concrete wall head on

You should →

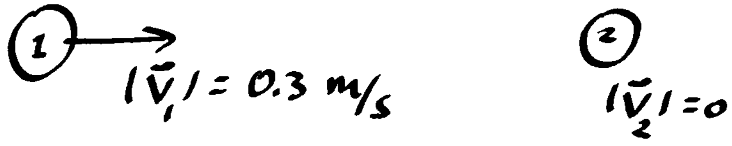
1. Hit the other car
2. Hit the wall
3. Hit Either one – it makes NO difference.

4. Consult your lecture Notes.

Example

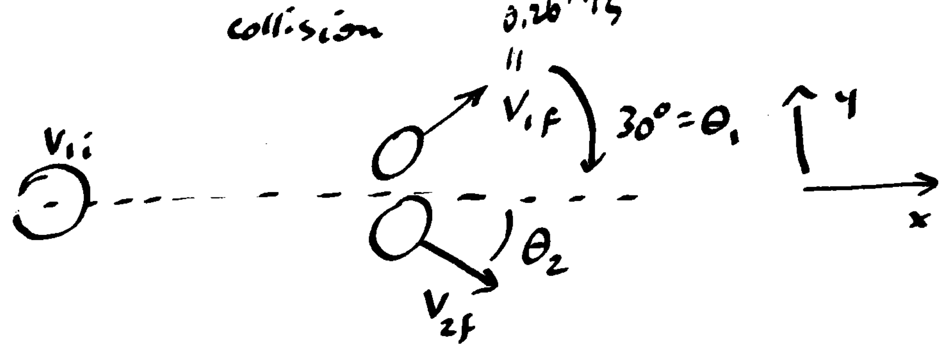
Consider The collision of two hockey Pucks each w/ Mass 0.1 kg

initially



collision

Final



$V_{1f} = 0.26$ m/s

What is the final ~~speed~~ velocity of the 2ND puck ?

$\Sigma \vec{P}_i = \Sigma \vec{P}_f$ ← vector eqn

same as $\left(\begin{array}{c} \Sigma P_{ix} = \Sigma P_{xf} \\ \text{-and-} \\ \Sigma P_{yi} = \Sigma P_{yf} \end{array} \right)$ Momentum Conserved for each component

x eqn

$$m_1 V_{1ix} + m_2 \underbrace{\cancel{V_{2ix}}}_{0} = m_1 \underbrace{V_{1fx}}_{V_{1f}\cos 30} + m_2 \underbrace{V_{2fx}}_{V_{2f}\cos\theta_2}$$

(I) $m_1 V_{1ix} = m_1 V_{1f} \cos 30 + m_2 V_{2f} \cos\theta_2$

y eqn

$$m_1 \cancel{V_{1iy}}^{\,0} + m_2 \cancel{V_{2iy}}^{\,0} = m_1 \underbrace{V_{1fy}}_{V_{1f}\sin 30} + m_2 \underbrace{V_{2fy}}_{-V_{2f}\sin\theta_2}$$

(II) $\quad V_{2f}\sin\theta_2 = V_{1f}\sin 30$

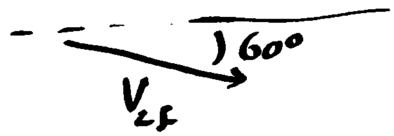
I $\quad (.1)(0.3) = (.1)(0.26)\cos 30 + (0.1)\,V_{2f}\cos\theta_2$

II $\quad V_{2f}\sin\theta_2 = (0.26)\sin 30$

2 eqns, 2 unknowns V_{2f}, θ_2

Should find $\quad |\vec{V}_{2f}| = 0.15$ m/s

$\theta_2 = 60°$ down from horizontal



Is the collision "elastic"?

initial KE $= \frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2}(0.1)(0.3)^2 = 4.5 \times 10^{-3}$ J

final KE $= \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$

$= \frac{1}{2}(0.1)(0.26)^2 + \frac{1}{2}(0.1)(0.15)^2 = 4.5 \times 10^{-3}$ J

Yes collision is elastic.

Kinetic Energy is conserved!