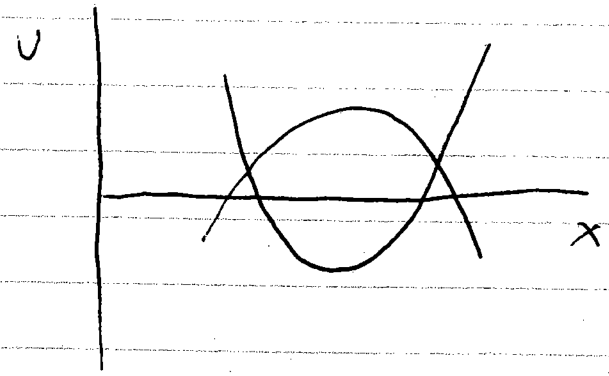
Last time -

$$F_x = -\frac{dU}{dx}$$



Stable vs. Unstable
Equilibrium

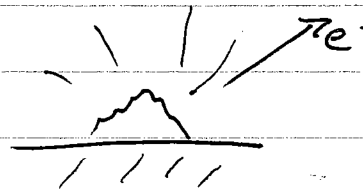
P. conservation

$$\sum \vec{P}_i = \sum \vec{P}_{final}$$ vector eqn.!

P demos

Historical Application of Momentum conservation

The Neutrino ~ 1930



β decay

Expect

$n \to p + e^-$ 2 body decays

example

${}^{40}_{19}K \to {}^{40}_{20}Ca$

TOTAL energy available

$$Q = K_1 + K_2 = \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 V_2^2$$

P cons. $M_1 V_1 = -M_2 V_2$

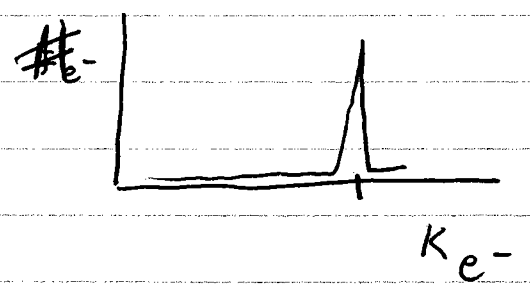
$$\frac{1}{2} M_1^2 v_1^2 = \frac{1}{2} M_2^2 V_2^2$$

$$M_1 K_1 = M_2 K_2$$

$$K_1 = \frac{M_2}{M_1 + M_2} Q \qquad K_2 = \frac{M_1}{M_1 + M_2} Q$$

For a specific reaction Q is fixed, M_1, M_2 also fixed

expect



$m_1 v_1$

$\longleftarrow O \longrightarrow$

$m_1 v_2$

Total Energy Available After overcoming Binding changes

$$Q = \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 V_2^2 = K_1 + K_2$$

$$m_1 v_1 = -M_2 V_2$$

$$v_1 = -\frac{M_2 V_2}{M_1}$$

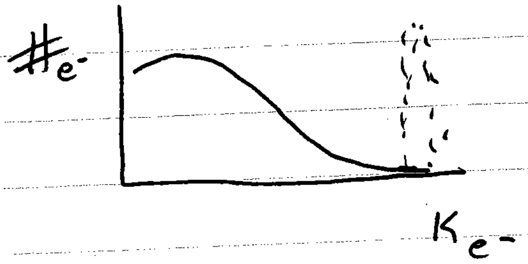
$$Q = -\frac{1}{2}\left(\frac{M_2}{M_1}\right)^2 M_1 V_2^2 + \frac{1}{2} M_2 V_2^2$$

$$= -\frac{1}{2}\frac{M_2^2 V_2^2}{M_1} + \frac{1}{2} M_2 V_2^2$$

$$Q = -K_2 \frac{M_2}{M_1} + K_2$$

$$\boxed{K_2 = \frac{M_1 Q}{M_1 + M_2}}$$

Actually see



Why? $\Rightarrow$ 3 body decay

$$n \rightarrow p + e^- + \bar{\nu}$$

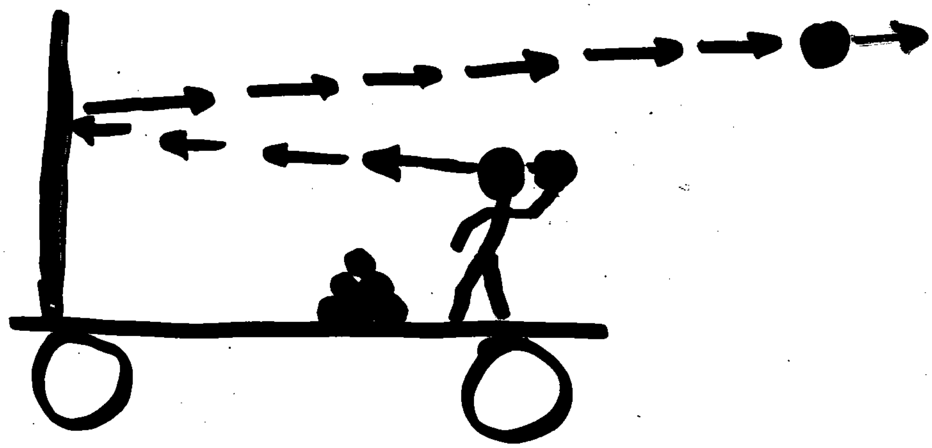
Pauli suggested the neutrino

Pauli 1931
Disc. Reines 1959
Solar ν Missing Davis 1968
1998 ν oscillations

1953

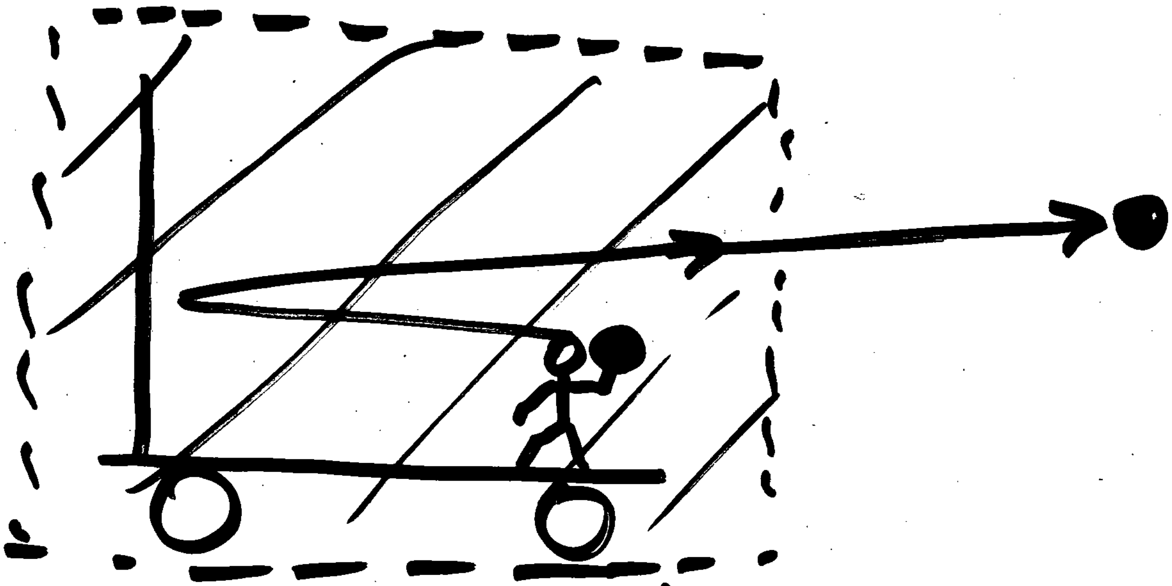
Thous light years Pb

Davis Koshiba



Cart initially at rest. No friction.
Balls thrown against wall as shown.

1. Cart moves to right.
2. Cart moves to left.
3. Cart remains stationary.

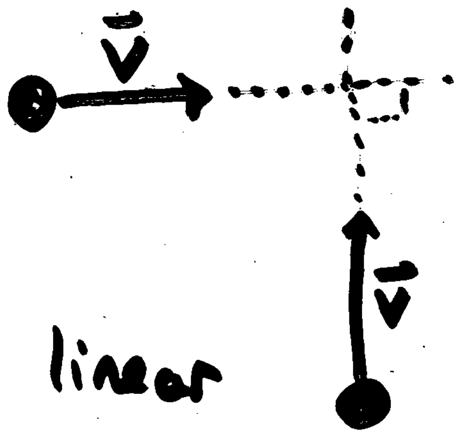


Think of Big box

Ball Thrown to Right

P cons. $\Longrightarrow$ cart Moves left

Two identical bodies of mass M move with equal speeds v as illustrated in the figure.



The magnitude of the linear momentum of the system is

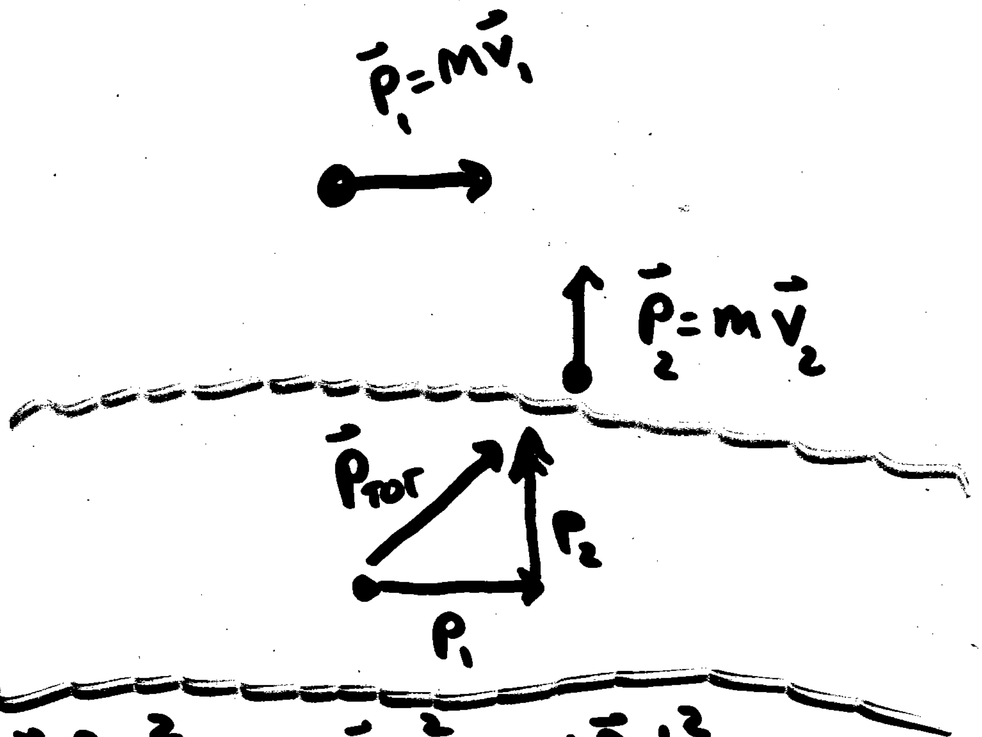
① $2Mv$

② Mv

③ $4Mv$

④ $\sqrt{2}\,Mv$

⑤ $4\sqrt{2}\,Mv$

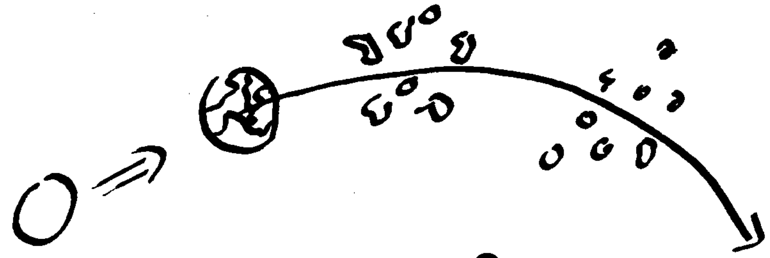


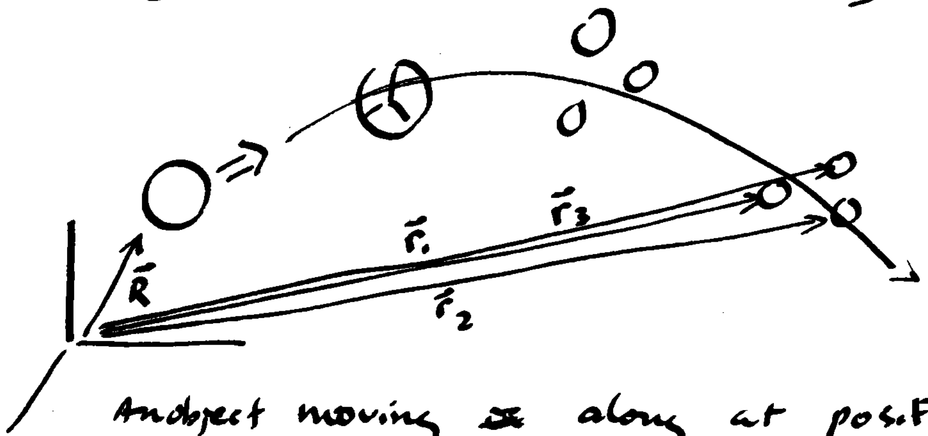
$$|\vec{P}_{TOT}|^2 = |\vec{P}_1|^2 + |\vec{P}_2|^2$$

$$P_{TOT}^2 = 2(mv)^2$$

$$P_{TOT} = \sqrt{2}\, mv \longrightarrow \text{Ans. 4}$$

Center-of-Mass





An object moving ~~x~~ along at position $R(t)$ breaks into 3 (or more) objects w/ positions described by vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$

Momentum Conservation

$$\Longrightarrow \quad \Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$$

$$M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt}$$

$$\frac{d}{dt}(M\vec{R}) = \frac{d}{dt}(m_1\vec{r}_1) + \frac{d}{dt}(m_2\vec{r}_2) + \frac{d}{dt}(m_3\vec{r}_3)$$

$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3$$

break into component

$$\left\{ \begin{aligned} MX &= m_1x_1 + m_2x_2 + m_3x_3 \\ MY &= m_1y_1 + m_2y_2 + m_3y_3 \\ MZ &= m_1z_1 + m_2z_2 + m_3z_3 \end{aligned} \right.$$

-or- $X = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M}$ These are Known as Center-of-Mass coordinates

Similar for Y, Z

Can consider the system of 3 bodies equivalent to One body of Mass $M = m_1 + m_2 + m_3$ ~~at a position~~ Concentrated at the center-of-mass position

Center-of-Mass coordinates:

Mass weighted Average position of a system of particles

consider N masses

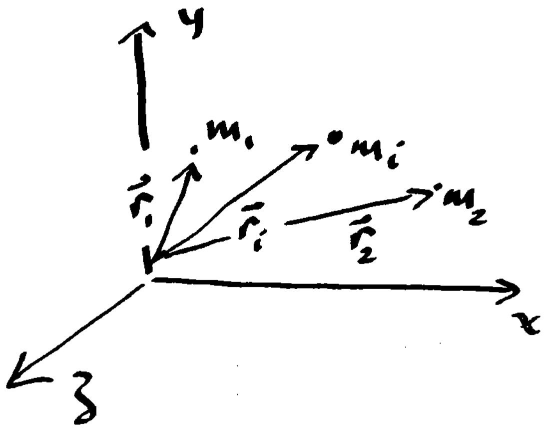
$$X_{c.m.} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_N x_N}{m_1 + m_2 + \cdots + m_N} = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i}$$

$$Y_{c.m.} = \frac{\sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i}$$

$$Z_{c.m.} = \frac{\sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_i}$$

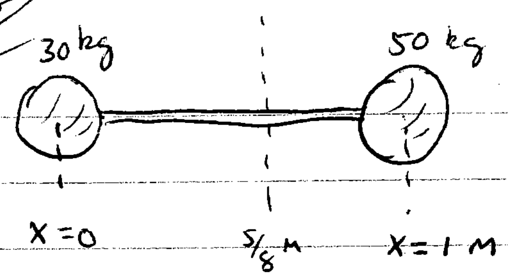
-or-

$$\vec{R}_{c.m.} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$



Easy to use for a system of discrete particles

Example

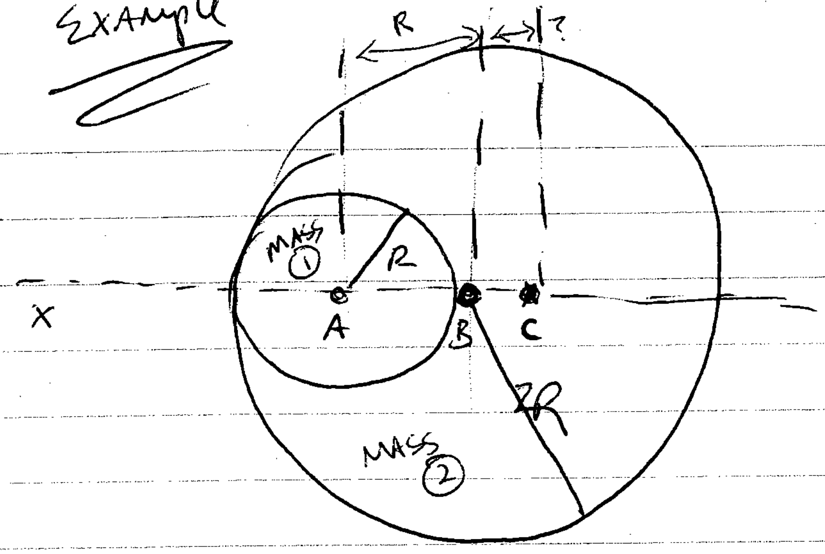


Where is the center of Mass of the system

$$X_{CM} = \frac{30(kg)(0\,m) + 50(kg)(1\,m)}{(30+50)\,kg} = \frac{50}{80} \text{ meter}$$

$$= \frac{5}{8} \text{ meter}$$

EXAMPLE

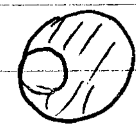


Circular metal plate w/ radius 2R

Circular plug of radius R is removed

Find the position of the center of mass of the plate from which the disk was removed

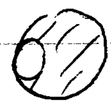
→ pt C in Drawing



X_B = Center of Mass of whole metal plate

~~C of M of disk + C of M~~ can be thought of as made of two Masses

① circular disk w/ Mass centered at A

②  w/ Mass centered at C

$x_B = 0 \Rightarrow -M_1 x_A = M_2 x_C$

$$X_B = \frac{M_1 X_A + M_2 X_C}{M_1 + M_2}$$

Let center of big plate be at $X=0$

$\Rightarrow X_B = 0$

$$X_C = -\frac{M_1 X_A}{M_2} = -\frac{[\pi R^2(\rho t)](-R)}{\pi(2R)^2(\rho t) - \pi R^2(\rho t)}$$

$\rho \equiv$ Volume Mass density

t = thickness

$(t)(a) <$ A = area

= Volume

$$\boxed{X_C = \frac{1}{3}R}$$